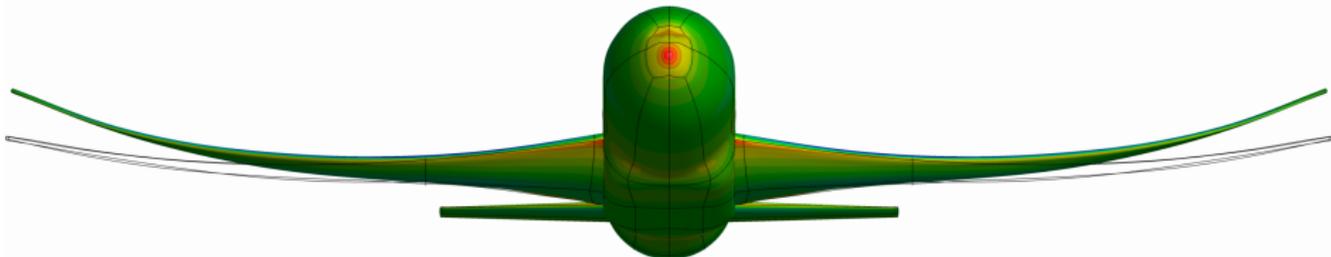


Adjoint-based Aerodynamic and Aerostructural Optimization of Transport Aircraft

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Advanced Modeling and Simulation Seminar Series
NASA Ames Research Center, February 7, 2017

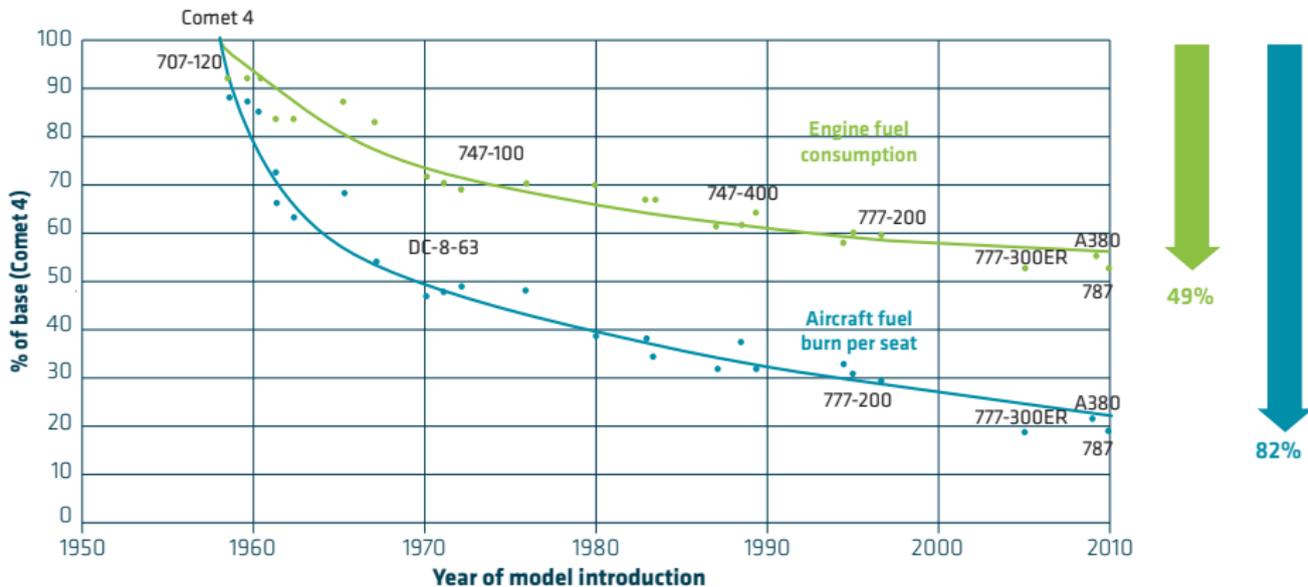
With over 90 000 flights daily, small improvements in commercial aircraft performance can have a huge impact



The Boeing 707 and the Boeing 787 do not appear to have vastly different designs...



Over 80% reduction in fuel burn of current generation compared to first generation jets



Where will the next 80% fuel burn reduction come from?

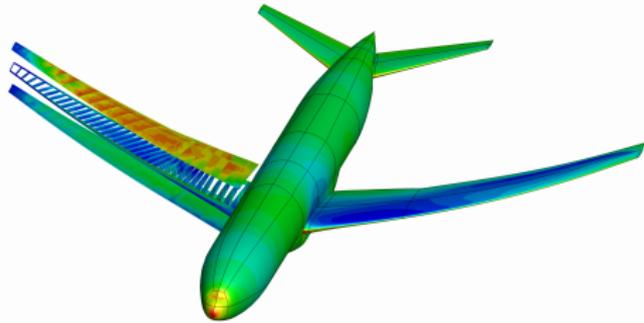


The next generation of transport aircraft will be more challenging to design

- High aspect ratio, flexible wings
- Coupled aero-propulsion effects from boundary layer ingestion
- Complex non-linear flutter behavior for truss-braced wings
- Unknown design space
- High risk



Challenges for high-fidelity optimization in aircraft design

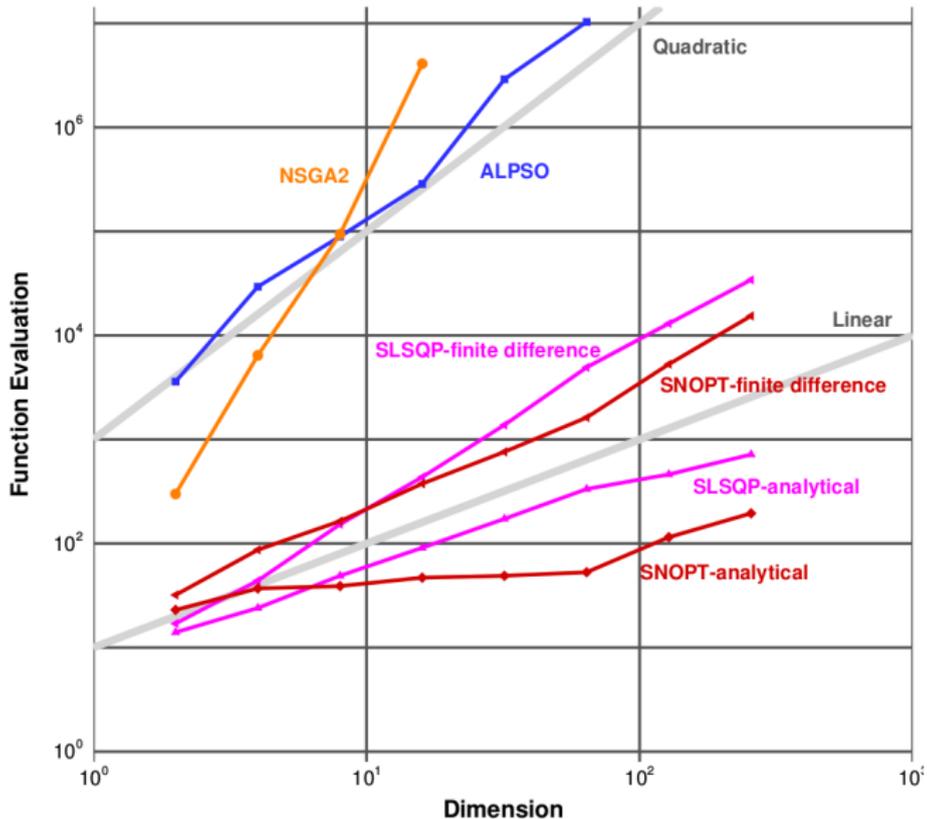


- Computational expensive objective and constraints
- Multiple high-coupled systems
- Large number of design variables, design points and constraints

- Motivation
- Computing derivatives
- Aerodynamic shape optimization
- Aerostructural design optimization
- Summary

- Motivation
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Gradient based optimization is our only hope for large numbers of design variables



Gradient-based optimization requires derivatives of the objective and constraints

$$\begin{aligned} \min \quad & f(x, y(x)) \\ \text{s.t.} \quad & h(x, y(x)) = 0 \\ & g(x, y(x)) \leq 0 \end{aligned}$$

x : design variables

y : state variables. Determined by solving
 $\mathcal{R}(x, y(x)) = 0$

Need to find $\frac{df}{dx}$ (and $\frac{dh}{dx}$, $\frac{dg}{dx}$)

Method for computing derivatives

Monolithic <i>Black boxes</i> <i>input and outputs</i>	Finite-differences $\frac{df}{dx_j} = \frac{f(x_j + h) - f(x)}{h} + \mathcal{O}(h)$
	Complex-step $\frac{df}{dx_j} = \frac{\text{Im} [f(x_j + ih)]}{h} + \mathcal{O}(h^2)$
Analytic <i>Governing eqns</i> <i>state variables</i>	Direct $\frac{df}{dx} = \frac{\partial f}{\partial x} - \underbrace{\frac{\partial f}{\partial y} \left[\frac{\partial R}{\partial y} \right]^{-1} \frac{\partial R}{\partial x}}_{-dy/dx}$
	Adjoint $\frac{df}{dx} = \frac{\partial f}{\partial x} + \underbrace{\frac{\partial f}{\partial y} \left[\frac{\partial R}{\partial y} \right]^{-1} \frac{\partial R}{\partial x}}_{\psi}$
Algorithmic differentiation <i>Lines of code</i> <i>code variables</i>	Forward $\begin{bmatrix} 1 & 0 & \dots & 0 \\ -\frac{\partial T_2}{\partial t_1} & 1 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial T_n}{\partial t_1} & \dots & -\frac{\partial T_n}{\partial t_{n-1}} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \dots & 0 \\ dt_2 & 1 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ dt_n & \dots & dt_{n-1} & 1 \end{bmatrix} = I = \begin{bmatrix} 1 - \frac{\partial T_2}{\partial t_1} & \dots & -\frac{\partial T_n}{\partial t_1} \\ 0 & 1 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 1 - \frac{\partial T_n}{\partial t_{n-1}} \end{bmatrix} \begin{bmatrix} dt_2 & \dots & dt_n \\ dt_1 & \dots & dt_1 \\ 0 & 1 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \dots & \dots & 1 & dt_n \\ 0 & \dots & 0 & 1 \end{bmatrix}$
	Reverse $\begin{bmatrix} 1 & 0 & \dots & 0 \\ -\frac{\partial T_2}{\partial t_1} & 1 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial T_n}{\partial t_1} & \dots & -\frac{\partial T_n}{\partial t_{n-1}} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \dots & 0 \\ dt_2 & 1 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ dt_n & \dots & dt_{n-1} & 1 \end{bmatrix} = I = \begin{bmatrix} 1 - \frac{\partial T_2}{\partial t_1} & \dots & -\frac{\partial T_n}{\partial t_1} \\ 0 & 1 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 1 - \frac{\partial T_n}{\partial t_{n-1}} \end{bmatrix} \begin{bmatrix} dt_2 & \dots & dt_n \\ dt_1 & \dots & dt_1 \\ 0 & 1 & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \dots & \dots & 1 & dt_n \\ 0 & \dots & 0 & 1 \end{bmatrix}$

Compute derivatives by linearizing the governing equations

Need df/dx , $f(x, y(x))$

$$\frac{df}{dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \frac{dy}{dx}$$

Derivative of the governing equations: $\mathcal{R}(x, y(x)) = 0$

$$\frac{d\mathcal{R}}{dx} = \frac{\partial \mathcal{R}}{\partial x} + \frac{\partial \mathcal{R}}{\partial y} \frac{dy}{dx} = 0 \rightarrow \frac{\partial \mathcal{R}}{\partial y} \frac{dy}{dx} = -\frac{\partial \mathcal{R}}{\partial x}$$

Substitute the result into the derivative equation

$$\frac{df}{dx} = \frac{\partial f}{\partial x} + \underbrace{\frac{\partial f}{\partial y} \left[\frac{\partial \mathcal{R}}{\partial y} \right]^{-1}}_{\psi} \frac{\partial \mathcal{R}}{\partial x}$$

Partial derivative terms evaluated using Automatic Differentiation (AD)

Solve the governing equations

$$\mathcal{R}(x, y(x)) = 0$$

Solve the adjoint equations for the particular function of interest, f

$$\left[\frac{\partial \mathcal{R}}{\partial y} \right]^T \psi = -\frac{\partial f}{\partial y}$$

and compute the derivatives

$$\frac{df}{dx} = \frac{\partial f}{\partial x} + \psi^T \frac{\partial \mathcal{R}}{\partial x}$$

Our requirements for using AD

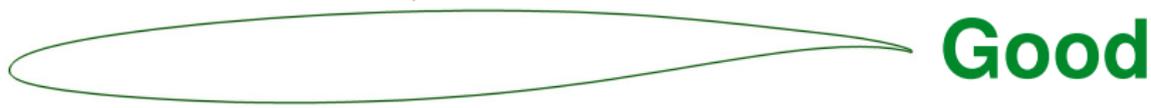
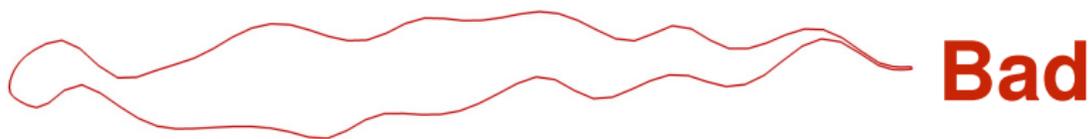
- Yield derivatives **consistent** with the flow solution and be verifiable with the complex step
- Require **no modification** to the original code.
- Require **no duplication** of the original code.
- Result in an **efficient adjoint** derivative computation.
- Have an **automatic implementation**.
- Incur **no nonlinear run-time penalty**
- **Low memory** footprint.

Adjoint approach with AD has evolved through four approaches

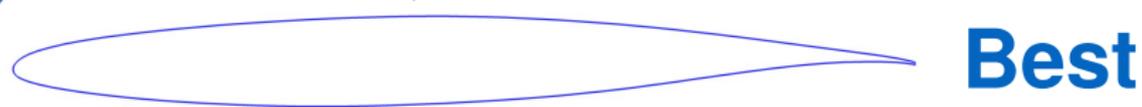
- 1 **Single Cell:** AD cell residual routine, loop over cells to assemble full Jacobian [2005]
- 2 **Forward mode coloring:** AD original residual routines using coloring for efficiency and store full Jacobian [2011]
- 3 **Full reverse mode:** AD master ghost routing that yields the desired transposed Jacobian-vector products in a matrix-free fashion [2014]
- 4 **Hybrid reverse mode:** AD individual non-linear routines and assemble the transposed Jacobian-vector productions manually [2015]

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Small changes in shape can make a big difference in performance



5% less
drag

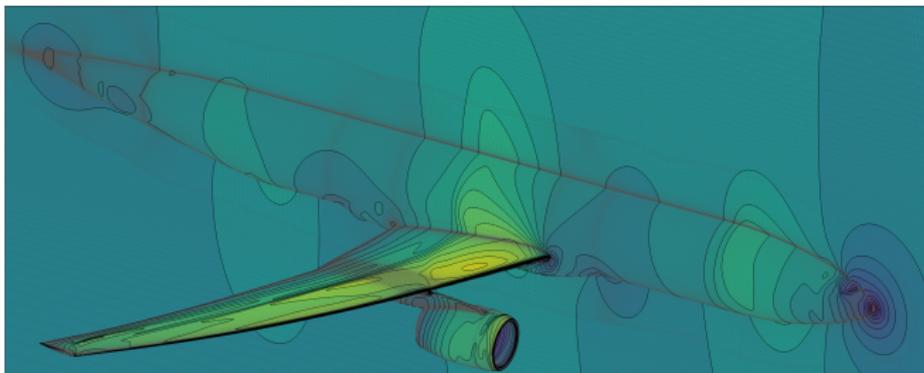


Transonic aerodynamic shape optimization requires a high-fidelity model

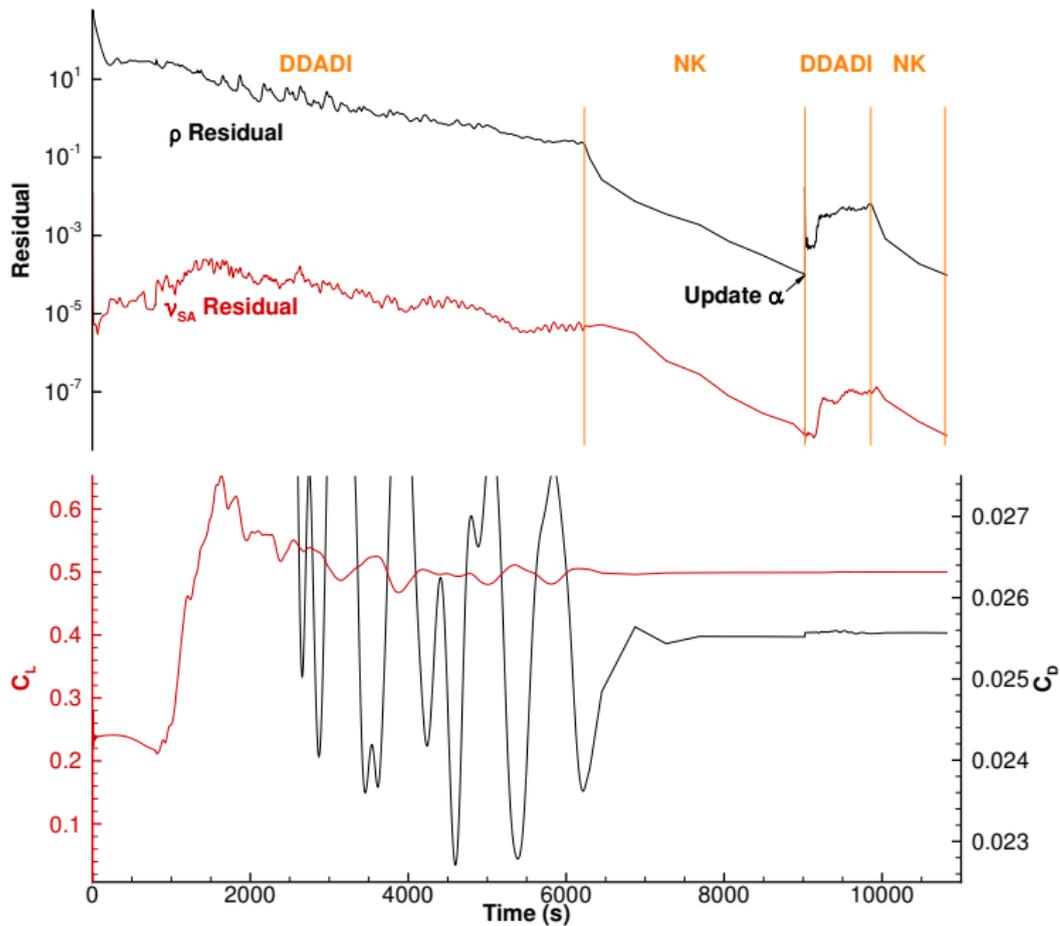


ADflow is well suited for optimization studies

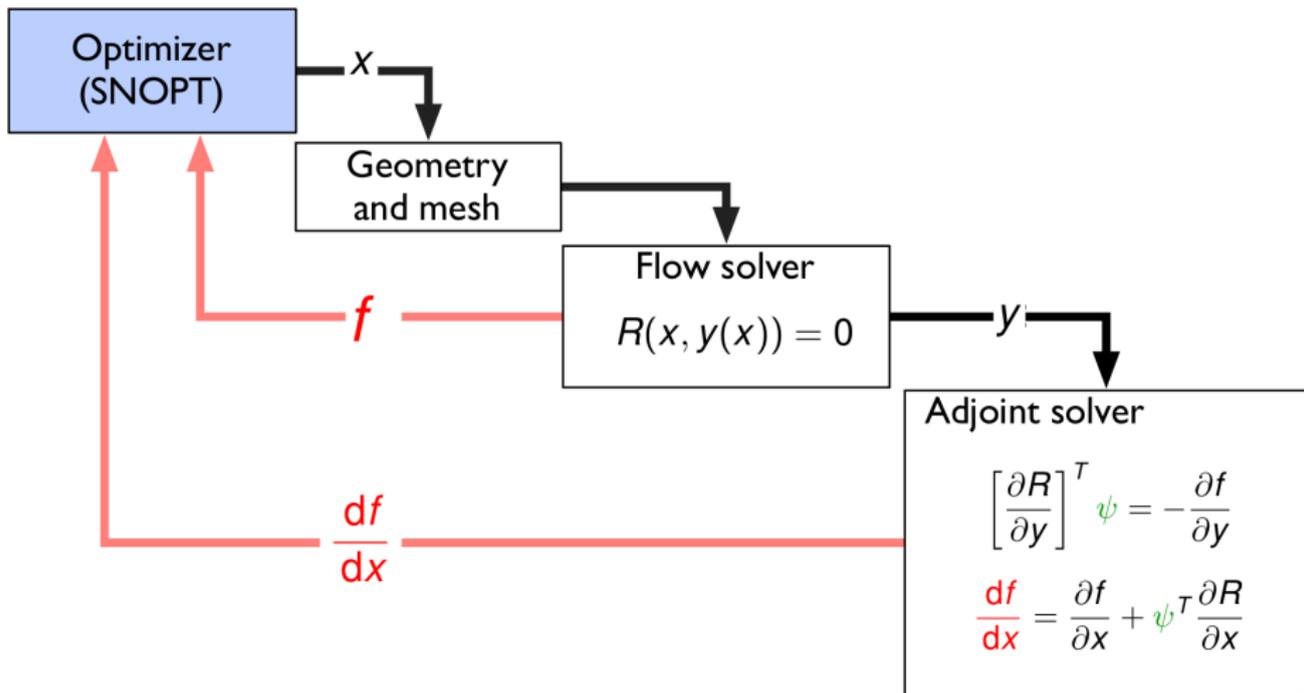
- Parallel, 2nd order, finite-volume multiblock/overset solver for the RANS equations
- SA and SST turbulence models
- Exact discrete adjoint implemented using AD
- Linearized turbulence model
- Newton–Krylov solution method for extremely rapid convergence



Newton–Krylov method especially useful for optimization

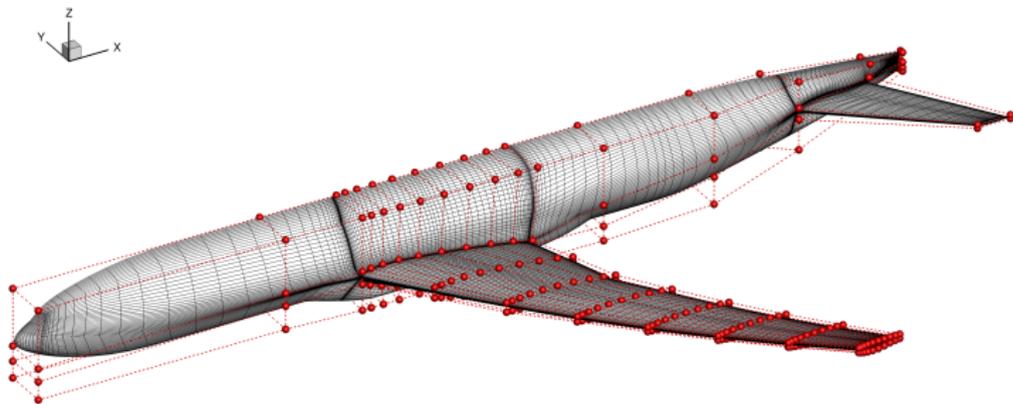


Design optimization requires more than just a flow solver

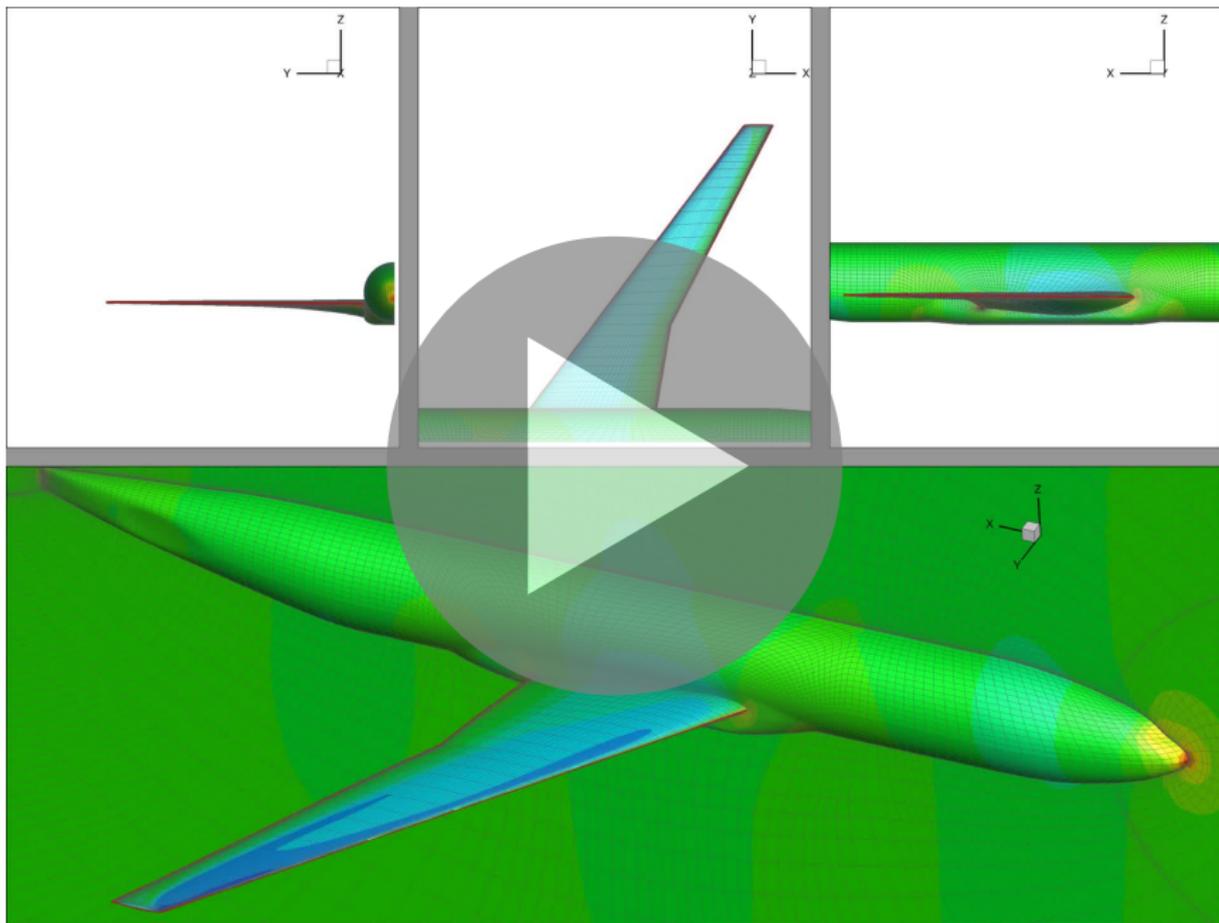


Multiblock free-form deformation volume approach for complex configurations

- Visualized as embedding an object in a clear, flexible, rubber-like material
- $\mathbf{R}^3 \rightarrow \mathbf{R}^3$ B-spline basis mapping
- Smooth with global and local shape control
- Common parametrization for all disciplines

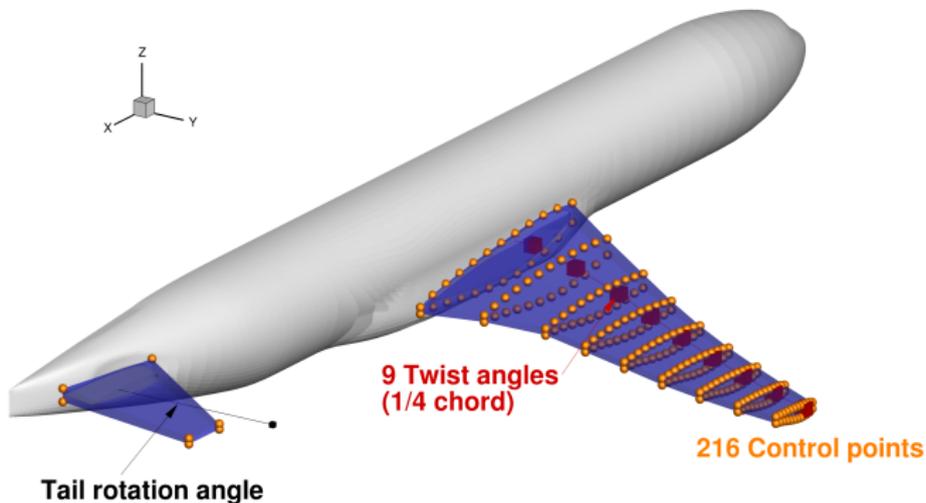


Fast mesh deformation handles large design changes



ADODG Case 5: The CRM Wing-Body-Tail Configuration

- Nominal operating point: Mach=0.85, $C_L=0.5$
- Off design conditions: M=0.85, $C_L=0.65$; M=0.89, $C_L=0.456$
- Flight Reynolds number: 43×10^6
- Weighted drag minimization at fixed lift
- Trimmed flight conditions with tail control variable
- 100% minimum thickness constraints
- No decrease in wing initial volume



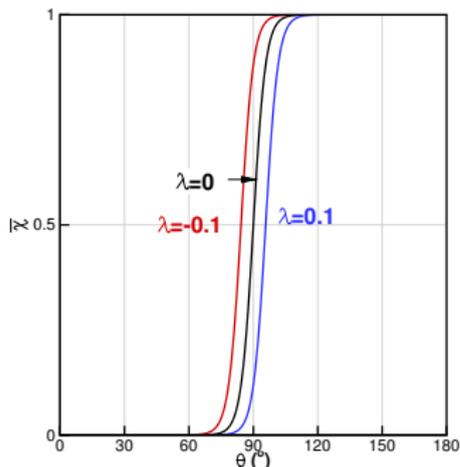
“Separation sensor” method for buffet onset prediction

- Separated flow correlates with lowering of the lift curve slope
- Integral of area where x -axis component skin friction is negative

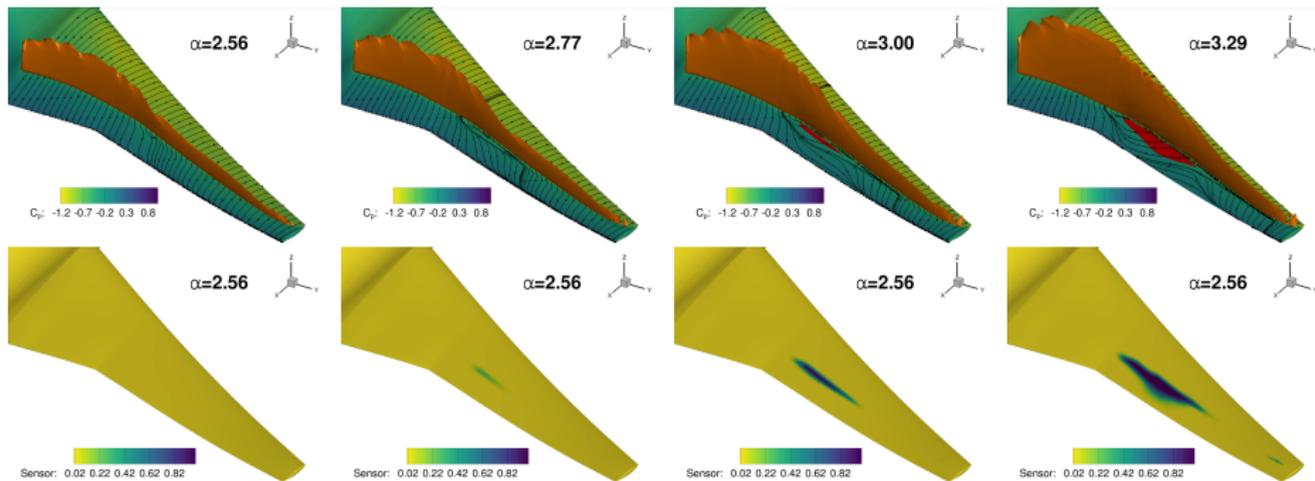
$$\text{Sep} = \frac{1}{A_{\text{ref}}} \iint_S \frac{1.0}{1.0 + e^{-2k(\chi - \lambda)}} dS$$

$$\chi = -\vec{V} \cdot \vec{V}_{\text{freestream}}$$

- Smooth Heaviside function ($k=10$)



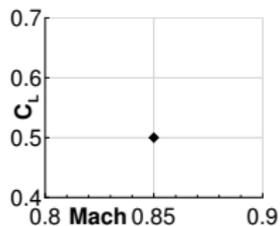
Buffet onset for the CRM at $M=0.85$



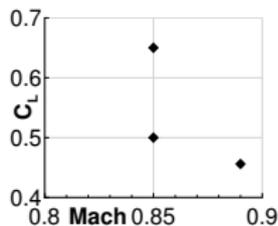
Red surface denotes flow in the negative stream-wise direction.
The brown isosurface is the Lovely–Haines shock sensor.

ADODG Case 5 problem statement

minimize	$\sum_{i=1}^N \mathcal{W}_i C_{D_i}$	Quantity
with respect to	Wing cross sectional shape	240
	Wing twist	9
	Angle of attack (α_i)	N
	Tail rotation angle (η_i)	N
subject to	$C_{L_i} - C_{L_i}^* = 0.0$	N
	$C_{M_{y_i}} = 0.0$	N
	$t_j \geq t_{j_{CRM}}$	750

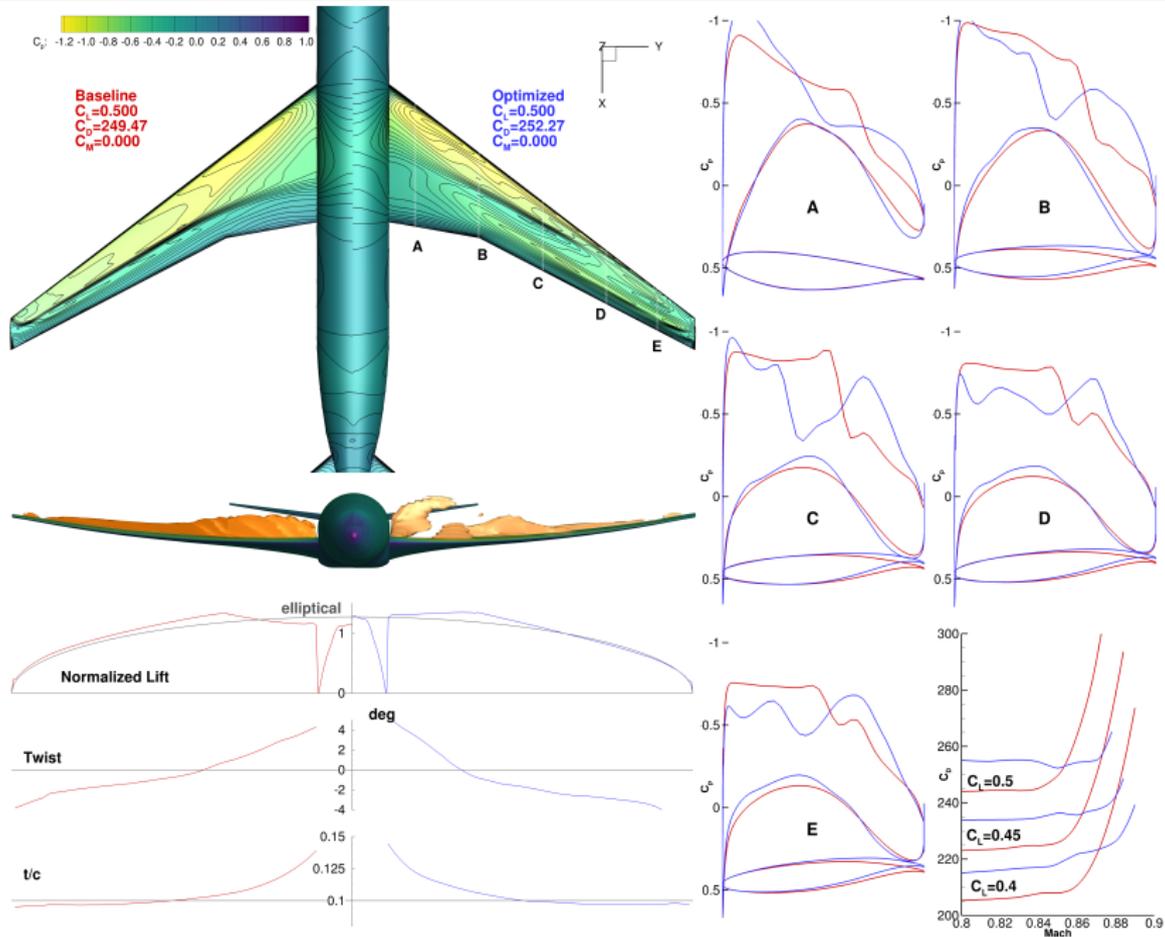


Case 5.1 (N=1)

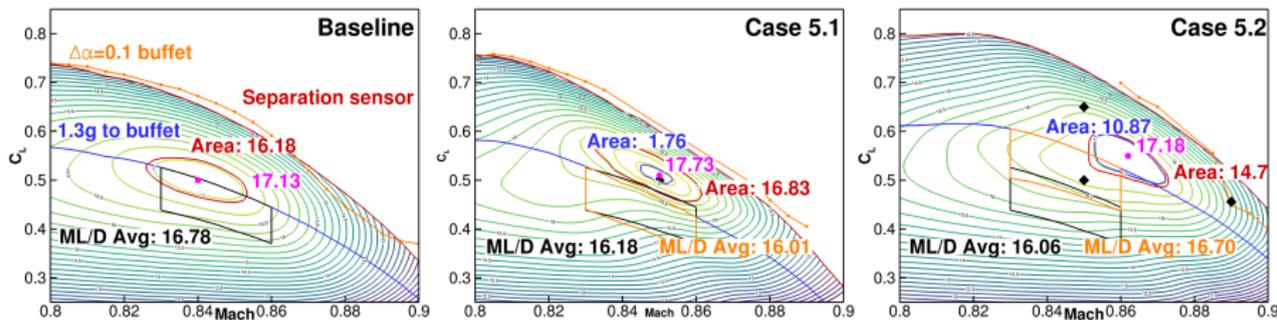


Case 5.2 (N=3)

Case 5.2: Off design conditions improve robustness



ML/D contours give clear indication of off-design performance



Buffet onset boundary

$\Delta\alpha = 0.1$ buffet onset boundary

1.3g margin to buffet

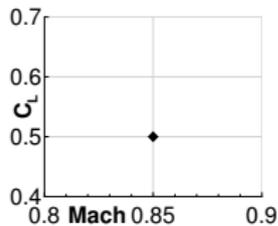
99% ML/D_{\max} of baseline

99% ML/D_{\max}

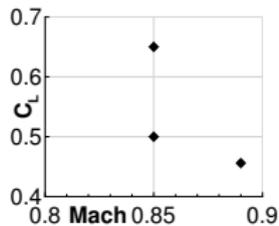
Absolute maximum ML/D
Integration region based on
actual 1.3g margin

Integration region based on
baseline 1.3g margin

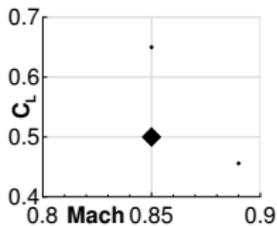
Off-design weighting or buffet-onset constraint?



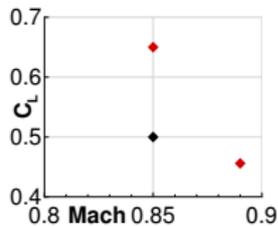
Case 5.1



Case 5.2

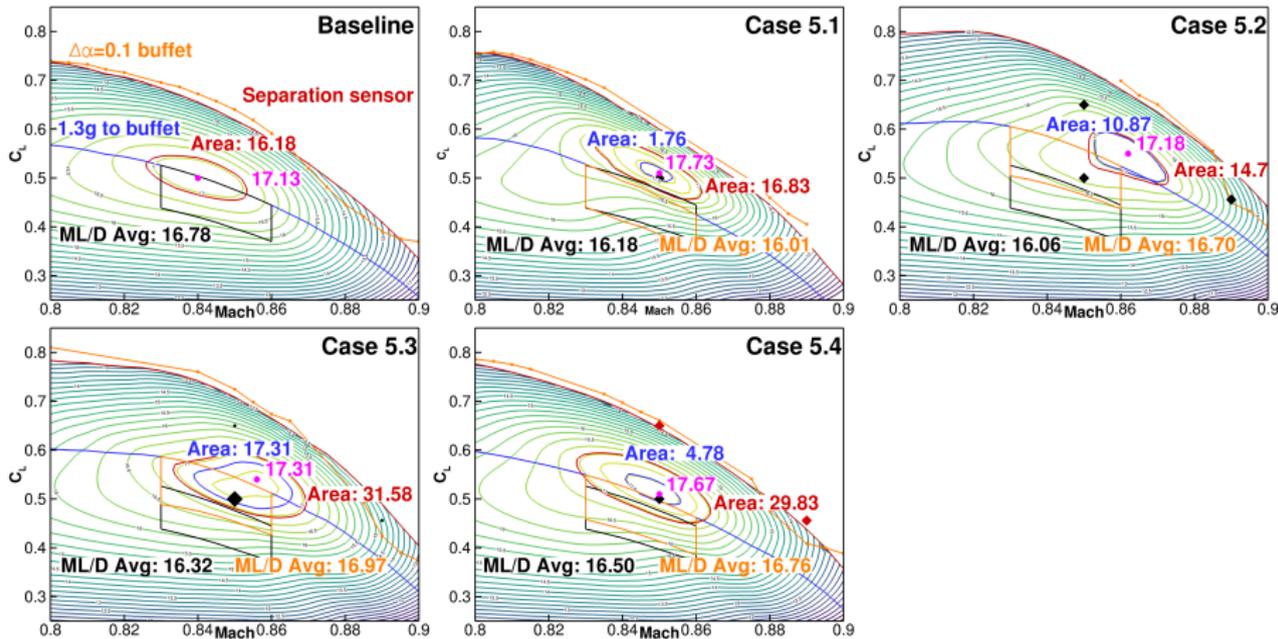


Case 5.3

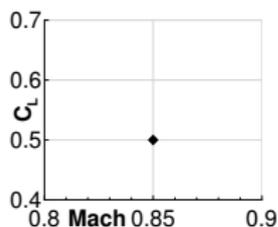


Case 5.4

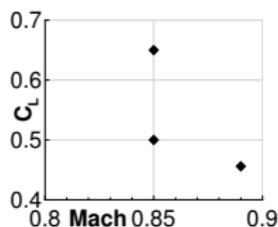
Case 5.3 and Case 5.4 still result in unsatisfactory designs



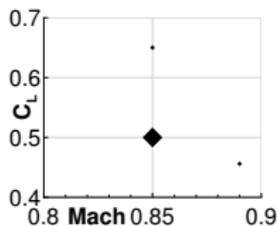
Will multipoint formulations help?



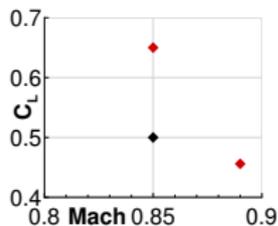
Case 5.1



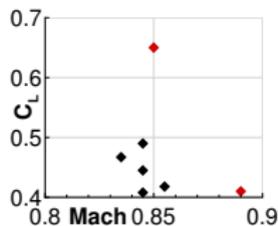
Case 5.2



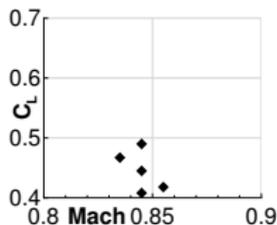
Case 5.3



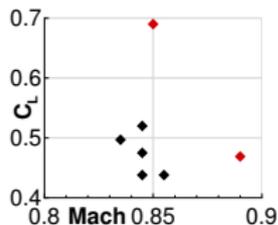
Case 5.4



Case 5.5

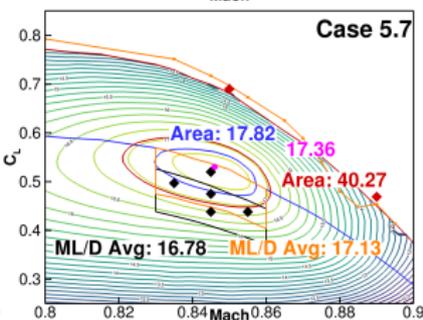
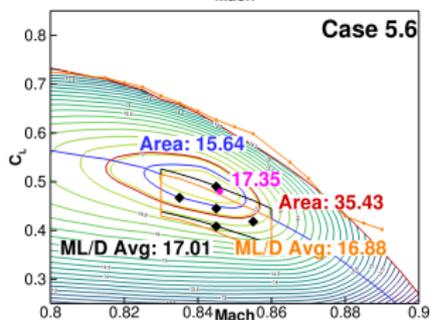
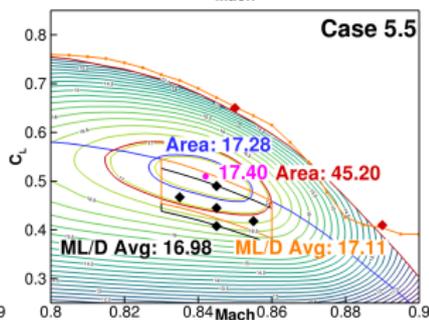
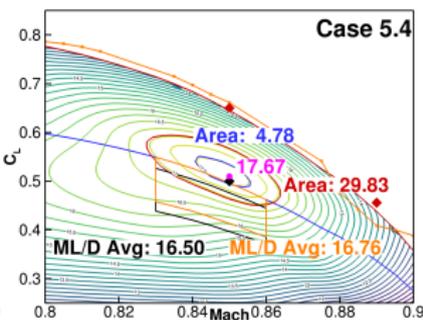
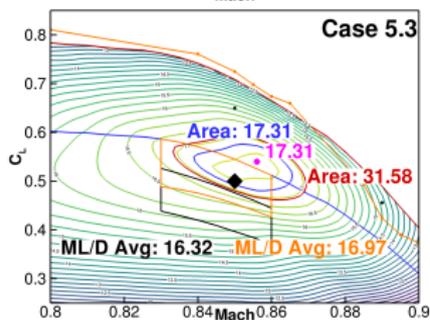
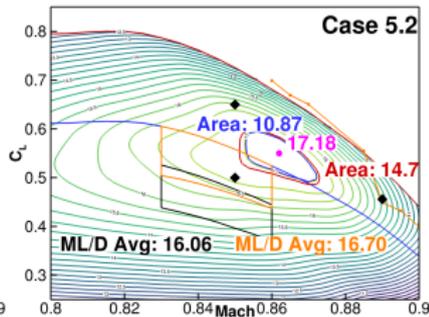
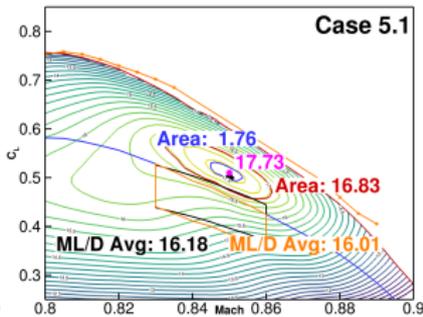
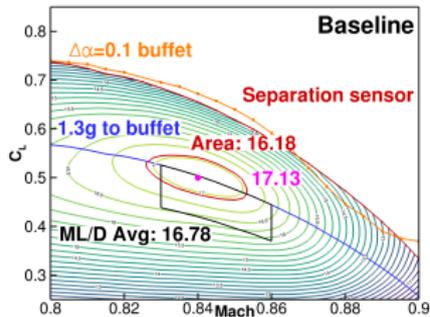


Case 5.6



Case 5.7

ML/D Maximization
with variable C_L .



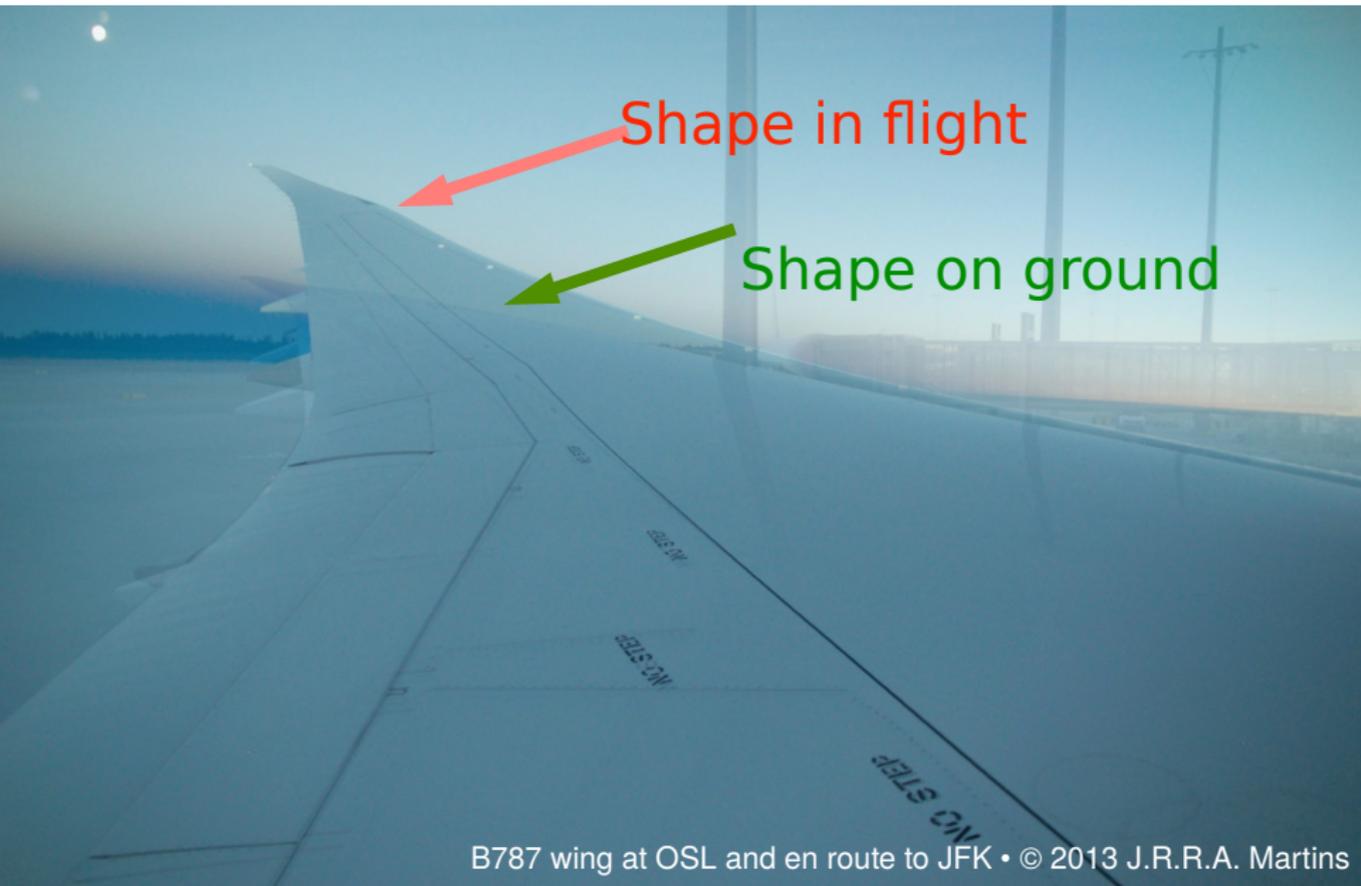
Computational cost breakdown in CPU-hours

- Intel E5540 CPUs running at 2.66 GHz

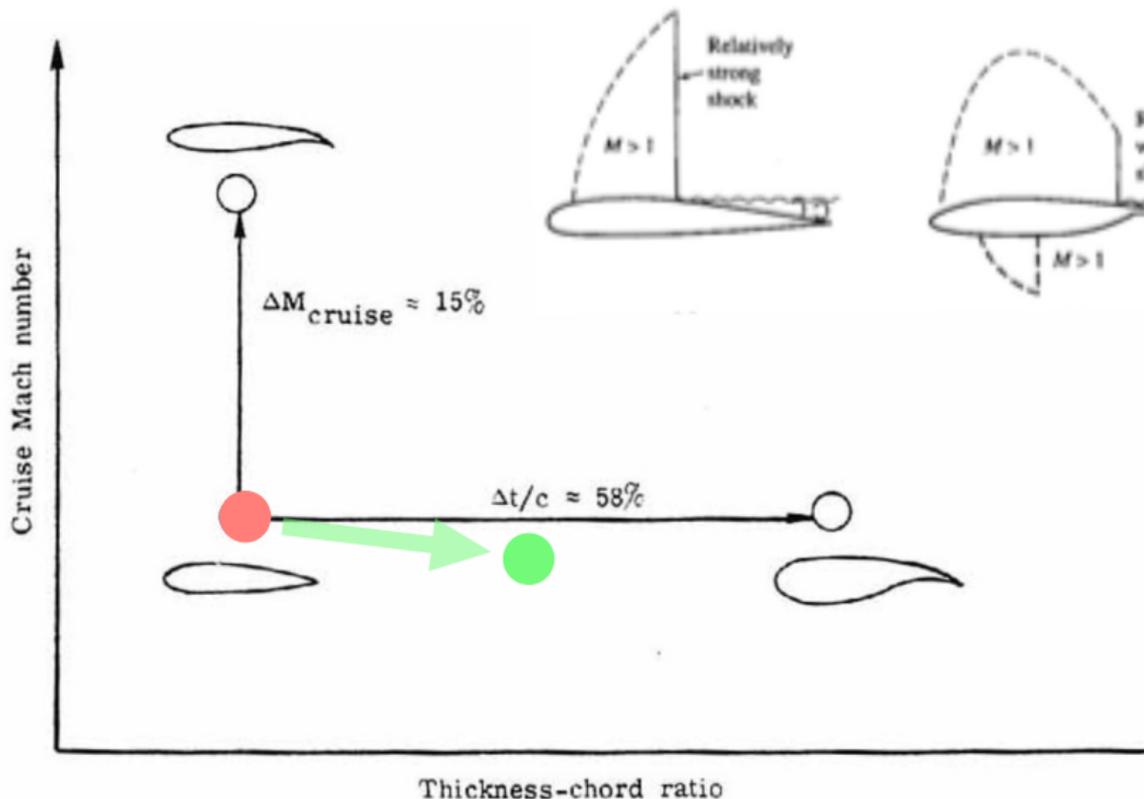
Case	L2 optimization	L1.5 optimization	Contour	Grid convergence	Total
Baseline	–	–	1 346	817	2 162
5.1	289	611	1 270	1 009	3 179
5.2	2 378	2 394	1 795	1 121	7 688
5.3	1 290	2 505	1 750	910	6 457
5.4	1 507	2 602	1 384	1 024	6 518
5.5	2,090	3 506	1 392	830	7 369
5.6	1,111	1 803	1 147	610	4 673
5.7	4,136	6 623	1 800	696	13 255
Total	12 802	19 567	11 886	7 019	51 303

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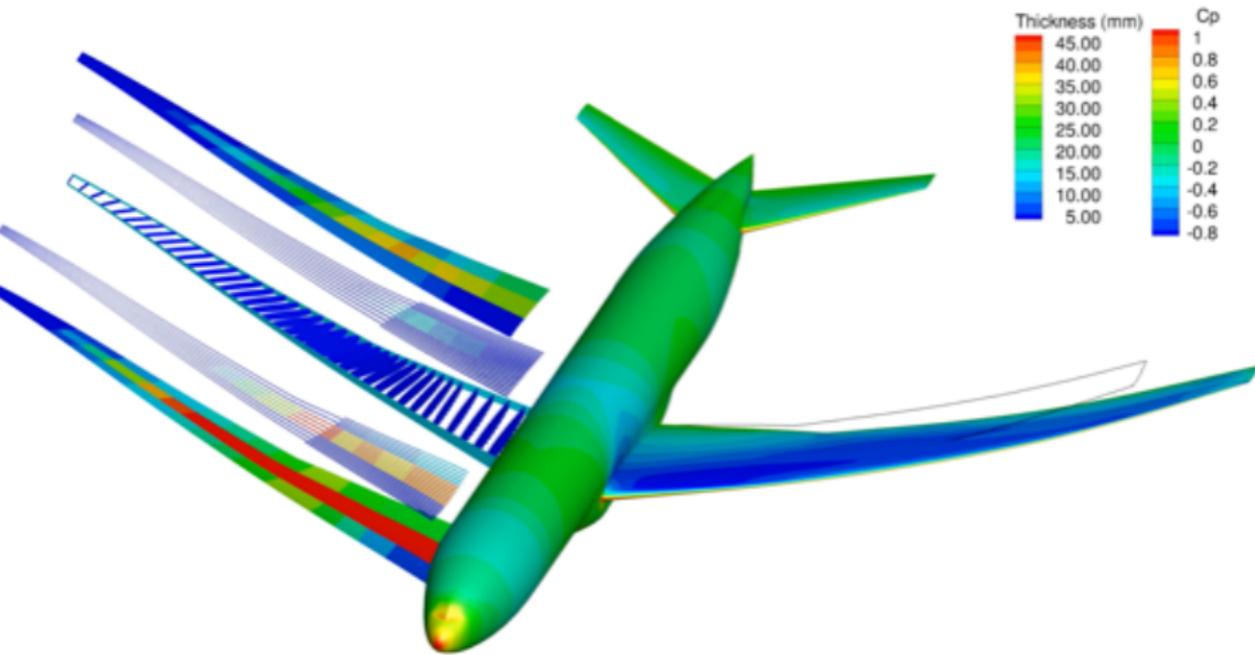
Wing design is more than just aerodynamics



Why you should not trust an aerodynamist (even a brilliant one) to make design decisions



Want to optimize both aerodynamic shape and structural sizing, with high fidelity



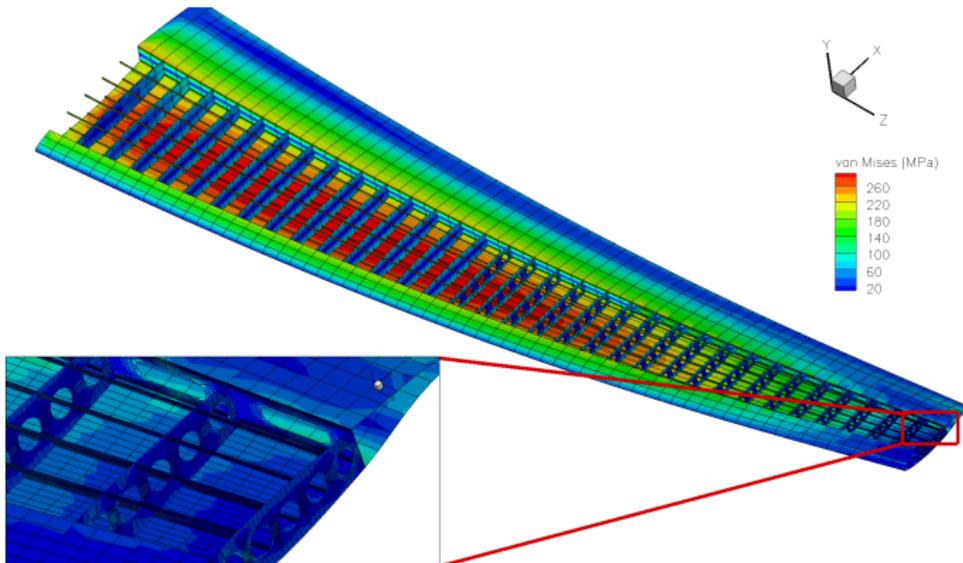
MDO for Aircraft Configuration with High-fidelity (MACH)

Python user script				
Setup up the problem: objective function, constraints, design variables, optimizer and solver options				
Optimizer interface <i>pyOptSparse</i> Common interface to various optimization software		Aerostructural solver <i>AeroStruct</i> Coupled solution methods and coupled derivative evaluation		Geometry modeler <i>DVGeometry/GeoMACH</i> Defines and manipulates geometry, evaluates derivatives
SNOPT	Other optimizers	Structural solver <i>TACS</i> Governing and adjoint equations	Flow solver <i>ADflow</i> Governing and adjoint equations	

- Underlying solvers are parallel and compiled
- Coupling through memory only
- Emphasis on clean Python user interface

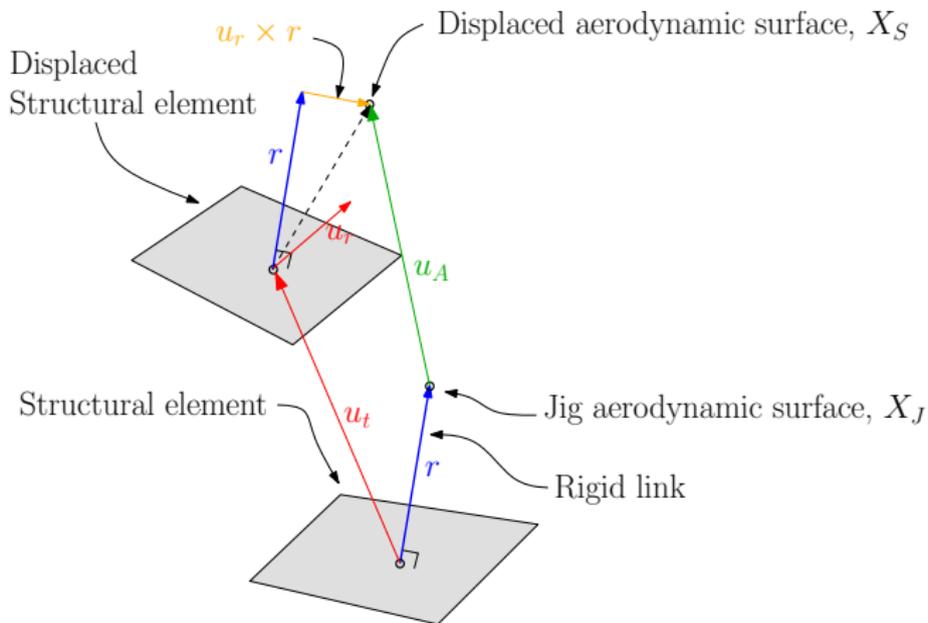
Structural Solver TACS: Toolkit for the Analysis of Composite Structures

- Linear finite element method with MITC shell elements
- Efficient parallel-direct solver for systems with millions of degrees of freedom
- Structural residuals: $\mathcal{S}(u) = 0$

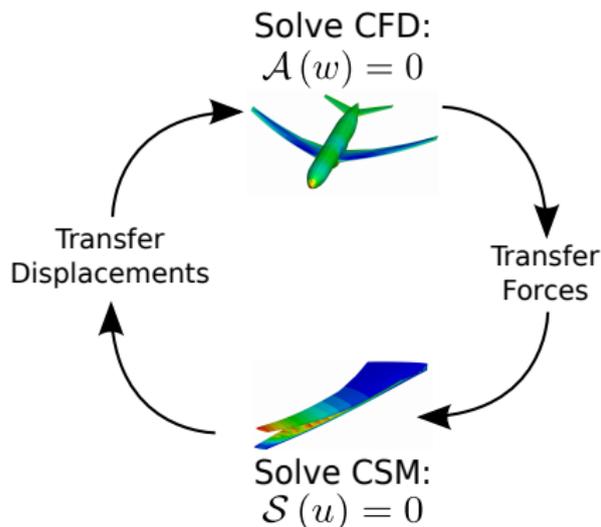


Load and displacement transfer

- CFD loads must be transferred to the CSM model
- Rigid link approach for non-matching surfaces
- Design variable dependent, but robust



Aerostructural solution techniques: Nonlinear Block Gauss–Seidel Method (NLBGS)

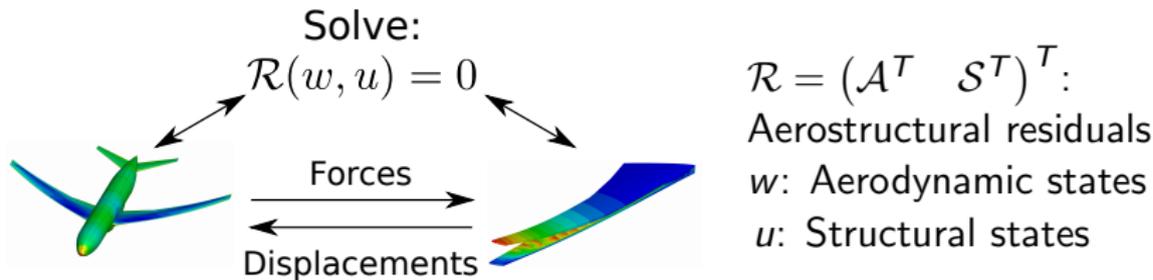


\mathcal{A} : Aerodynamic residuals
 w : Aerodynamic states
 \mathcal{S} : Structural residuals
 u : Structural states

- Convergence can be accelerated with Aitken acceleration

$$\theta \leftarrow \theta \left(1 - \frac{(\Delta u^{(k)} - \Delta u^{(k-1)}) \cdot \Delta u^{(k)}}{\|(\Delta u^{(k)} - \Delta u^{(k-1)})\|^2} \right)$$
$$u^{(k+1)} \leftarrow u^{(k)} + \theta \Delta u^{(k)}$$

Aerostructural solution techniques: Coupled Newton–Krylov Method (CNK)



- Monolithic solution strategy – full aerostructural problem treated simultaneously
- Newton update:

$$\begin{bmatrix} \frac{\partial \mathcal{A}}{\partial w} & \frac{\partial \mathcal{A}}{\partial u} \\ \frac{\partial \mathcal{S}}{\partial w} & \frac{\partial \mathcal{S}}{\partial u} \end{bmatrix} \begin{bmatrix} \Delta w \\ \Delta u \end{bmatrix} = - \begin{bmatrix} \mathcal{A}(w) \\ \mathcal{S}(u) \end{bmatrix}$$

- Matrix-free FGMRES with Block–Jacobi preconditioner
- With wrapped codes and direct state/residual variable access, very little code modification!

Adjoint method can be extended to aerostructural system

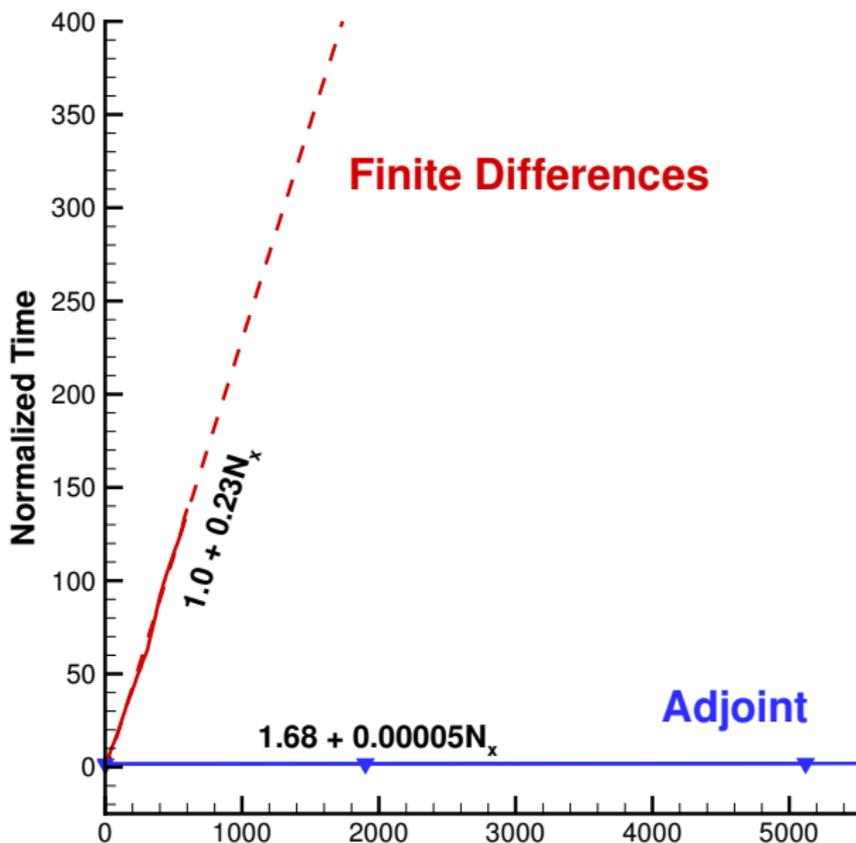
- Adjoint sensitivities are imperative for high-fidelity aerostructural optimization with large numbers of design variables
- For each function of interest, I , solve for the coupled adjoint vector, $\Psi = (\psi^T \quad \phi^T)^T$:

$$\begin{bmatrix} \frac{\partial \mathcal{A}}{\partial w} & \frac{\partial \mathcal{A}}{\partial u} \\ \frac{\partial \mathcal{S}}{\partial w} & \frac{\partial \mathcal{S}}{\partial u} \end{bmatrix}^T \begin{bmatrix} \psi \\ \phi \end{bmatrix} = \begin{bmatrix} \frac{\partial I}{\partial w} & \frac{\partial I}{\partial u} \end{bmatrix}^T$$

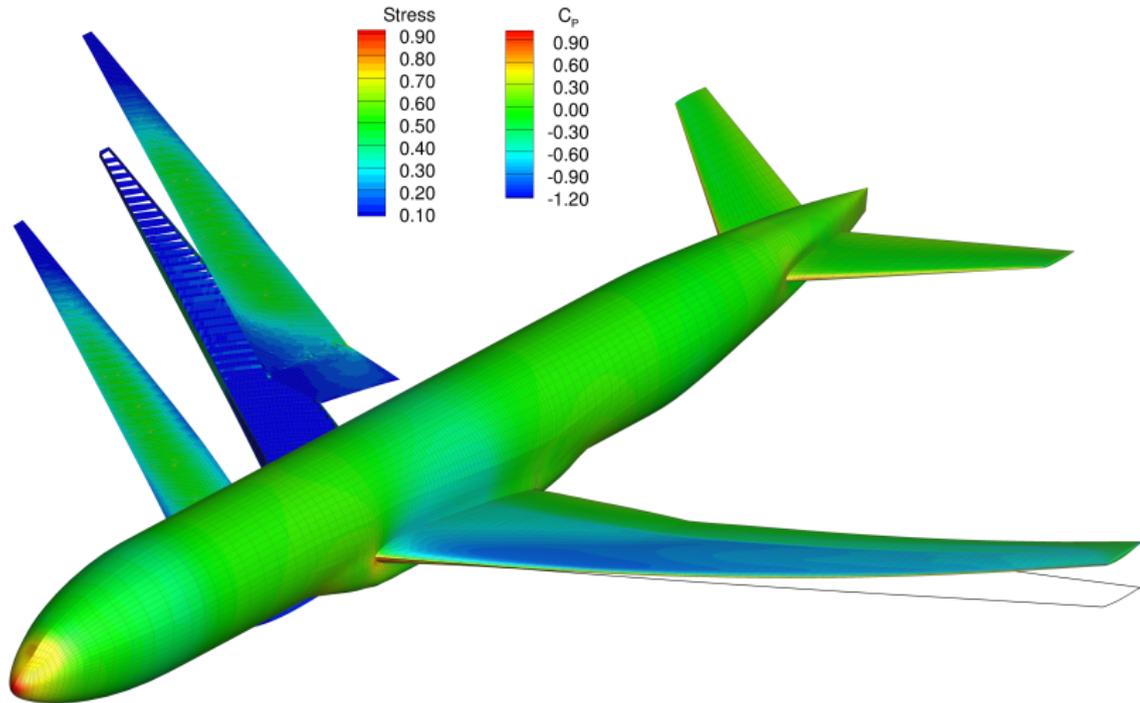
- Adjoint vector is independent of the number design variables
- Solve one adjoint for each function
- Total sensitivity with respect to design variables, x , can then be obtained with

$$\frac{dI}{dx} = \frac{\partial I}{\partial x} - \psi^T \left(\frac{\partial \mathcal{A}}{\partial x} \right) - \phi^T \left(\frac{\partial \mathcal{S}}{\partial x} \right)$$

Adjoint method efficiently computes gradients with respect to thousands of variables



Let's do aerostructural optimization!

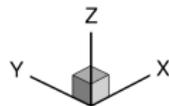


NASA-Michigan undeformed Common Research Model (uCRM)

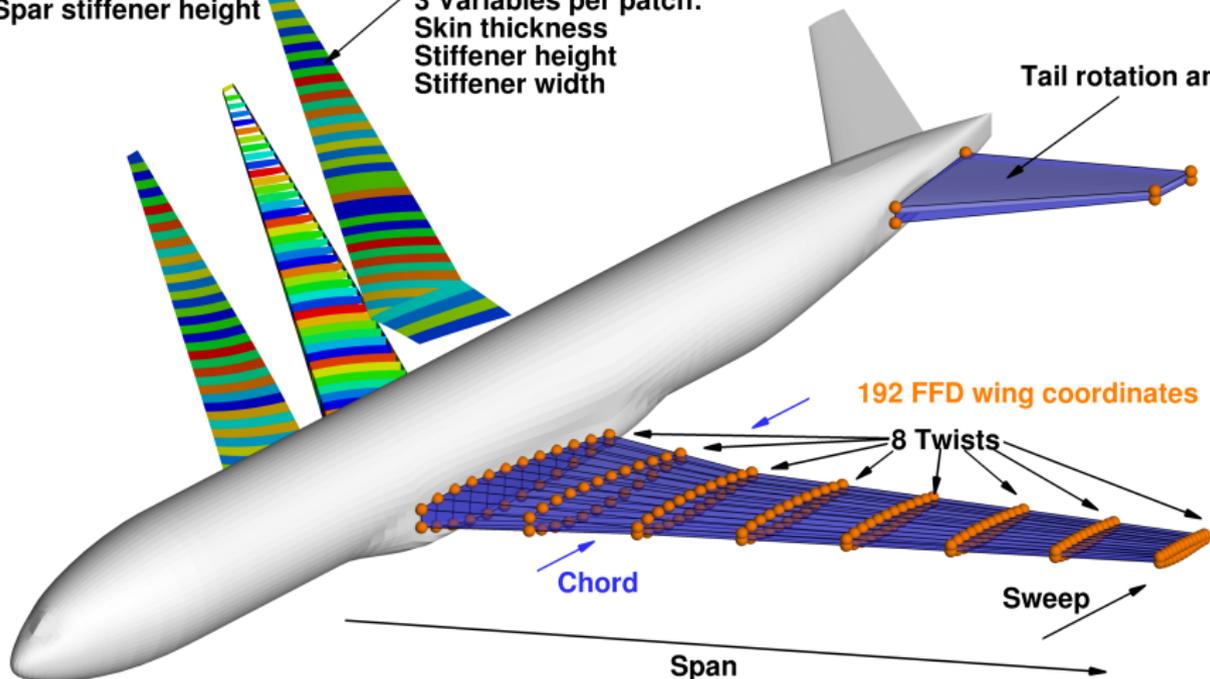
Optimize with respect to 972 “aerodynamic” and structural sizing variables

Upper skin pitch
Lower skin pitch
Rib stiffener pitch
Rib stiffener height
Spar stiffener pitch
Spar stiffener height

3 Variables per patch:
Skin thickness
Stiffener height
Stiffener width



Tail rotation angle



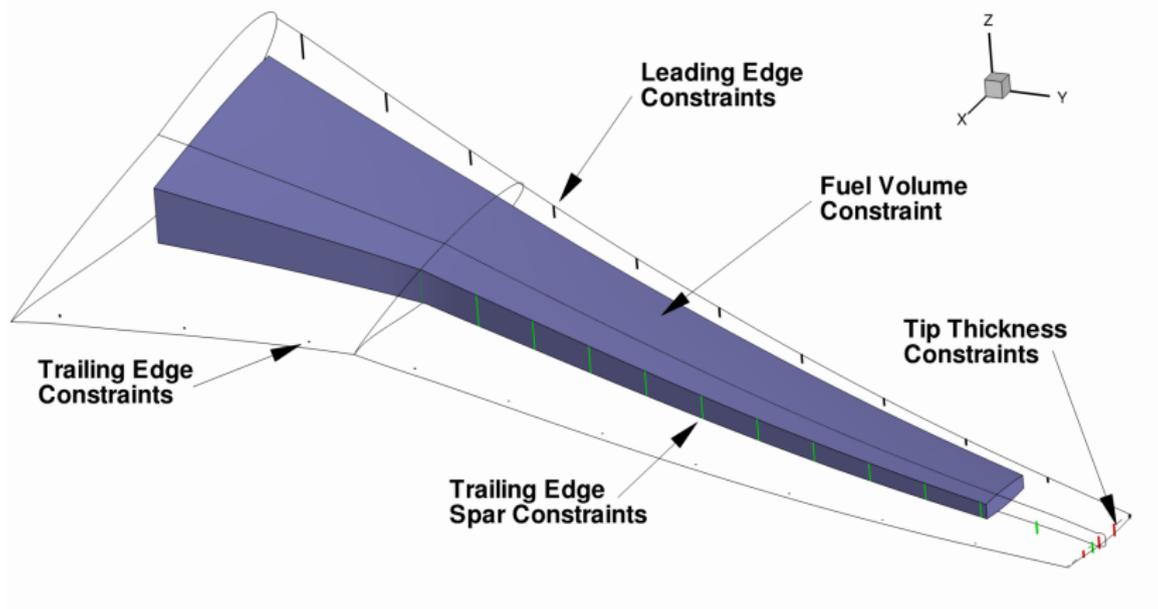
Optimization objective and design variables

	Function/variable	Description	Quantity
minimize	$\sum_i^N \mathcal{W}_i FB$		
with respect to	x_{span}	Wing span	1
	x_{sweep}	Wing sweep	1
	x_{chord}	Wing chord	1
	x_{twist}	Wing twist	7
	$x_{airfoil}$	FFD control points	192
	x_{α_i}	Angle of attack at each flight condition	12
	x_{η_i}	Tail rotation angle at each flight condition	12
	$x_{throttle_i}$	Throttle setting for each cruise flight condition	7
	$x_{altitude}$	Cruise altitude	1
	X_{CG}	CG position	1
	$x_{skin\ pitch}$	Upper/lower stiffener pitch	2
	$x_{spar\ pitch}$	LE/TE Spar stiffener pitch	2
	x_{ribs}	Rib thickness	45
	$x_{panel\ thick}$	Panel thickness skins/spars	172
	$x_{stiff\ thick}$	Panel stiffener thickness skins/spars	172
	$x_{stiff\ height}$	Panel stiffener height skins/spars	172
	$x_{panel\ length}$	Panel length skin/spars	172
		Total design variables	972

Constraints

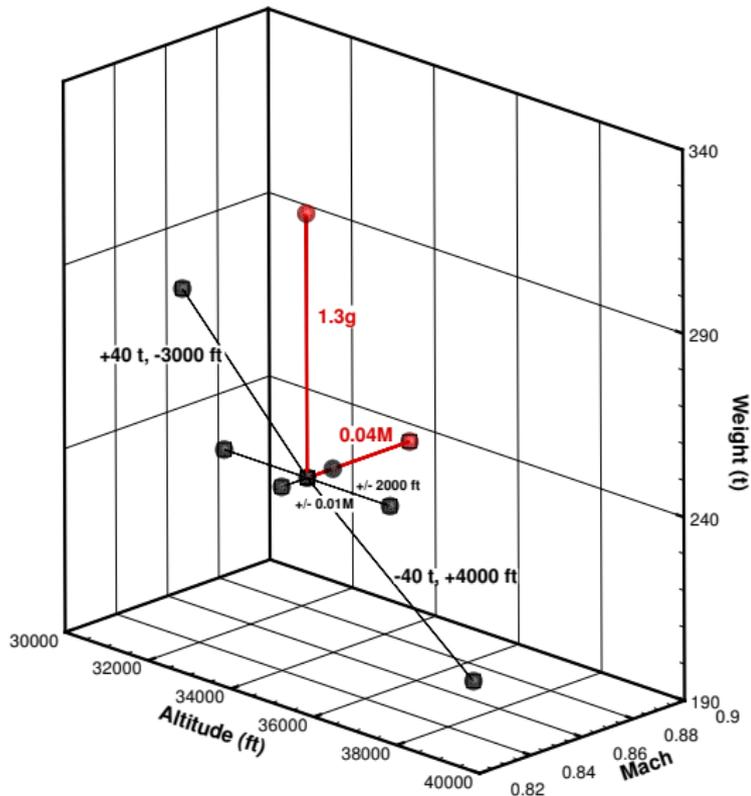
	Function/variable	Description	Quantity
subject to	$L = n_i W$	Lift constraint	12
	$C_{M_{y_i}} = 0.0$	Trim constraint	12
	$T = D$	Thrust constraint	7
	$1.08D - T_{\max} < 0$	Excess thrust constraint	7
	$t_{LE}/t_{LE_{init}} \geq 1.0$	Leading edge radius	20
	$t_{TE}/t_{TE_{init}} \geq 1.0$	Trailing edge thickness	20
	$\mathcal{V}_{wing} > \mathcal{V}_{fuel}$	Minimum fuel volume	1
	$x_{CG} - 1/4MAC = 0$	CG location at 1/4 chord MAC	1
	$L_{panel} - x_{panel\ length} = 0$	Target panel length	172
	$KS_{stress} \leq 1$	2.5 g Yield stress	4
	$KS_{buckling} \leq 1$	2.5 g Buckling	3
	$KS_{buckling} \leq 1$	-1.0 g Buckling	3
	$KS_{buckling} \leq 1$	1.78 g Yield stress	3
	$KS_{buckling} \leq 1$	1.78 g Buckling	4
	$ x_{panel\ thick_i} - x_{panel\ thick_{i+1}} \leq 0.005$	Skin thickness adjacency	168
	$ x_{stiff\ thick_i} - x_{stiff\ thick_{i+1}} \leq 0.005$	Stiffener thickness adjacency	168
	$ x_{stiff\ height_i} - x_{stiff\ height_{i+1}} \leq 0.005$	Stiffener height adjacency	168
	$x_{stiff\ thick} - x_{panel\ thick} < 0.005$	Maximum stiffener-skin difference	172
	$\Delta z_{TE,upper} = -\Delta z_{TE,lower}$	Fixed trailing edge	8
	$\Delta z_{LE,upper} = -\Delta z_{LE,lower}$	Fixed leading edge	8
		Total constraints	961

Fewer geometric constraints are required for aerostructural optimization



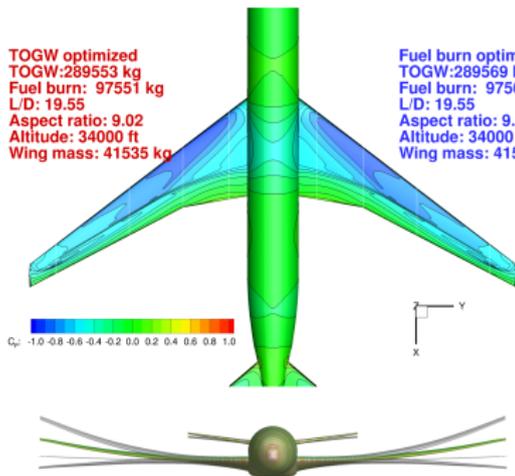
Considering multiple flight conditions is required for a practical design

- 7 cruise conditions for performance
- 2 off-design buffet onset conditions
- 3 maneuver conditions for structural constraints
- All flight conditions trimmed

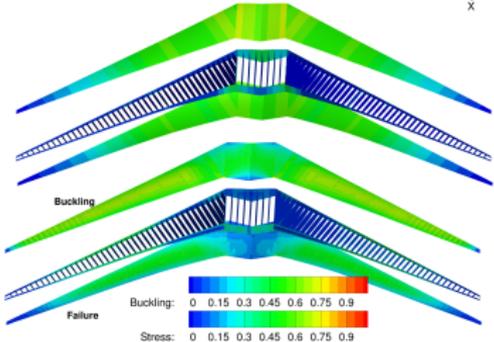


TOGW optimized
 TOGW:289553 kg
 Fuel burn: 97551 kg
 L/D: 19.55
 Aspect ratio: 9.02
 Altitude: 34000 ft
 Wing mass: 41535 kg

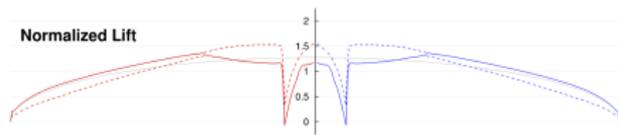
Fuel burn optimized
 TOGW:289569 kg
 Fuel burn: 97567 kg
 L/D: 19.55
 Aspect ratio: 9.02
 Altitude: 34000 ft
 Wing mass: 41535 kg



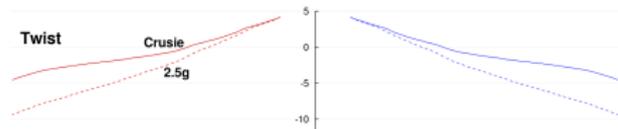
Equivalent Thickness (in): 0.2 0.725 1.25



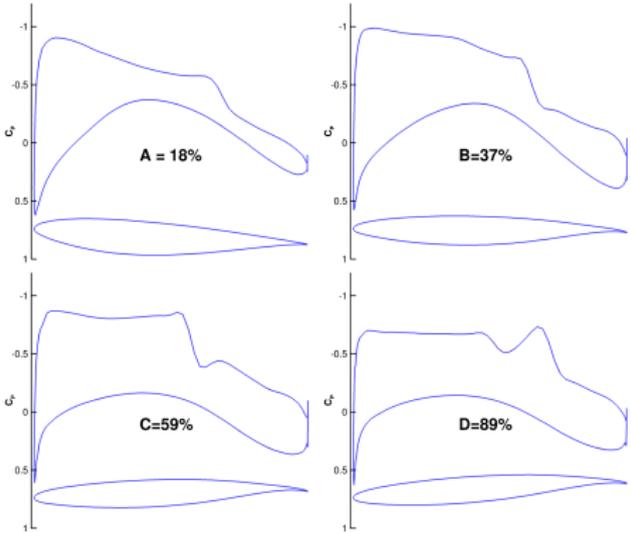
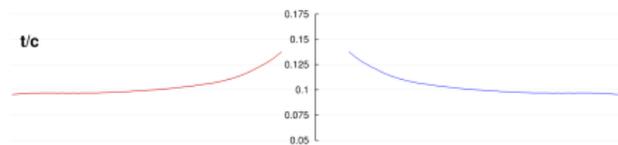
Normalized Lift



Twist

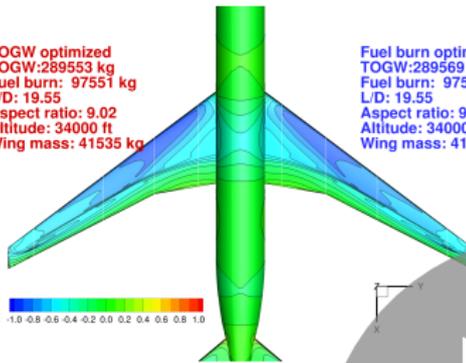


t/c

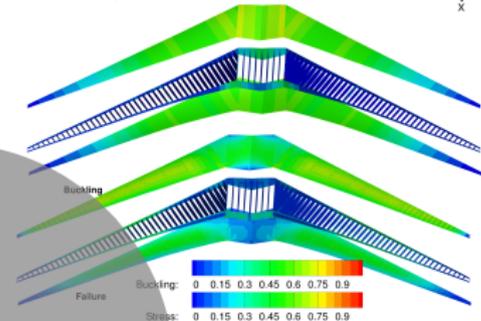


TOGW optimized
 TOGW:289553 kg
 Fuel burn: 97551 kg
 L/D: 19.55
 Aspect ratio: 9.02
 Altitude: 34000 ft
 Wing mass: 41535 kg

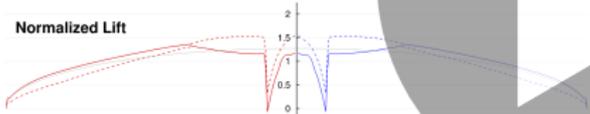
Fuel burn optimized
 TOGW:289569 kg
 Fuel burn: 97567 kg
 L/D: 19.55
 Aspect ratio: 9.02
 Altitude: 34000 ft
 Wing mass: 41535 kg



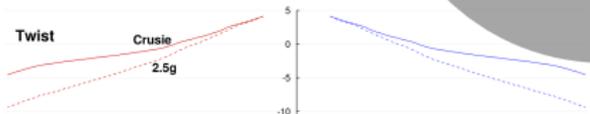
Equivalent Thickness (in): 0.2 0.725 1.25



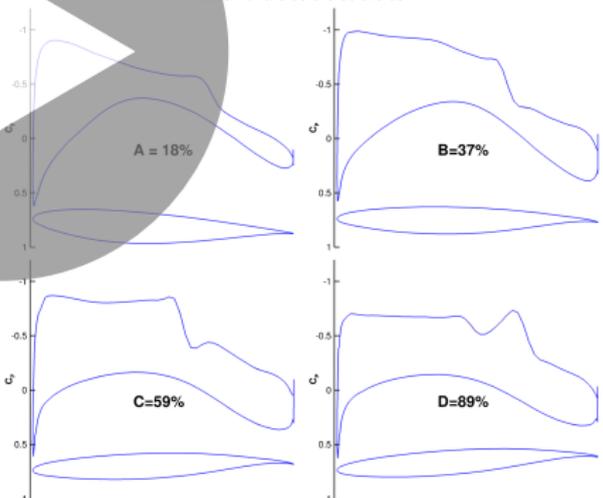
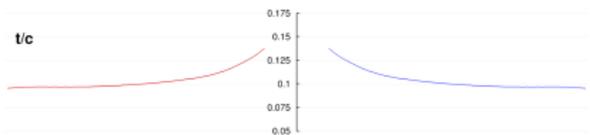
Normalized Lift



Twist

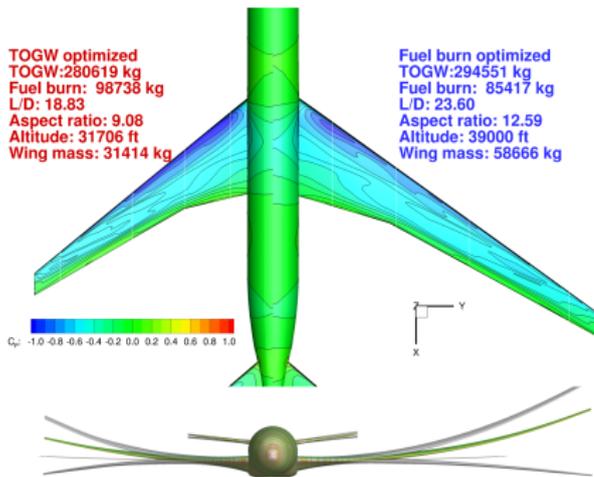


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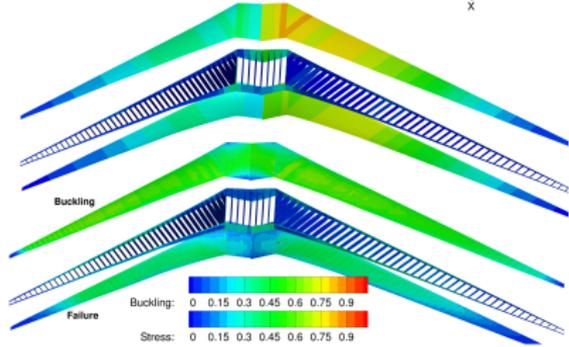


TOGW optimized
 TOGW:280619 kg
 Fuel burn: 98738 kg
 L/D: 18.83
 Aspect ratio: 9.08
 Altitude: 31706 ft
 Wing mass: 31414 kg

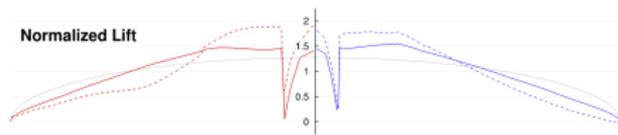
Fuel burn optimized
 TOGW:294551 kg
 Fuel burn: 85417 kg
 L/D: 23.60
 Aspect ratio: 12.59
 Altitude: 39000 ft
 Wing mass: 58666 kg



Equivalent Thickness (in): 0.2 0.725 1.25

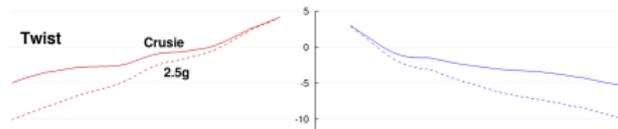


Normalized Lift

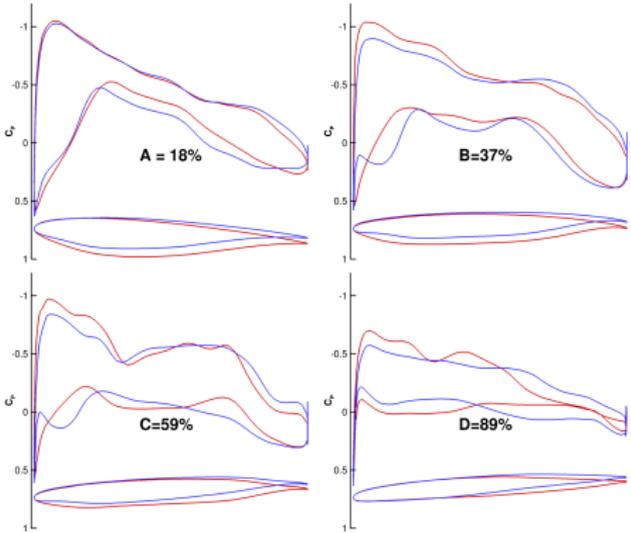
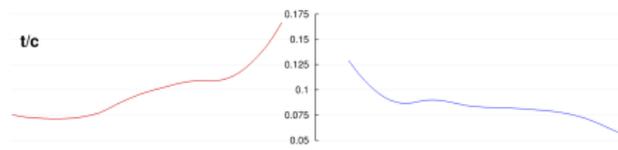


Twist

Crusie
2.5g



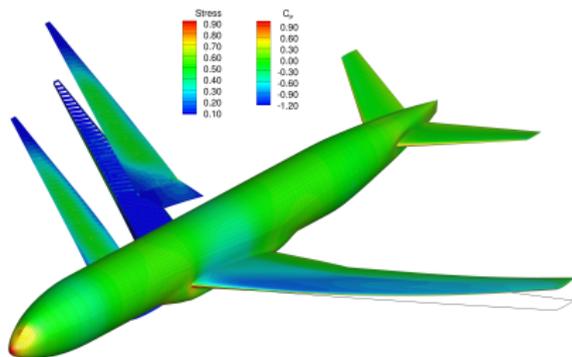
t/c



- Motivation
- Computing derivatives
- Aerodynamic shape optimization
- Aerostructural design optimization
- **Summary**

Summary

- Efficient and accurate gradient computations via the adjoint method and automatic differentiation
- Dual challenge of expensive function evaluations with large number of design variables
- Aerodynamic shape optimization limits design space
- Extend the adjoint method to multiple disciplines
- Aerostructural design optimization with respect to 1000 design variables
- Still much more work to do!



Relevant publications



Graeme J. Kennedy and Joaquim R. R. A. Martins.

A parallel finite-element framework for large-scale gradient-based design optimization of high-performance structures.

Finite Elements in Analysis and Design, 87:56–73, September 2014.



Gaetan K. W. Kenway, Graeme J. Kennedy, and Joaquim R. R. A. Martins.

Scalable parallel approach for high-fidelity steady-state aeroelastic analysis and derivative computations.

AIAA Journal, 52(5):935–951, May 2014.



Gaetan K. W. Kenway and Joaquim R. R. A. Martins.

Multipoint high-fidelity aerostructural optimization of a transport aircraft configuration.

Journal of Aircraft, 51(1):144–160, January 2014.



Gaetan K. W. Kenway and Joaquim R. R. A. Martins.

Buffet onset constraint formulation for aerodynamic shape optimization.

AIAA Journal, 2017.

(In press).



Gaetan W. K. Kenway and Joaquim R. R. A. Martins.

High-fidelity aerostructural optimization considering buffet onset.

In *Proceedings of the 16th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, Dallas, TX, June 2015.

AIAA 2015-2790.



Joaquim R. R. A. Martins and John T. Hwang.

Review and unification of methods for computing derivatives of multidisciplinary computational models.

AIAA Journal, 51(11):2582–2599, November 2013.

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