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Advanced Modeling & Simulation Seminar Series

An ODE-based Wall Model for Turbulent Flow Simulations

Marsha J. Berger

Courant Institute

New York University, NY, NY

Michael J. Aftosmis

Computational Aerosciences Branch

NASA Ames Research Center, Moffett Field, CA

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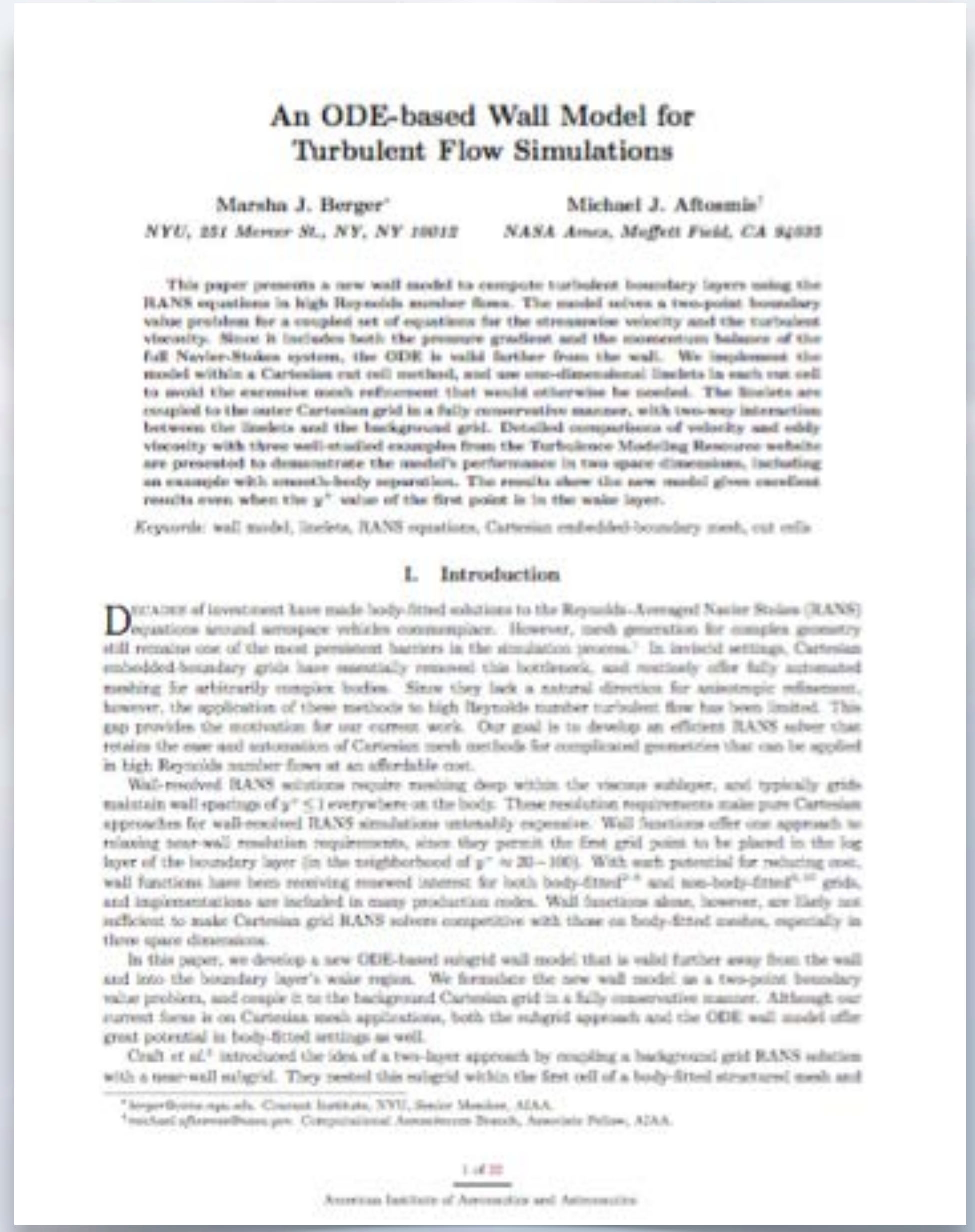


An ODE-based Wall Model for Turbulent Flow Simulations

Marsha J. Berger — NYU, 251 Mercer St., NY, NY 10012

Michael J. Aftosmis — NASA Ames, Moffett Field, CA 94035

This paper presents a new wall model to compute turbulent boundary layers using the RANS equations in high Reynolds number flows. The model solves a two-point boundary value problem for a coupled set of equations for the streamwise velocity and the turbulent viscosity. Since it includes both the pressure gradient and the momentum balance of the full Navier-Stokes system, the ODE is valid farther from the wall. We implement the model within a Cartesian cut cell method, and use one-dimensional linelets in each cut cell to avoid the excessive mesh refinement that would otherwise be needed. The linelets are coupled to the outer Cartesian grid in a fully conservative manner, with two-way interaction between the linelets and the background grid. Detailed comparisons of velocity and eddy viscosity with three well-studied examples from the Turbulence Modeling Resource website are presented to demonstrate the model's performance in two space dimensions, including an example with smooth-body separation. The results show the new model gives excellent results even when the y^+ value of the first point is in the wake layer.

Keywords: wall model, linelets, RANS equations, Cartesian embedded-boundary mesh, cut cells

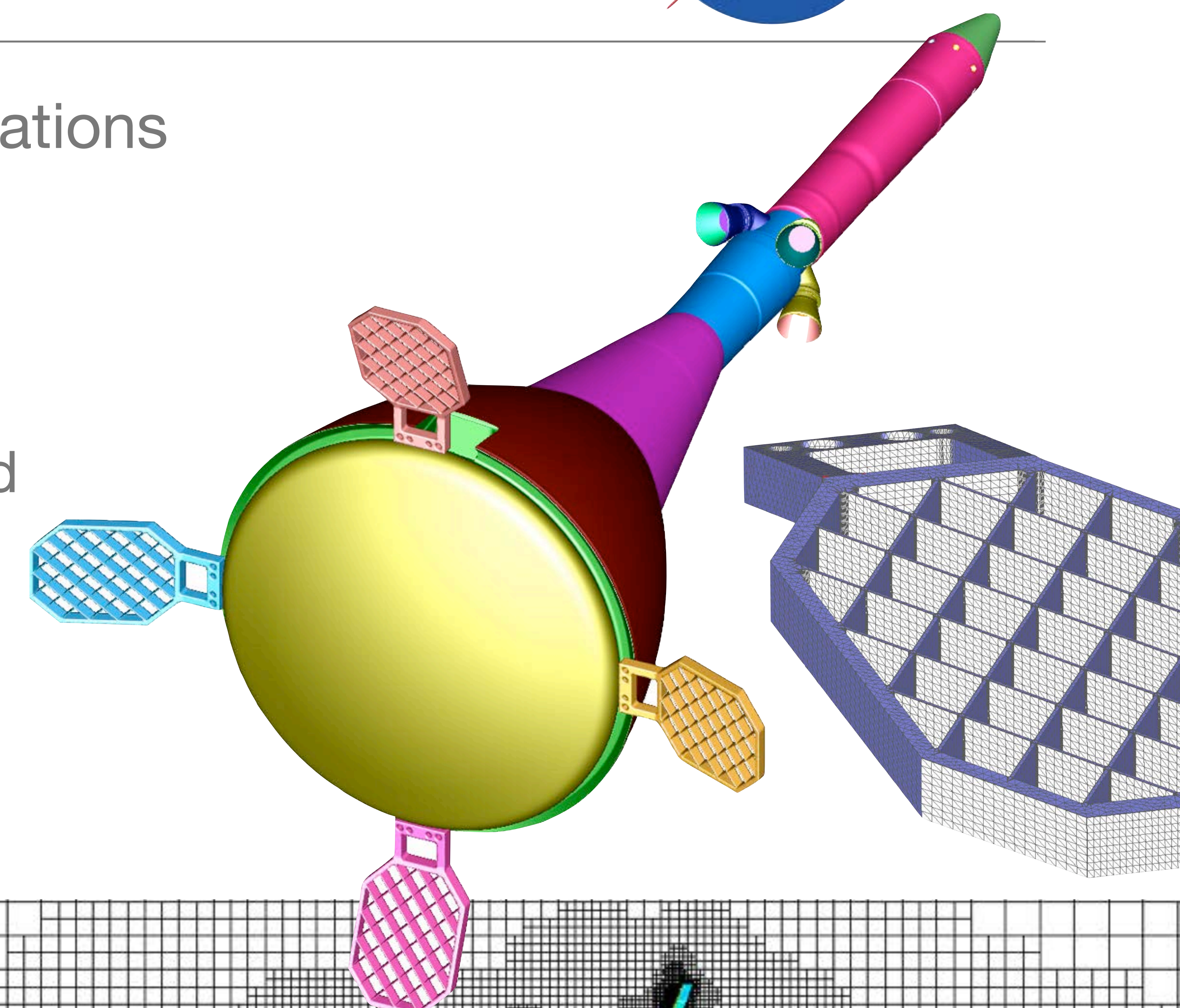
I. Introduction

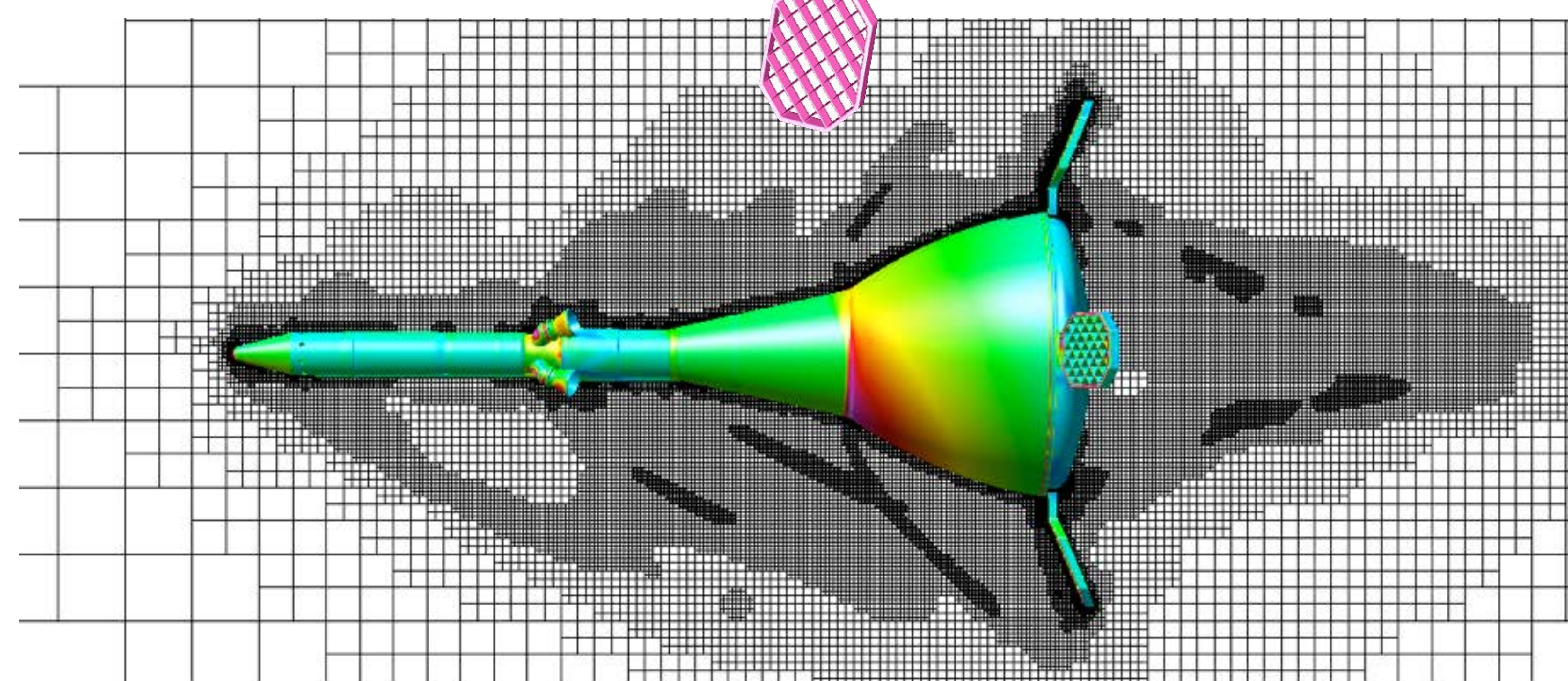
23 February, 2017

Motivation

Fully Automated Reynolds-Averaged Navier-Stokes Simulations

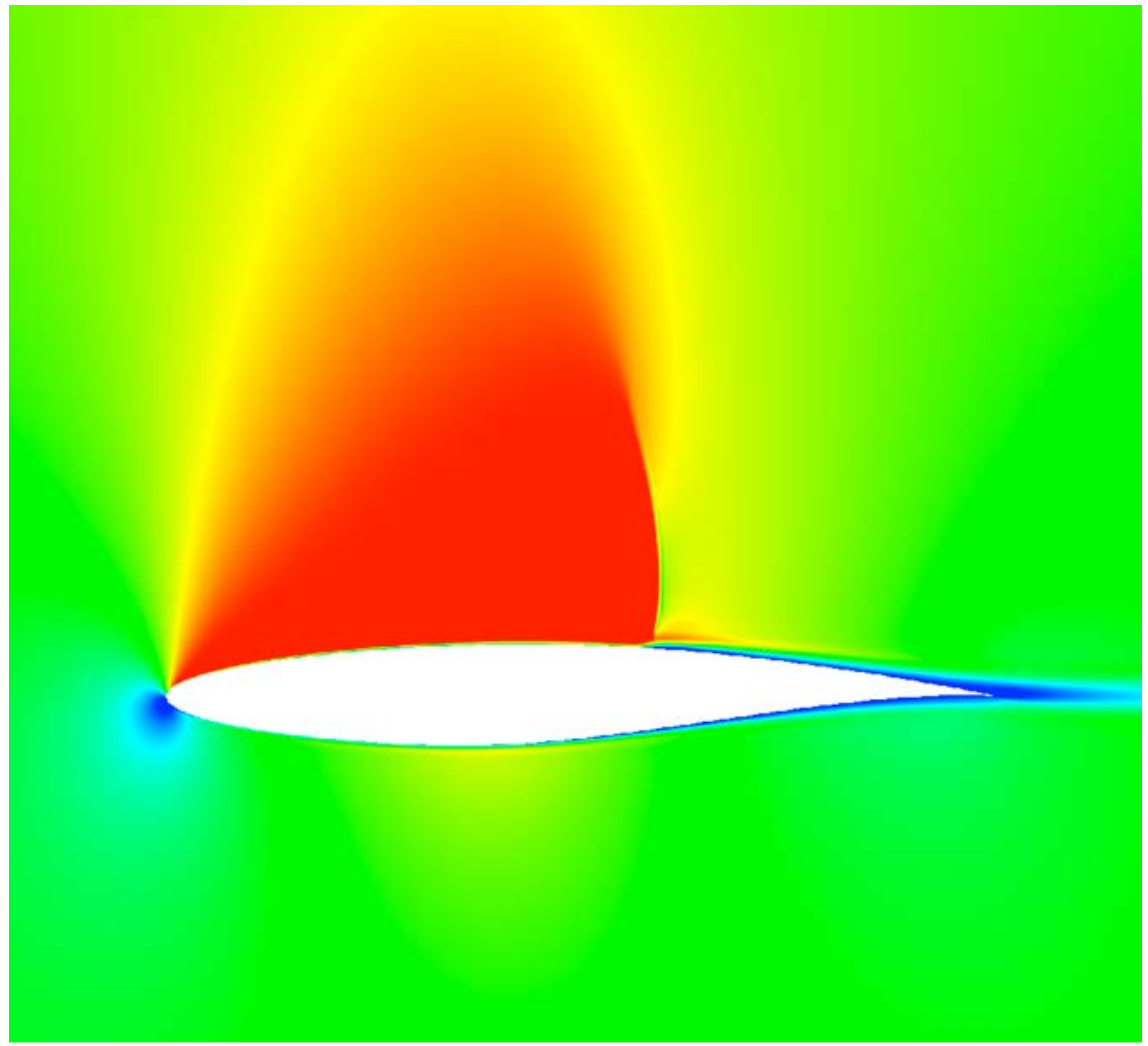
- Mesh generation for complex geometry continues to be the biggest bottleneck in the RANS simulation process
- Fully automated Cartesian methods routinely used for inviscid simulations about arbitrarily complex geometry
- These methods lack of an obvious & robust way to achieve near wall anisotropy
- **Goal:** Extend these methods for RANS simulation without sacrificing automation, at an affordable cost
- Note: Nothing here is limited to Cartesian methods, and much becomes simpler in a body-fitted setting.





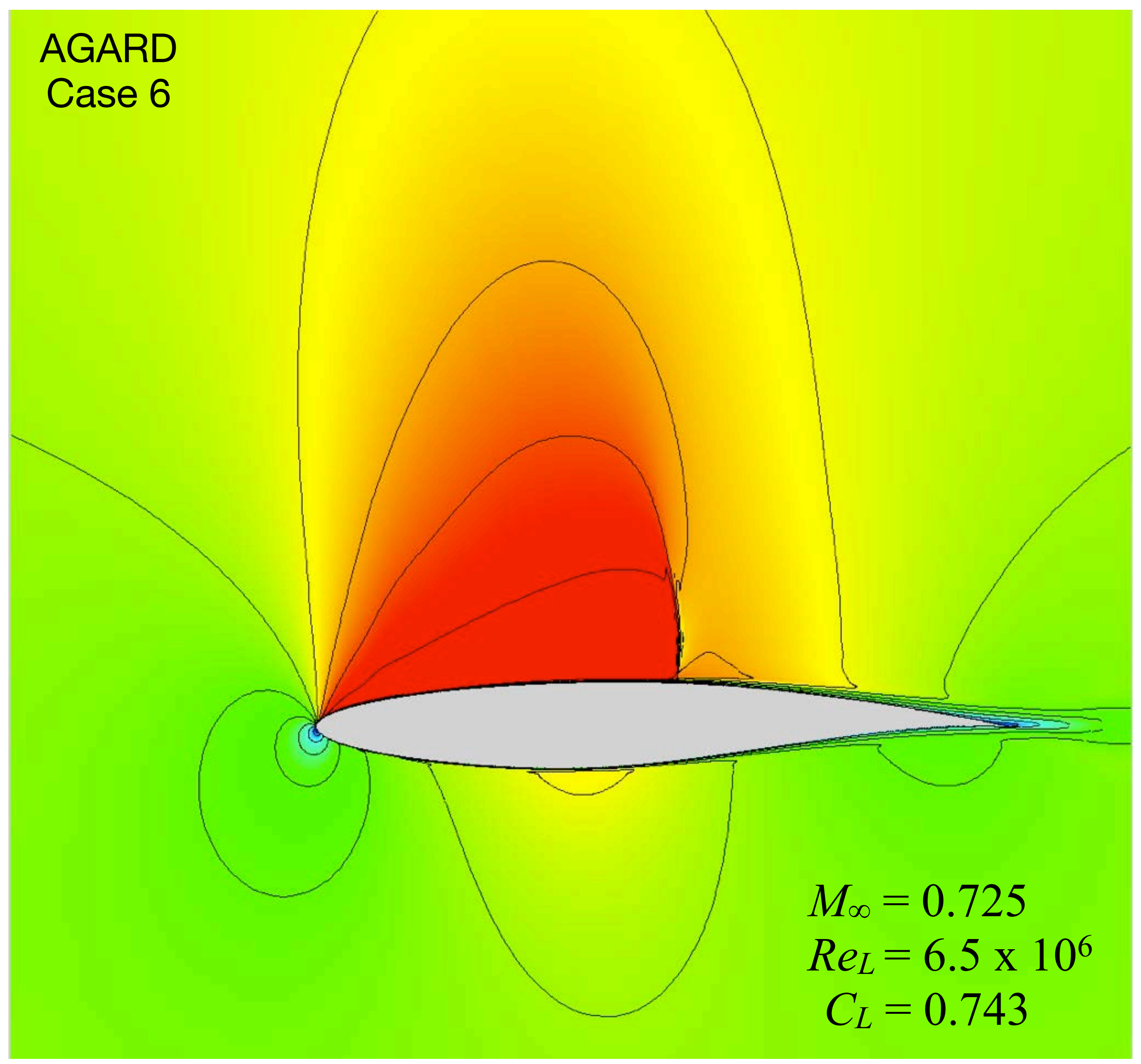
Outline

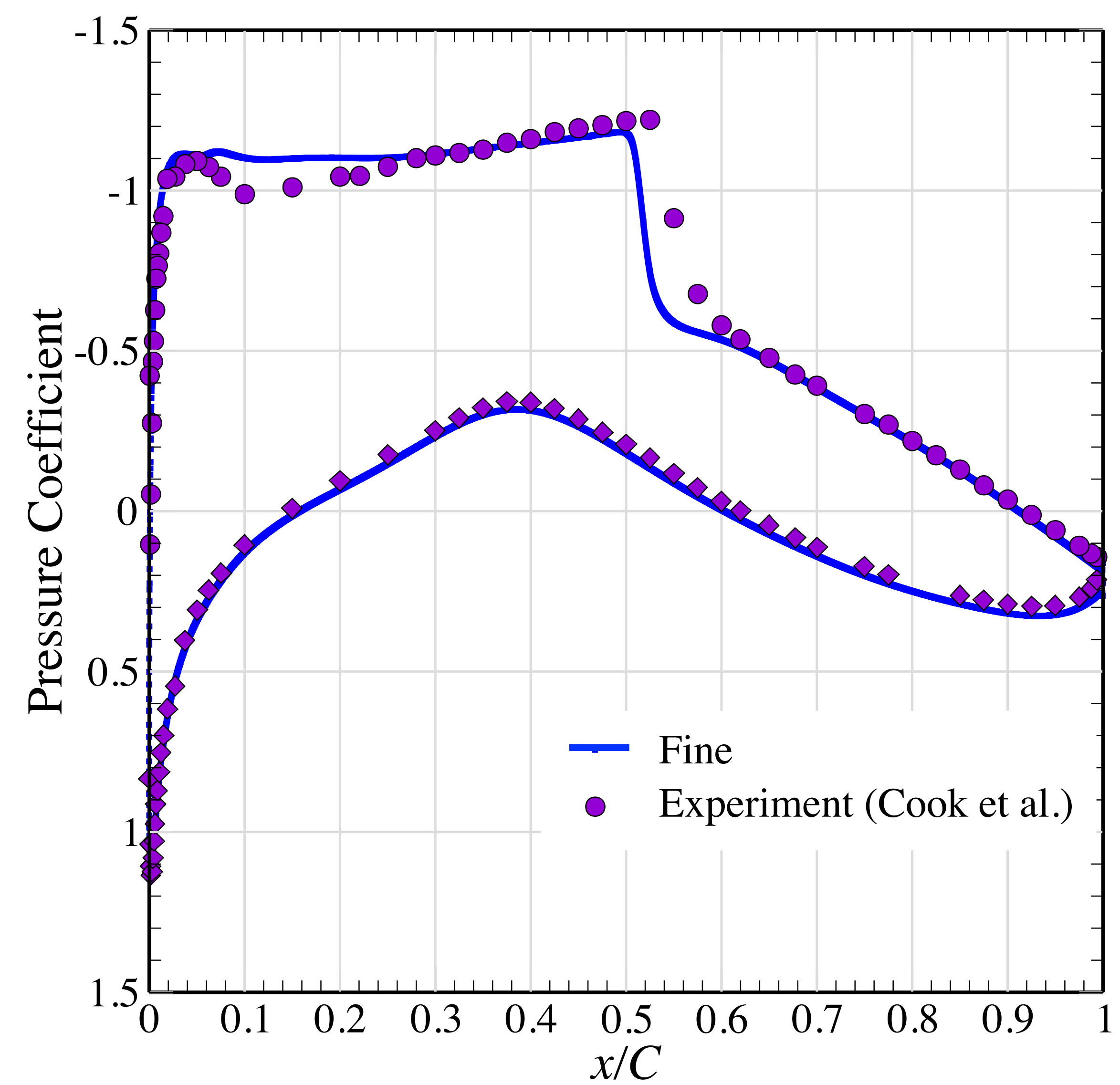
- Previous work & analytic wall functions
- ODE-based wall models
  - A New ODE wall model
  - Model Summary
- Numerical examples and V&V
- Conclusions

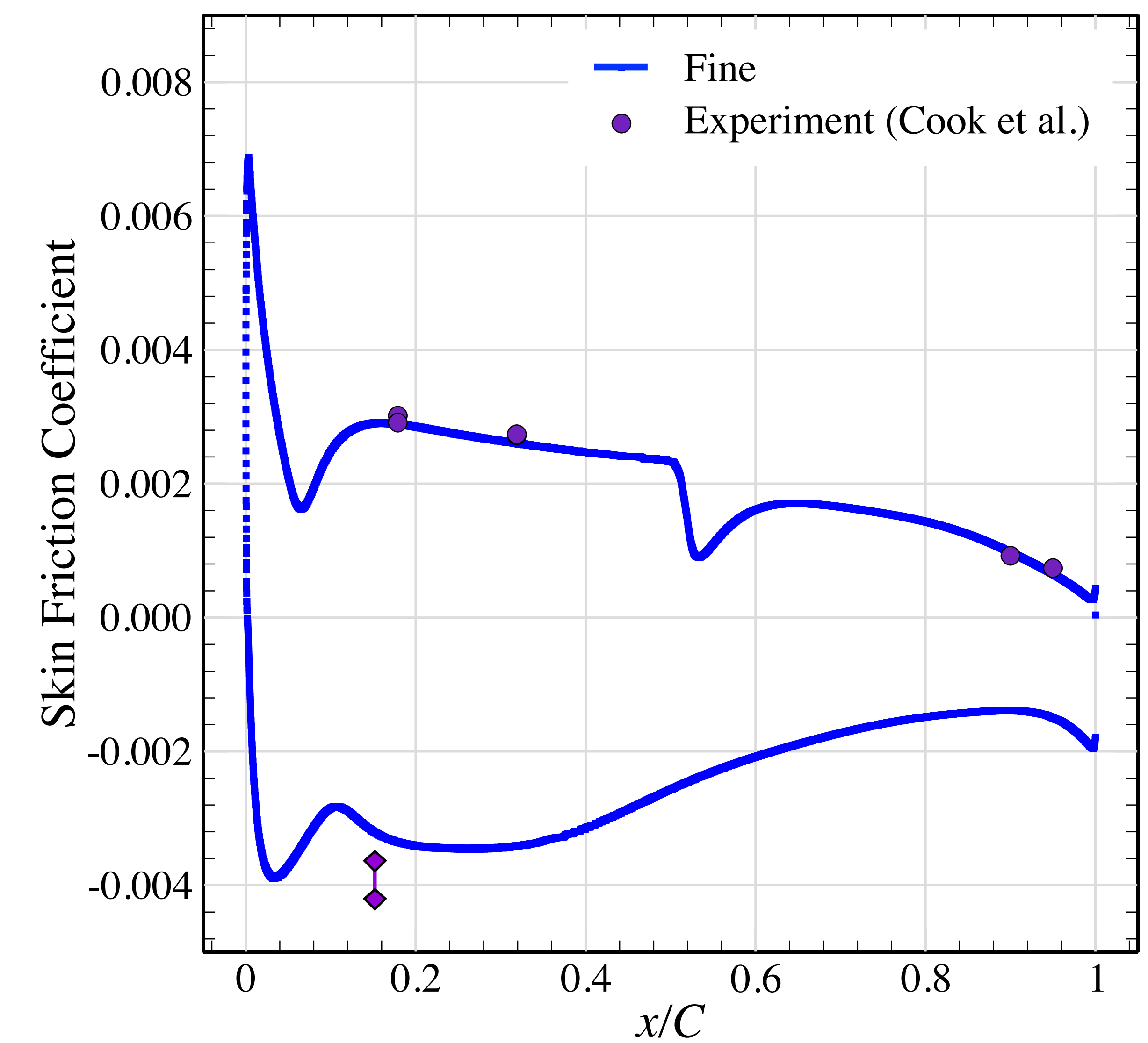


Previous Work

Analytic wall functions







- Pure Cartesian cut-cell approach, solve full RANS everywhere
- Coupled analytic wall functions with cut-cell Cartesian meshes in 2012*
- Results comparable to body-fitted methods using wall functions

Conclusion: *Cartesian RANS is viable, but wall functions alone are probably not sufficient to make the approach cost competitive*

* *AIAA 2012-1301, "Progress Towards a Cartesian Cut-Cell Method for Viscous Compressible Flow", Berger, Aftosmis & Allmaras*

Previous Work

Analytic wall functions

- Thin-layer form of streamwise momentum for RANS eqs.

$$\frac{\partial}{\partial y}\left((\mu + \mu_t)\frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x} + \rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]$$

- The diffusion model assumes that velocity is small and ZPG

$$\frac{\partial}{\partial y}\left((\mu + \mu_t)\frac{\partial u}{\partial y}\right) = 0$$

Previous Work

Analytic wall functions

- Thin-layer form of streamwise momentum for RANS eqs.

$$\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right)=\frac{\partial p}{\partial x}+\rho\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]$$

- The diffusion model assumes that velocity is small and ZPG

$$\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right)=0$$

- Wall functions typically close the system with a mixing-length model for eddy viscosity: $\nu_t \sim$ distance to the wall

$$\mu_t=\rho\nu_t=\rho\kappa\nu y^+$$

Previous Work

Analytic wall functions

- Thin-layer form of streamwise momentum for RANS eqs.

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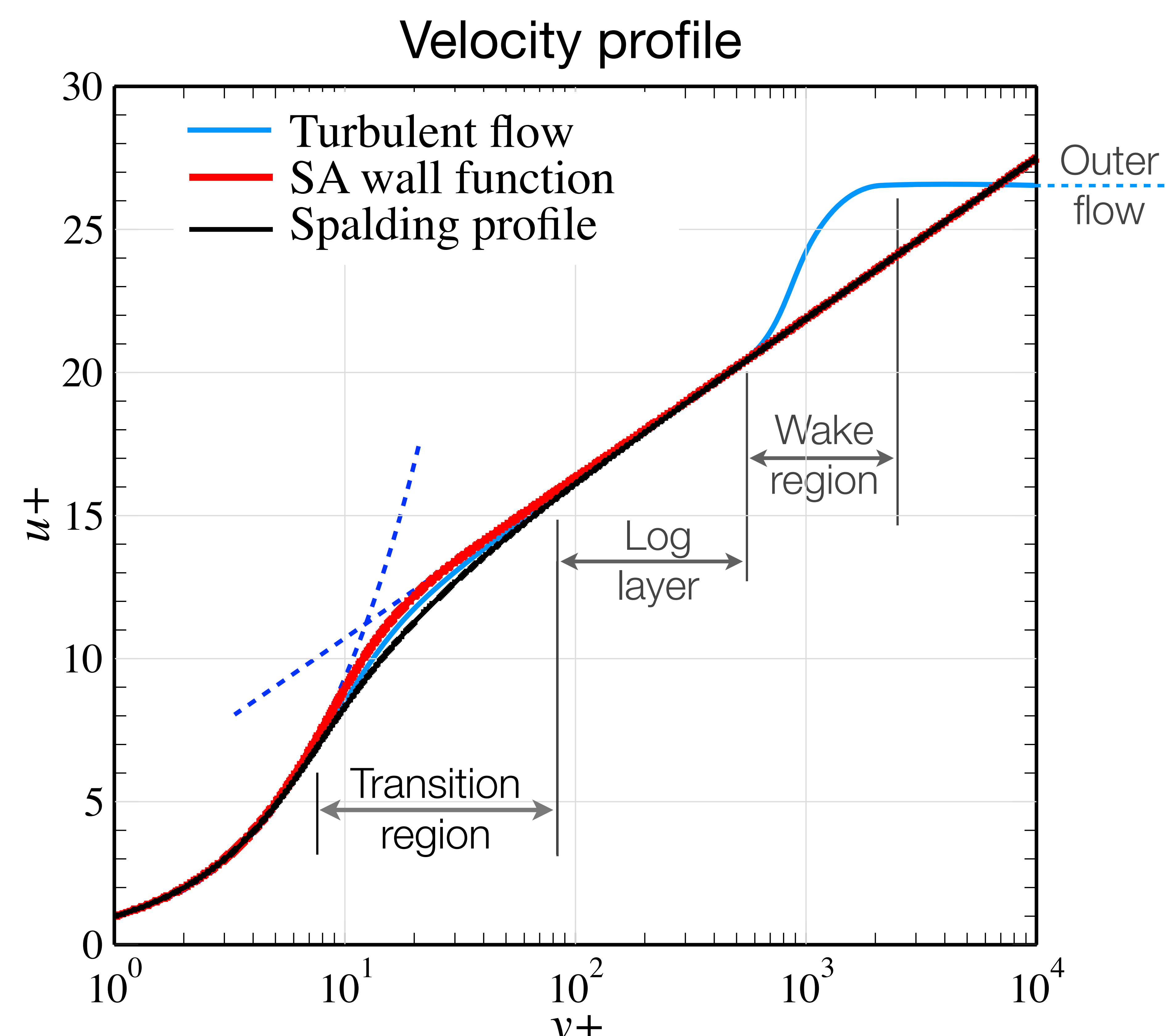
- Wall functions typically close the system with a mixing-length model for eddy viscosity: $\nu_t \sim$ distance to the wall

$$\mu_t=\rho\nu_t=\rho\kappa\nu y^+$$

- Excellent prediction of velocity data up through the log layer



Previous Work

Analytic wall functions

- Spalding model:

$$y^+(u^+) = u^+ + e^{-\kappa B}\left(e^{\kappa u^+} - 1 - \kappa u^+ - \frac{1}{2}(\kappa u^+)^2 - \frac{1}{6}(\kappa u^+)^3\right)$$

- SA wall function (2012[†]):

  Derived using a limiting form of SA turbulence model and integrating the diffusion model

$$\begin{aligned} u^+(y^+) \;=\; \bar{B} \;+\; & c_1 \log((y^+ + a_1)^2 + b_1^2) \;-\; c_2 \log((y^+ + a_2)^2 + b_2^2) \\ -\; & c_3 \,\mathrm{arctan2}(y^+ + a_1, b_1) \;-\; c_4 \,\mathrm{arctan2}(y^+ + a_2, b_2) \end{aligned}$$

- Prefer SA wall function, since it gives direct relationship for velocity as a function of distance

- Knowing u at a point F, iterate to find u_τ, so that $u^+(y_F^+) = u_F^+ = u_\tau u_F$

[†] *AIAA 2012-1301, "Progress Towards a Cartesian Cut-Cell Method for Viscous Compressible Flow", Berger, Aftosmis & Allmaras*

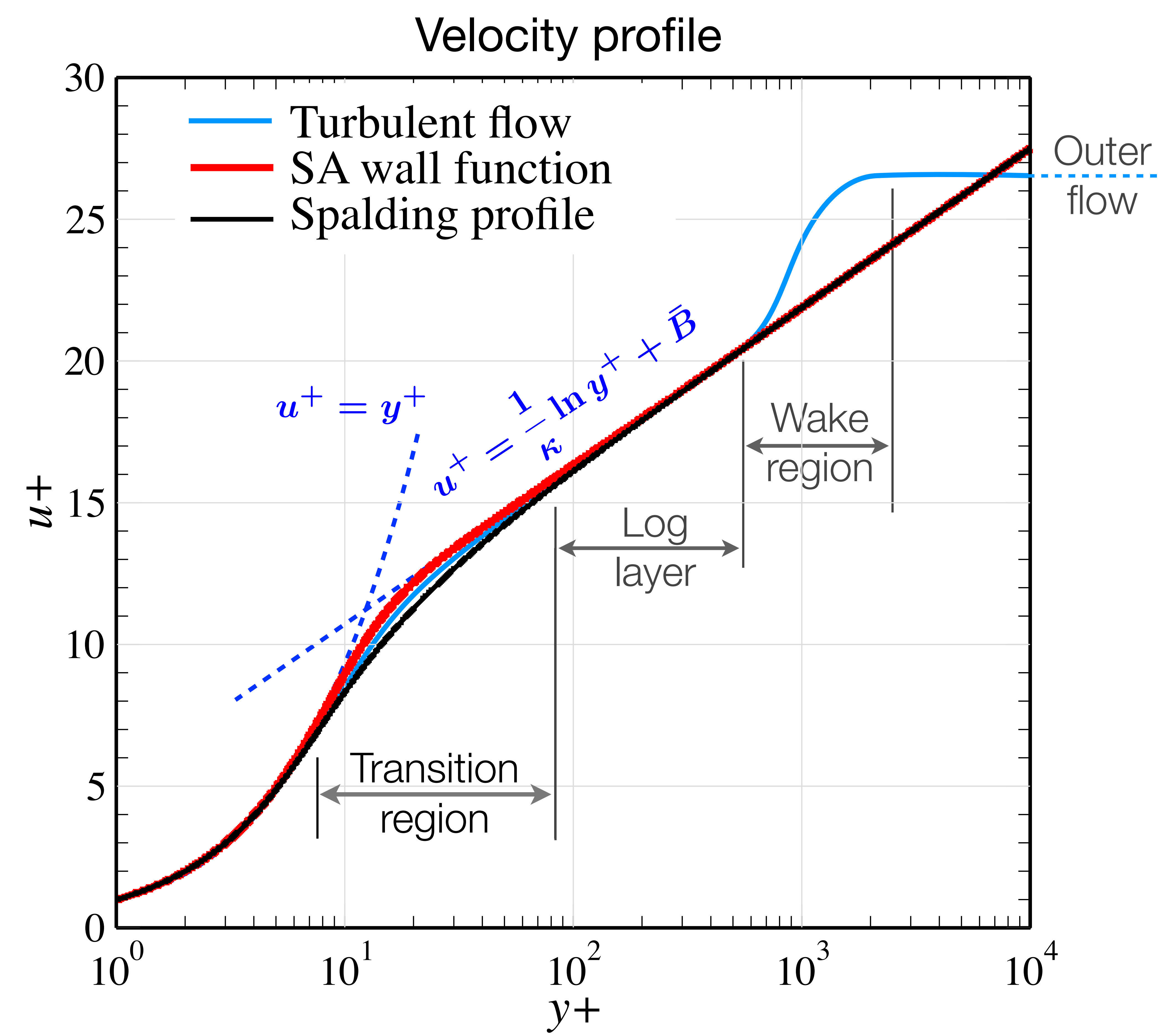
Previous Work

Analytic wall functions

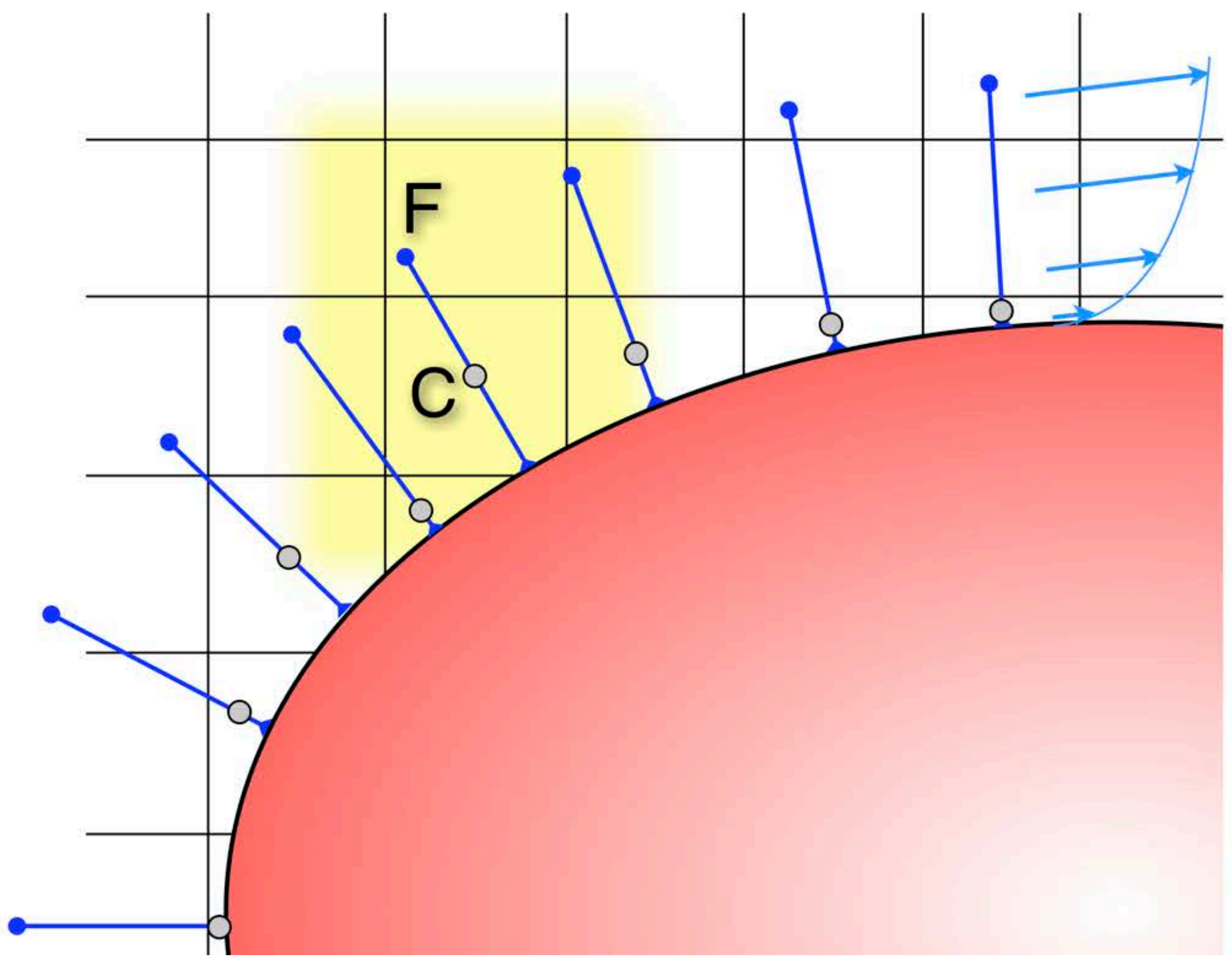
- Spalding model: $y^+(u^+) = \ldots$
- SA wall function (2012): $u^+(y^+) = \ldots$

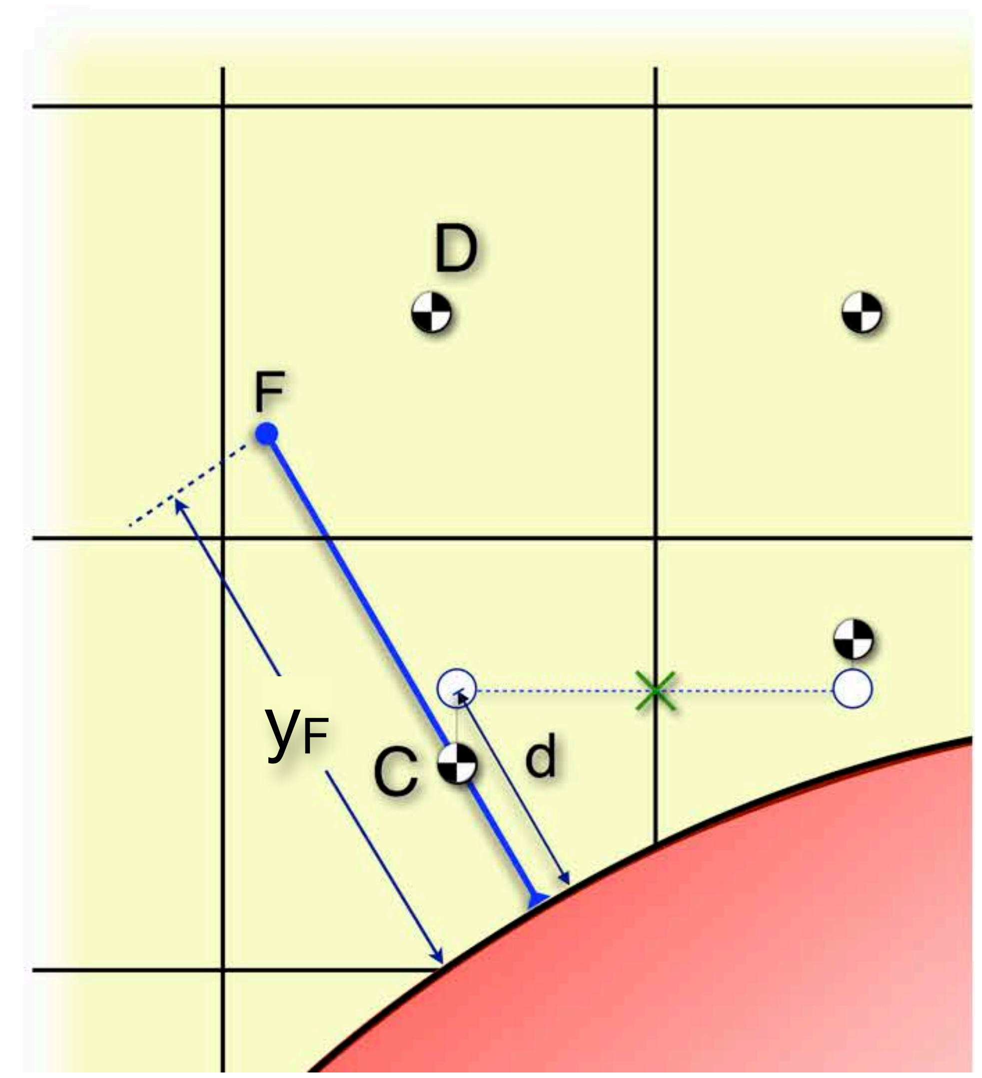
Both:

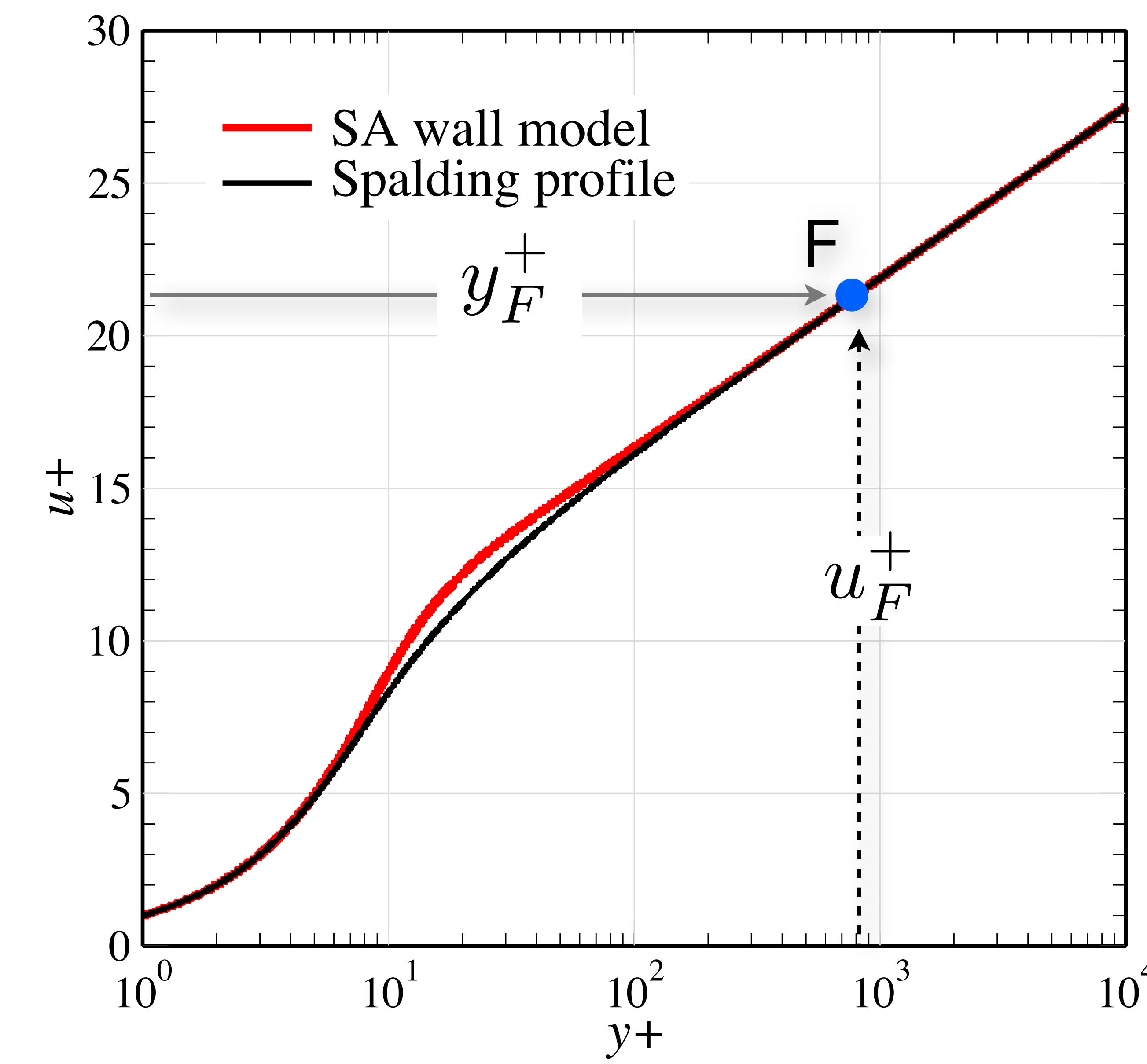
- Are good approximations and give accurate wall shear stress when anchored to the log-layer
- Are inappropriate beyond the log layer
  - Predict exponentially increasing velocity
  - Don't consider pressure gradients
  - Ignore the wake → don't match outer flow



Coupling and Forcing Point Construction





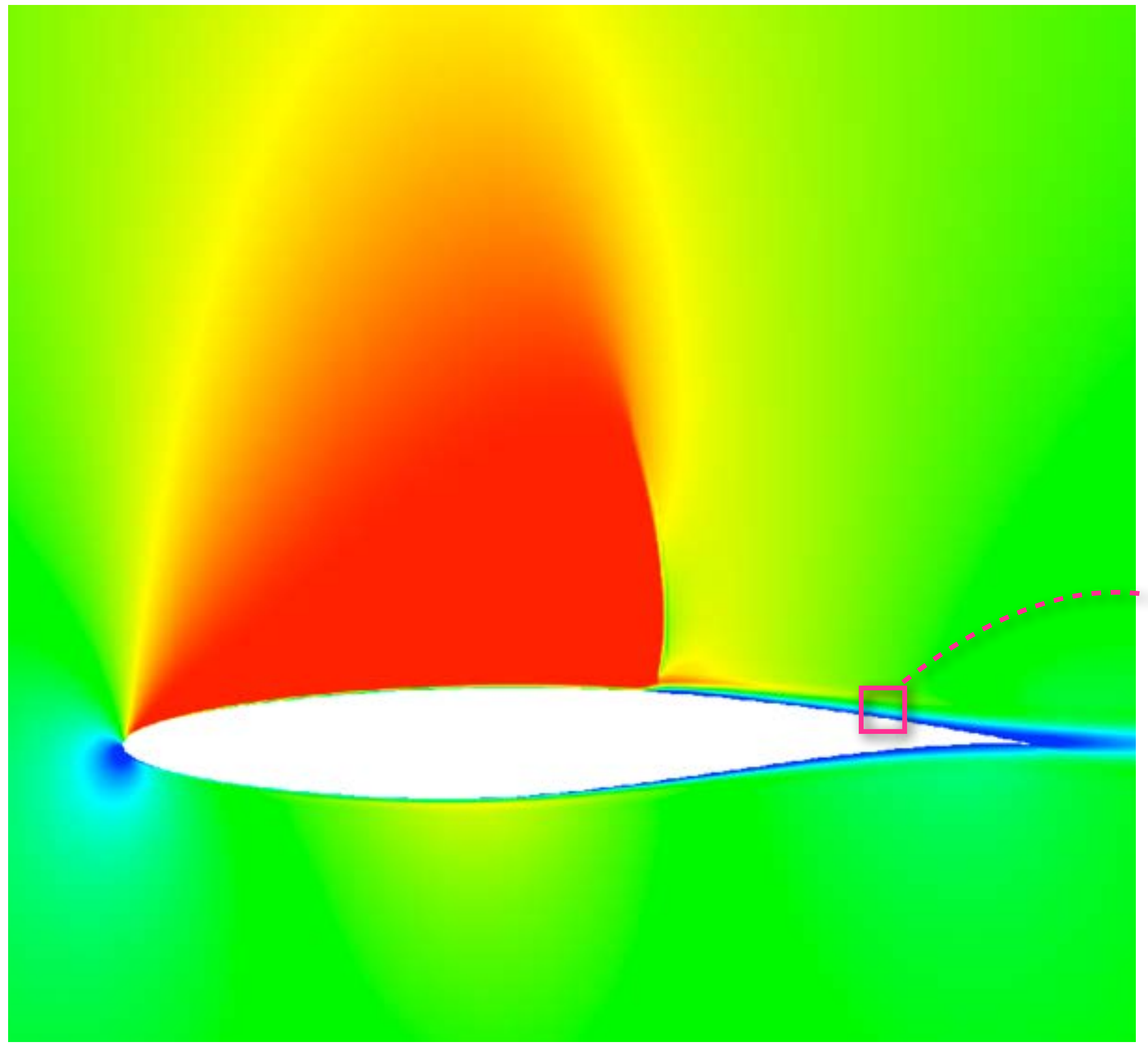


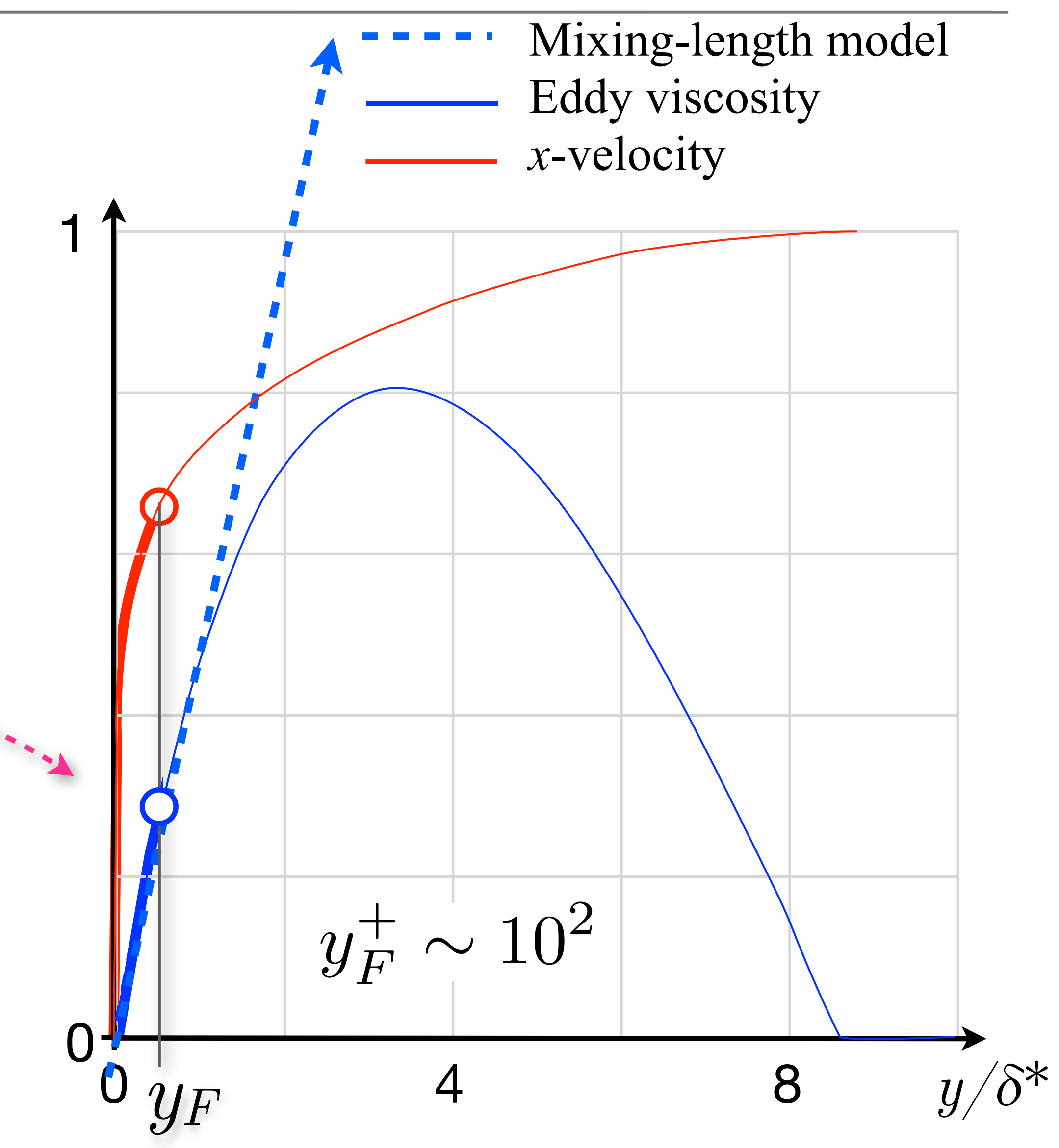
- Construct forcing points at uniform distance from wall
- Interpolate data to point *F* from cell centered solution on outer grid
- With velocity & distance at forcing point, *F*, use wall function to find u_τ and wall shear, $\tau_{\text{wall}} = \rho u_\tau^2$
- Feed back to outer meshes via viscous fluxes in the cut cells

Good wall functions gone bad

Thick boundary layer

- $y_F^+ \approx 10^2$
- Forcing point in log layer
- Mixing length model gives good estimate eddy viscosity
- Analytic wall function is appropriate





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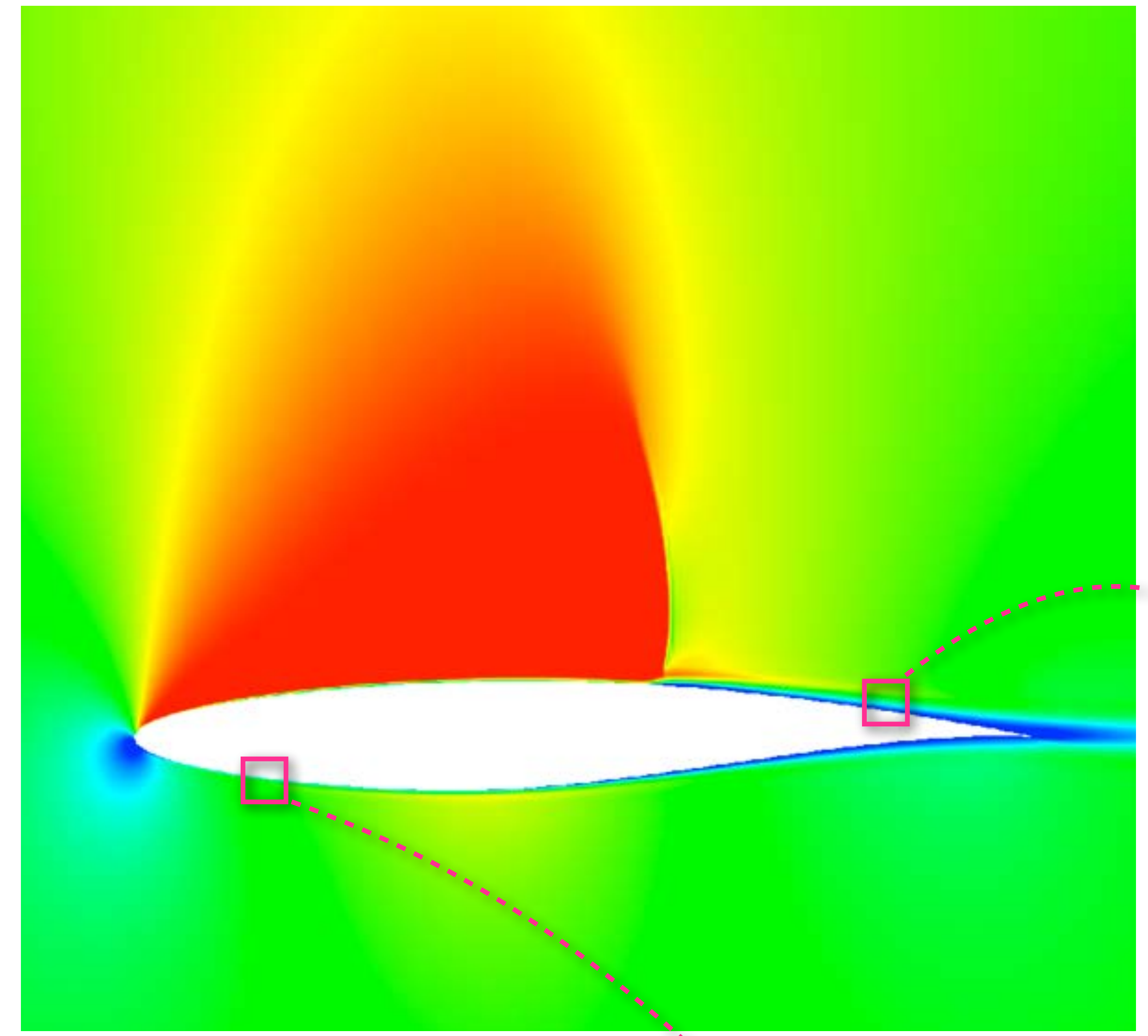
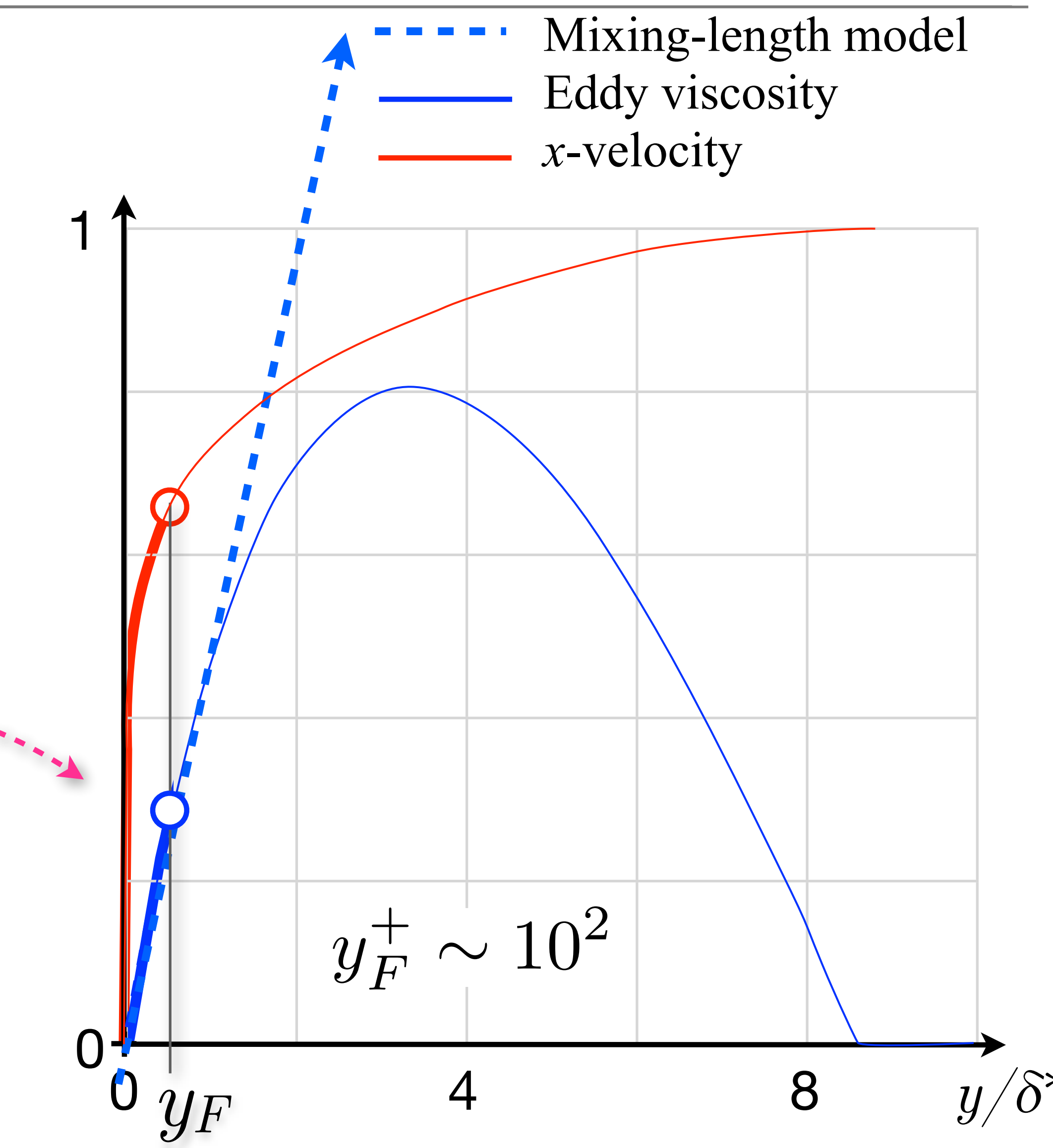
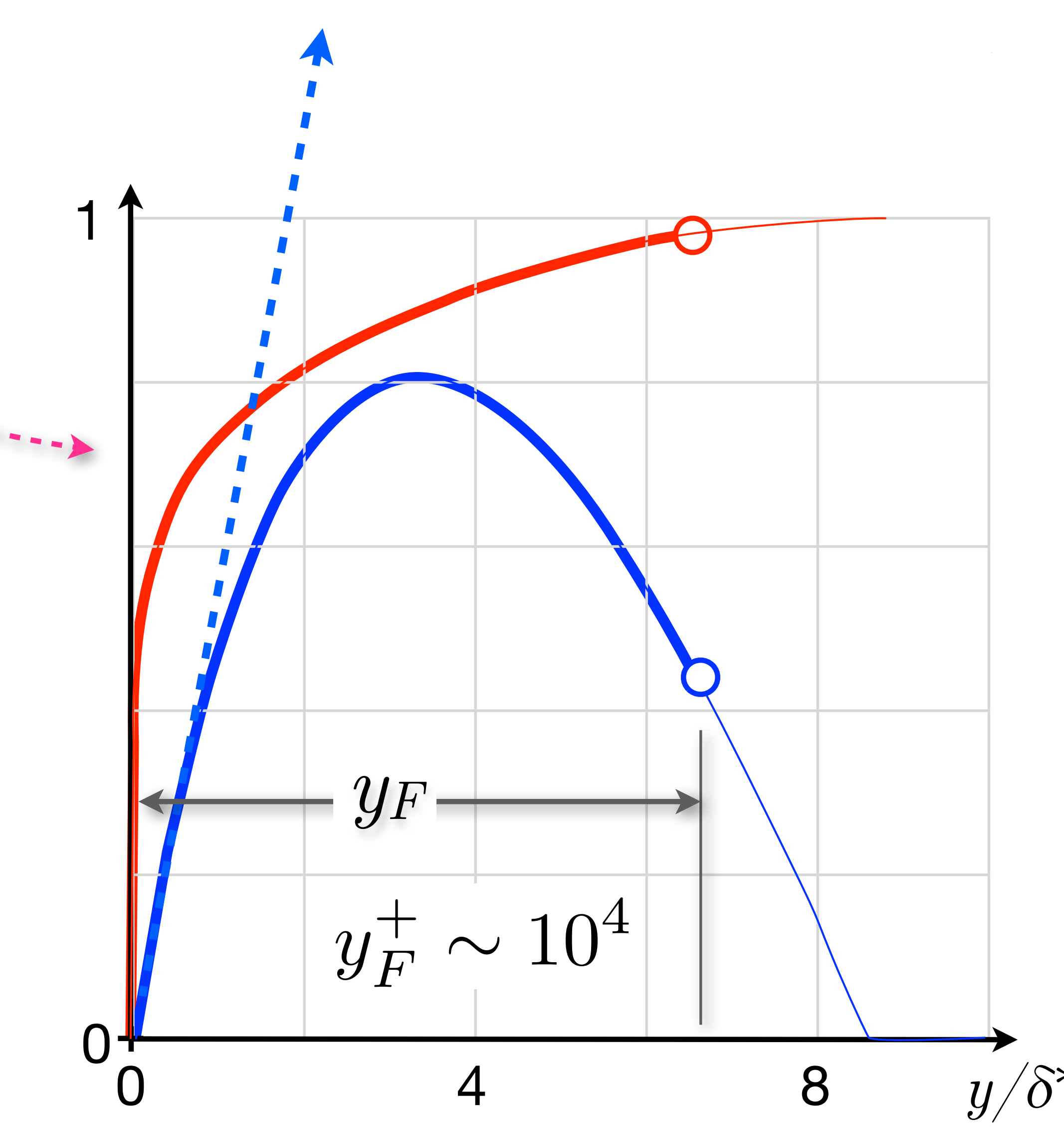




Thin boundary layer

- At the same distance from the wall, $y_F^+ \approx 10^4$
- Forcing point is now in the wake layer
- Eddy viscosity highly non-linear
- Mixing length model is a poor approximation, analytic wall function inappropriate



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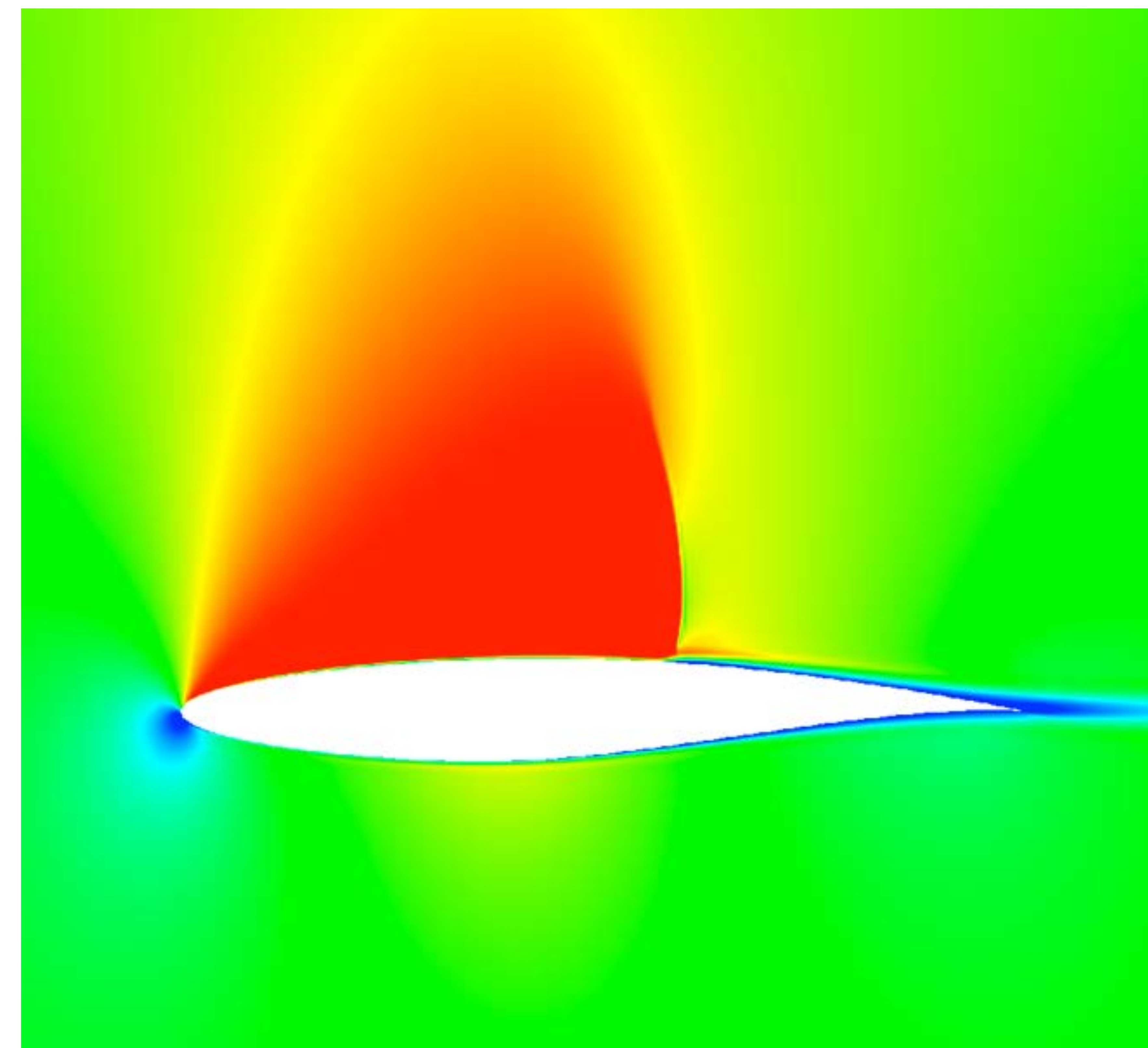
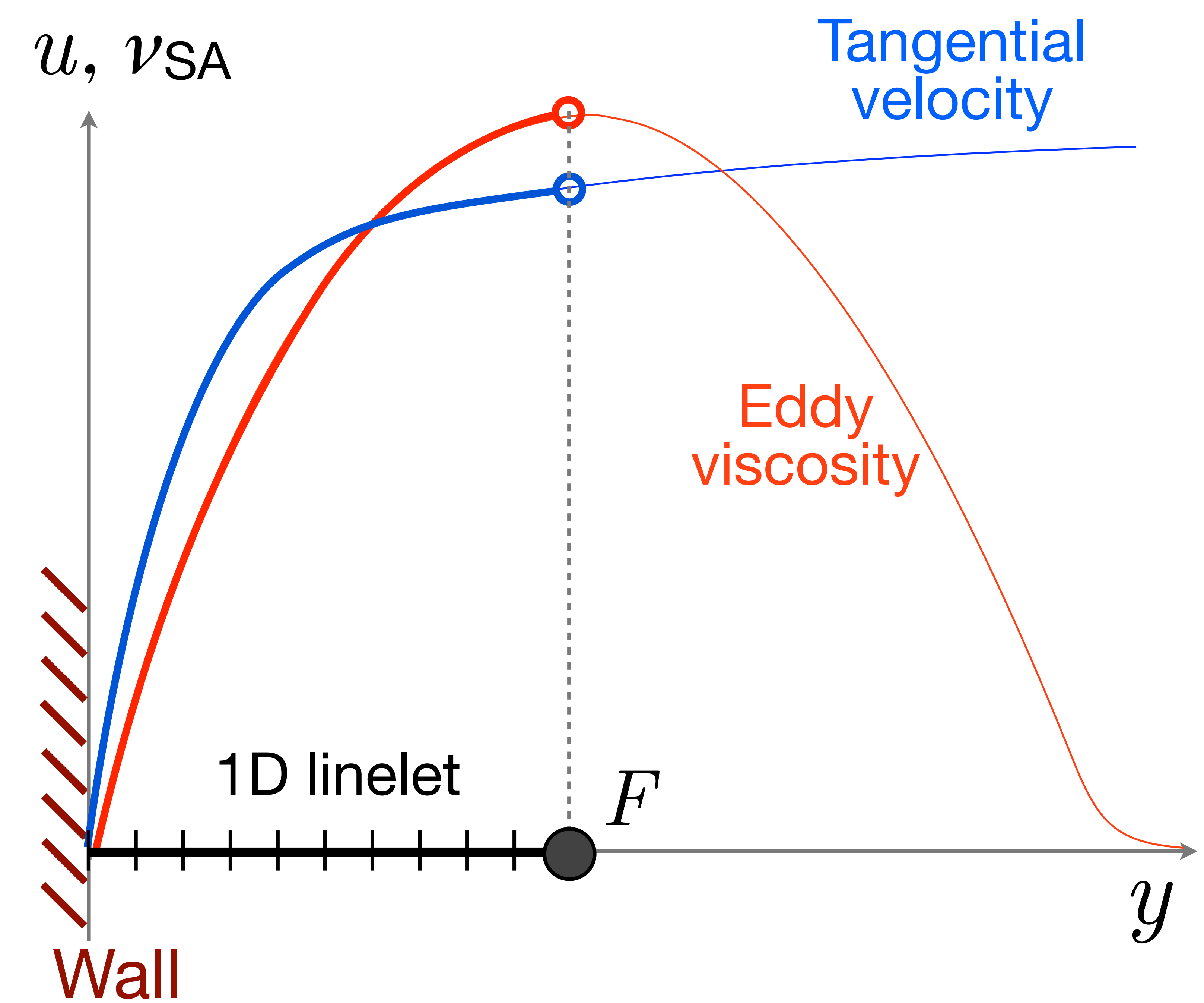


ODE-Based Wall Models

Proposed by several authors* in last decade

- Solve ODE on 1D "linelet" normal to surface
- Solve:
  - Diffusion eq. for streamwise momentum
  - Turbulence model in wall-normal direction
- Produces a system of 2-point, 2nd-order BVPs
- Coupling: *Just like an analytic wall function*



* see: *Kalitzin et al., J. Comp. Phys.,* ***204****, 2005,   Bond & Blottner, Intl. J. Num. Methods Fluids,* ***66****, 2011,   or Capizzano, AIAA J.* ***54****(2), 2016*

ODE-Based Wall Models

SA-BVP: Diffusion equation coupled with wall-normal SA turbulence model

$x-$momentum:
(diffusion equation)

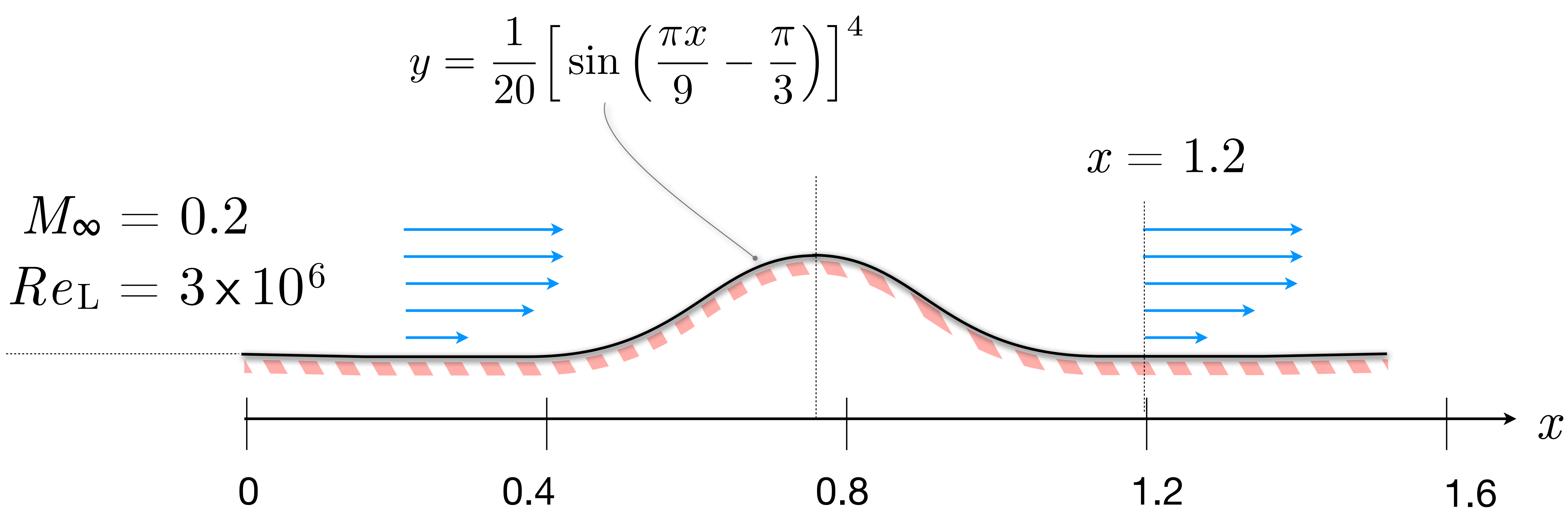
$$\frac{\partial}{\partial y}\left((\mu + \mu_t)\frac{\partial u}{\partial y}\right) = 0$$

SA model on linelet:

$$\frac{\partial}{\partial y}\left((\nu + \widetilde{\nu})\ \frac{\partial \widetilde{\nu}}{\partial y}\right) = \underbrace{-c_{b2}\left(\frac{\partial \widetilde{\nu}}{\partial y}\right)^2}_{\text{wall-normal diffusion}} + \text{Production} - \text{Destruction}$$

ODE-Based Wall Models

Compare SA-BVP with SA wall function on turbulent bump in channel

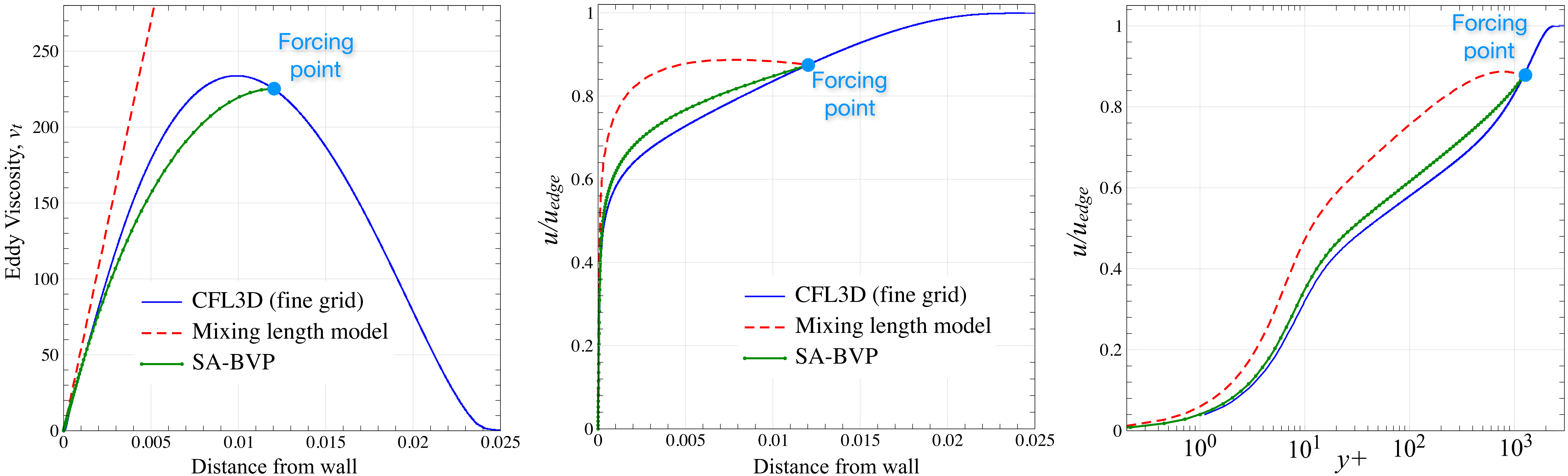


x–momentum: $$\frac{\partial}{\partial y}\left((\mu + \mu_t)\frac{\partial u}{\partial y}\right) = 0$$

SA model on linelet: $$\frac{\partial}{\partial y}\left((\nu + \widetilde{\nu})\ \frac{\partial \widetilde{\nu}}{\partial y}\right) = -c_{b2}\left(\frac{\partial \widetilde{\nu}}{\partial y}\right)^2 + \text{Production} - \text{Destruction}$$

ODE-Based Wall Models

Compare SA-BVP with SA wall function on turbulent bump in channel @ $x = 1.2$



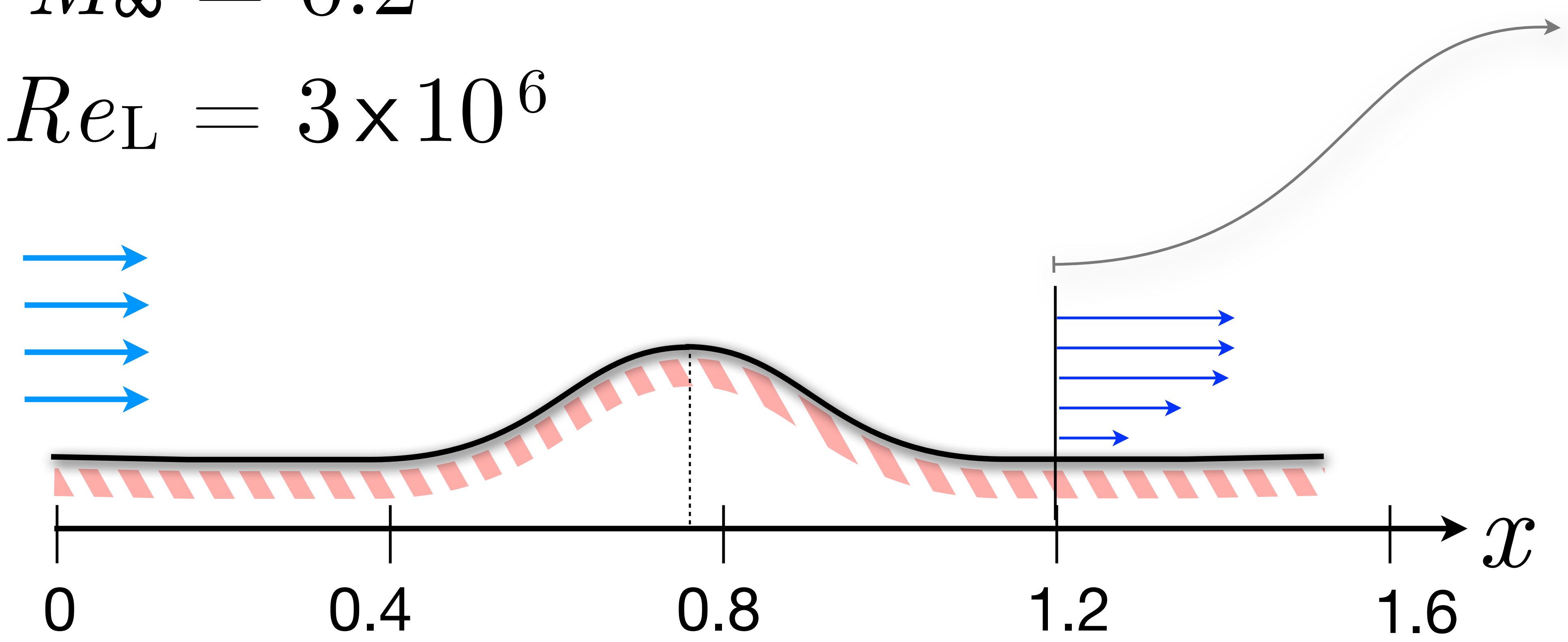
- Forcing point well out in wake layer, $y = 0.012$, $u = 0.85 u_{\text{edge}}$
- Mixing length eddy viscosity inappropriate, so diffusion model alone does poorly
- Improved eddy viscosity makes a significant difference

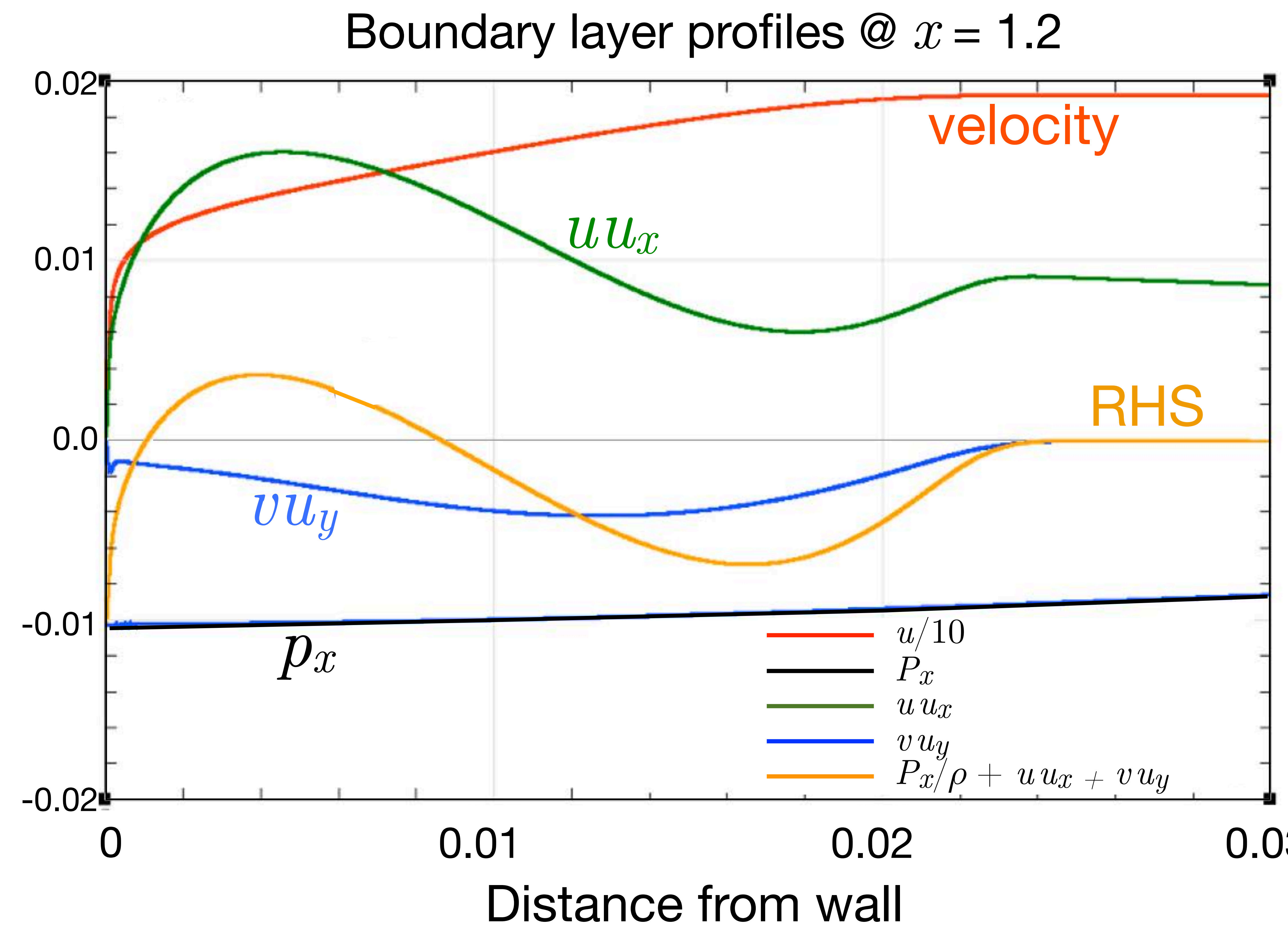
ODE-Based Wall Models

Streamwise momentum in the wake

$$M_\infty = 0.2$$

$$Re_L = 3\times10^6$$





- Thin layer streamwise momentum: $((\mu + \mu_t)u_y)_y = p_x + \rho(uu_x + vu_y)$
- Examine magnitude of terms as we move away from the wall

ODE-Based Wall Models

Streamwise momentum in the wake

- At the wall, we have

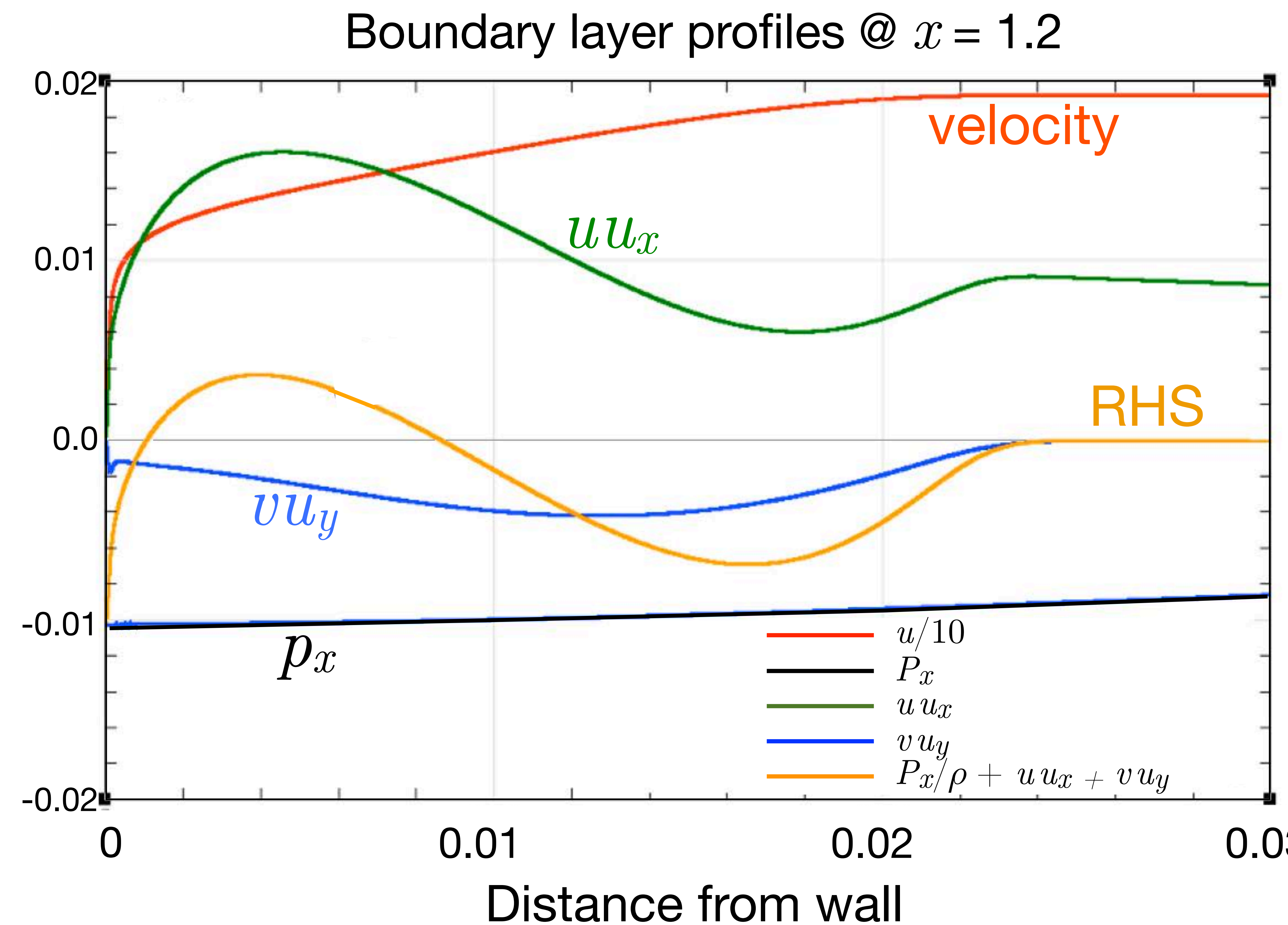
$$((\mu + \mu_t)u_y)_y = p_x$$

- Outside the boundary layer we approach:

$$0 = p_x + \rho(uu_x + vu_y)$$

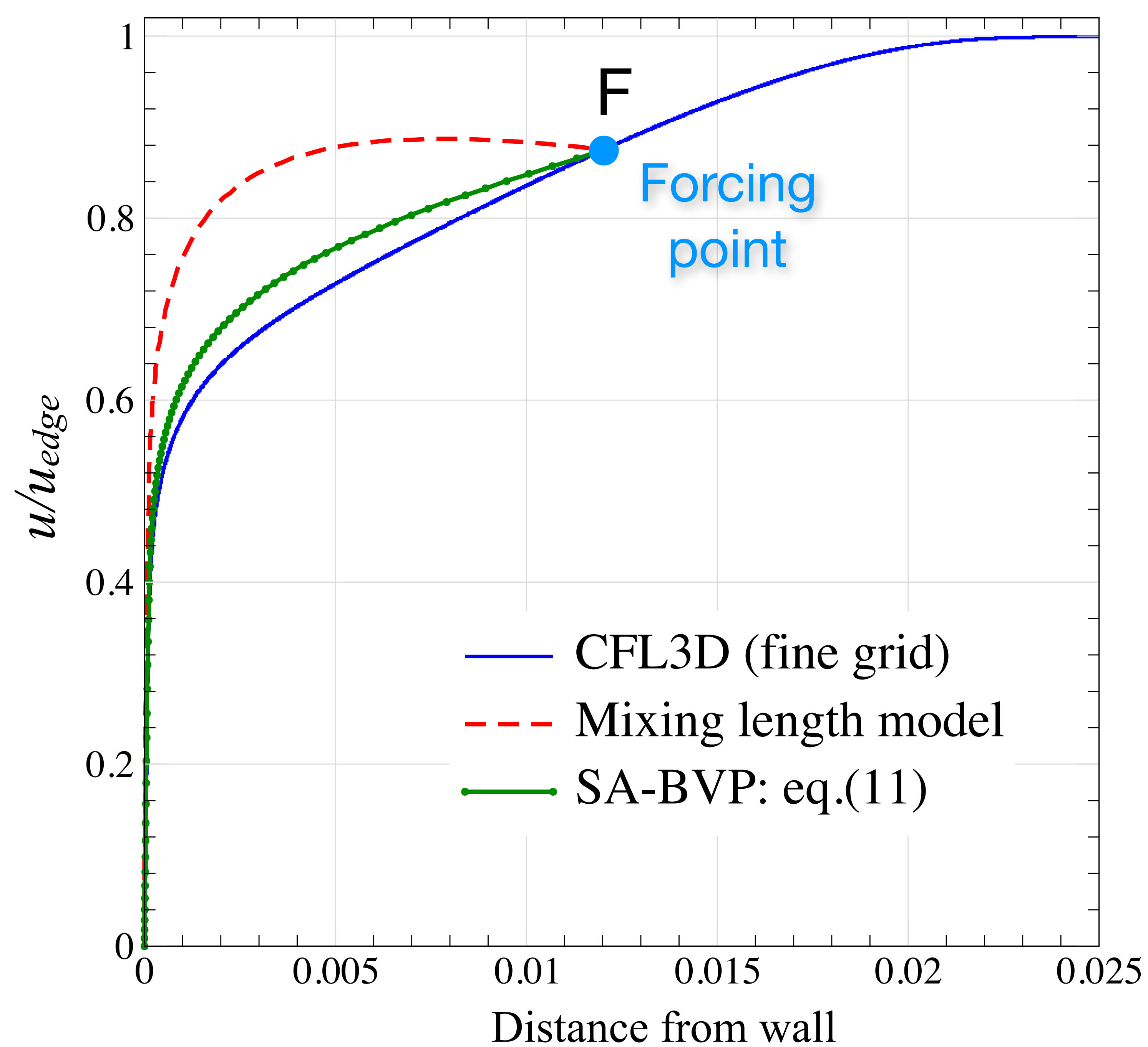
- In between we have the full streamwise momentum eq.

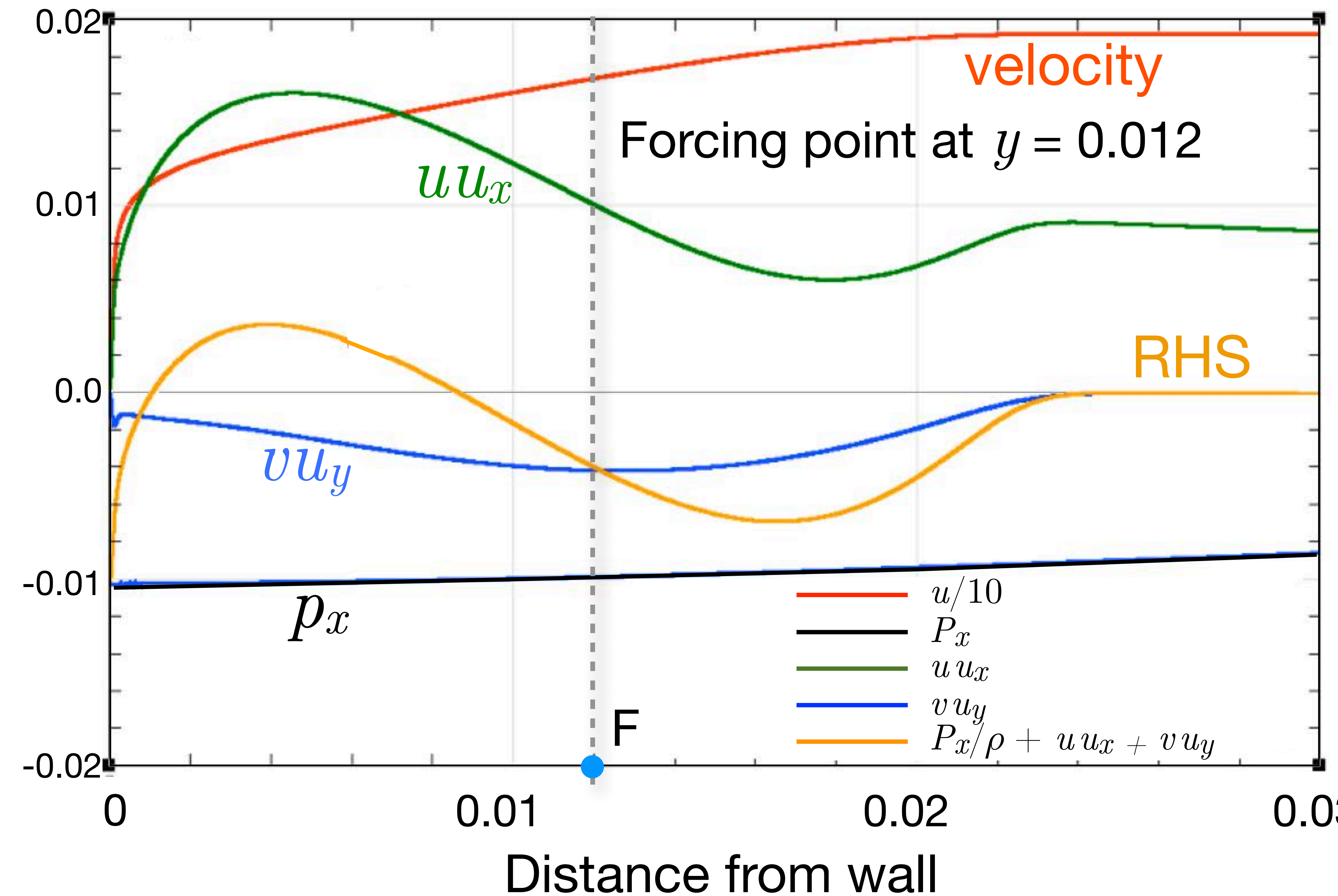
$$((\mu + \mu_t)u_y)_y = p_x + \rho(uu_x + vu_y)$$



ODE-Based Wall Models

Streamwise momentum in the wake





- Forcing point @ $y = 0.012$ is in the wake. The convective balance has similar magnitude as p_x ➛ Need to include!

$$((\mu + \mu_t)u_y)_y = p_x + \rho(uu_x + vu_y)$$

A New ODE-based Wall Model

The *bvp4* wall model

- The complete *bvp4* model becomes

$$\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right)=\frac{\partial p}{\partial x}+\psi(y)\rho\left[u_F\frac{\partial u}{\partial x}\bigg|_F+v_F\frac{\partial u}{\partial y}\bigg|_F\right]$$

$$\frac{\partial}{\partial y}\left((\nu+\widetilde{\nu})\ \frac{\partial\widetilde{\nu}}{\partial y}\right)=\text{wall-normal diffusion}+\text{Production}-\text{Destruction}$$

- 1D ODE's for streamwise momentum & SA turbulence model
- Includes pressure gradient and model for the streamwise convective balance

ODE-Based Wall Models

Including the convective balance

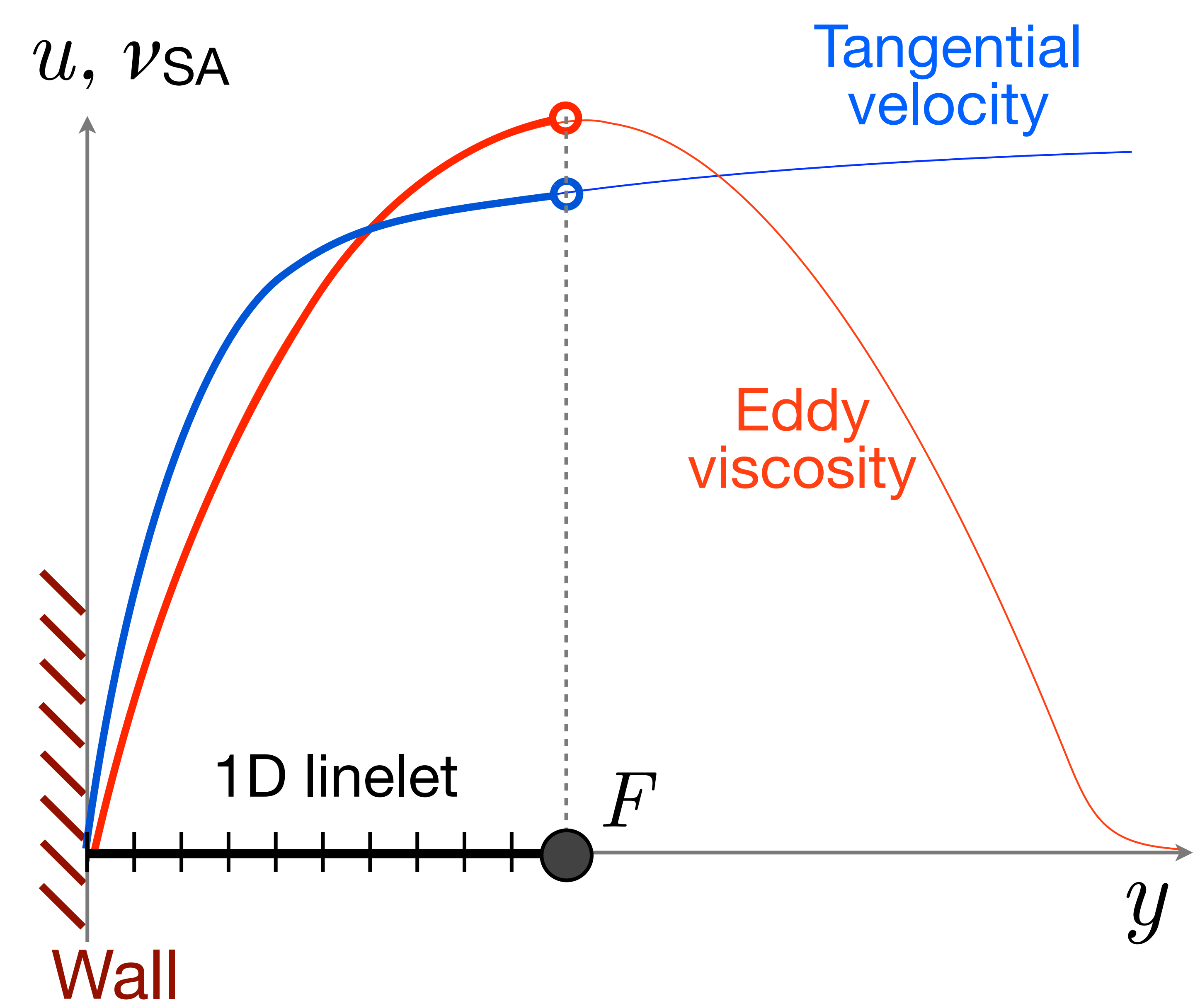
- At the wall, velocity is zero, $\therefore$ convective balance is zero

$$((\mu + \mu_t) u_y)_y = p_x$$

- But at the forcing point, it has the same magnitude as p_x

$$((\mu + \mu_t) u_y)_y = p_x + \rho(u u_x + v u_y)\big|_F$$

- Computing wall-normal variation of convective balance introduces streamwise coupling, and means computing the wall-normal velocity & interpolating derivatives
  – *prefer not to do this*



- Instead, we *choose* to model the wall normal variation of the convective balance
  – keeps the stencil local
  – well behaved on irregular meshes

ODE-Based Wall Models

Including the convective balance

- Introduce a cutoff function, $\psi(y)$, to shut down the convective balance approaching the wall

$$((\mu + \mu_t)u_y)_y = p_x + \psi(y)\left[\rho(uu_x + vu_y)\right]_F$$

- Require: $\psi(0) = 0$, and $\psi(y_F) = 1$

  Desire: Scales like velocity, since through the log-layer the convective balance roughly follows velocity

ODE-Based Wall Models

Including the convective balance

- Introduce a cutoff function, $\psi(y)$, to shut down the convective balance approaching the wall

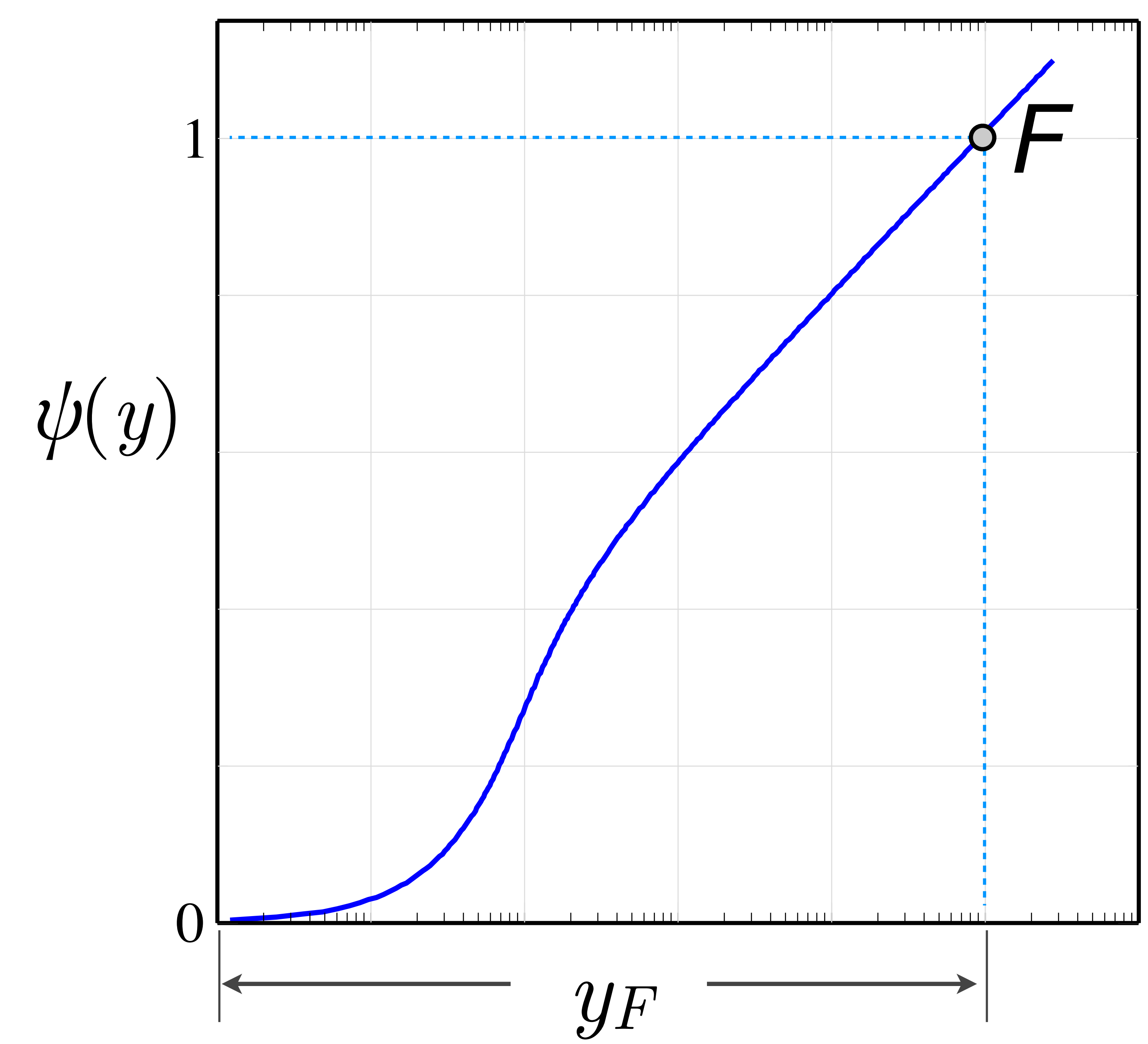
$$((\mu + \mu_t)u_y)_y = p_x + \psi(y)\left[\rho(uu_x + vu_y)\right]_F$$

- Require: $\psi(0) = 0$, and $\psi(y_F) = 1$

  Desire: Scales like velocity, since through the log-layer the convective balance roughly follows velocity

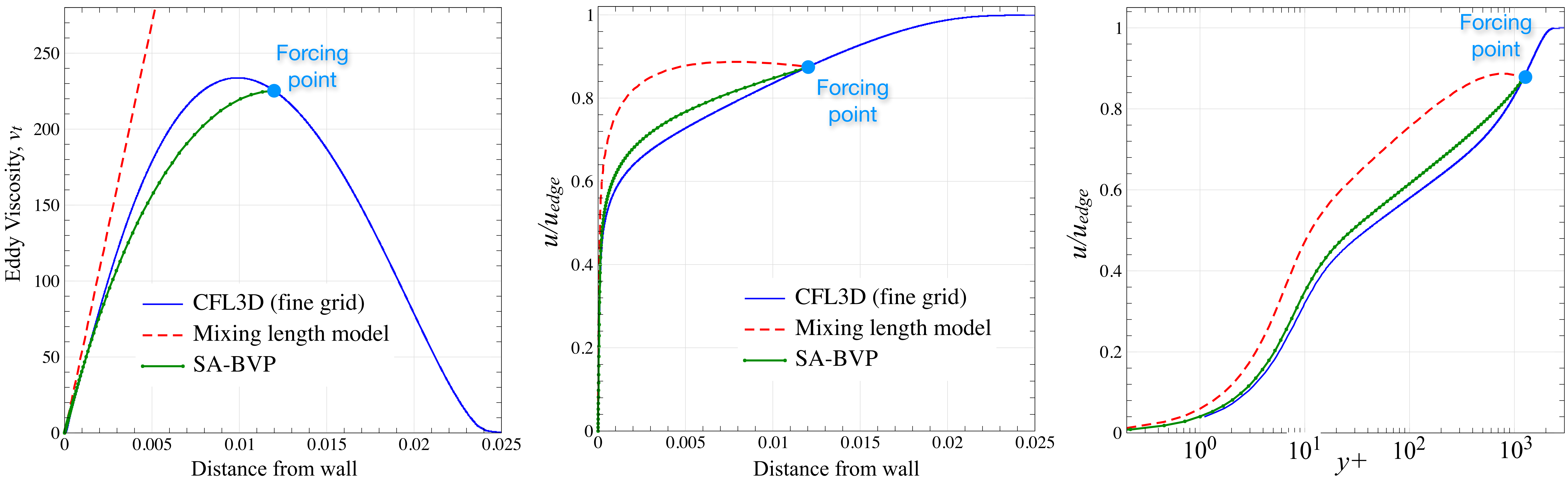
- Choose: $\psi(y) = \dfrac{u_{SA}(y)}{u_{SA\,F}} = \dfrac{u^+_{SA}(y)}{u^+_{SA\,F}}$

- Blends momentum in exponentially in log region and linearly in laminar sublayer, and gives full convective balance at the forcing point, *F*



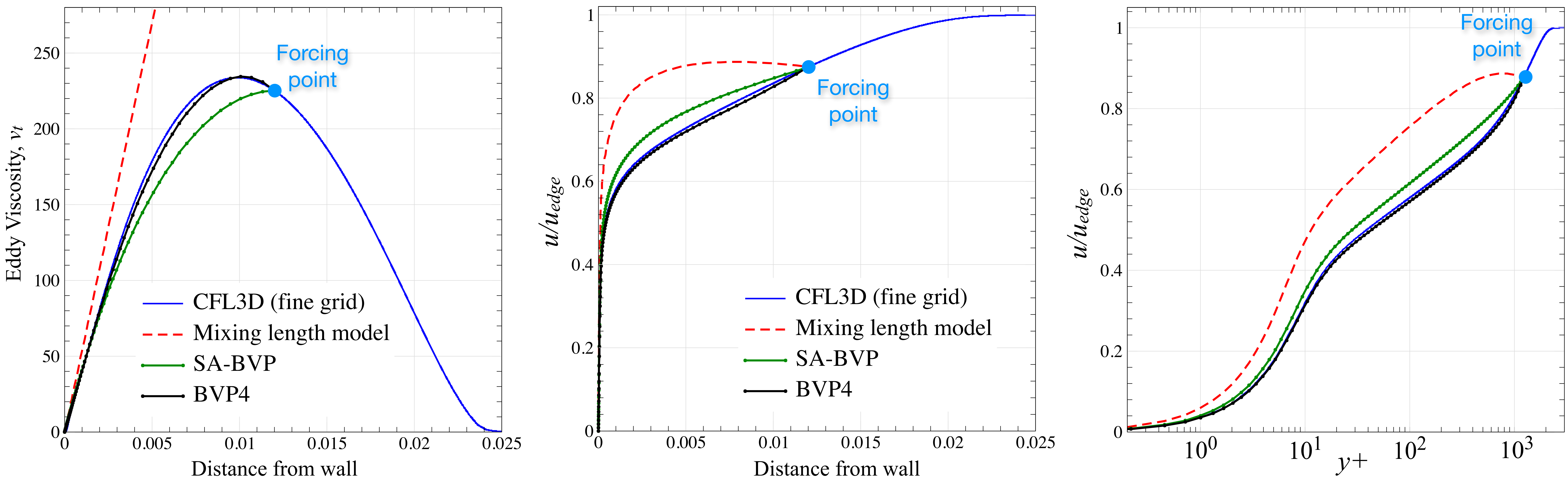
A New ODE-based Wall Model

bvp4 wall model: Include streamwise convective balance and pressure gradient



A New ODE-based Wall Model

bvp4 wall model: Include streamwise convective balance and pressure gradient



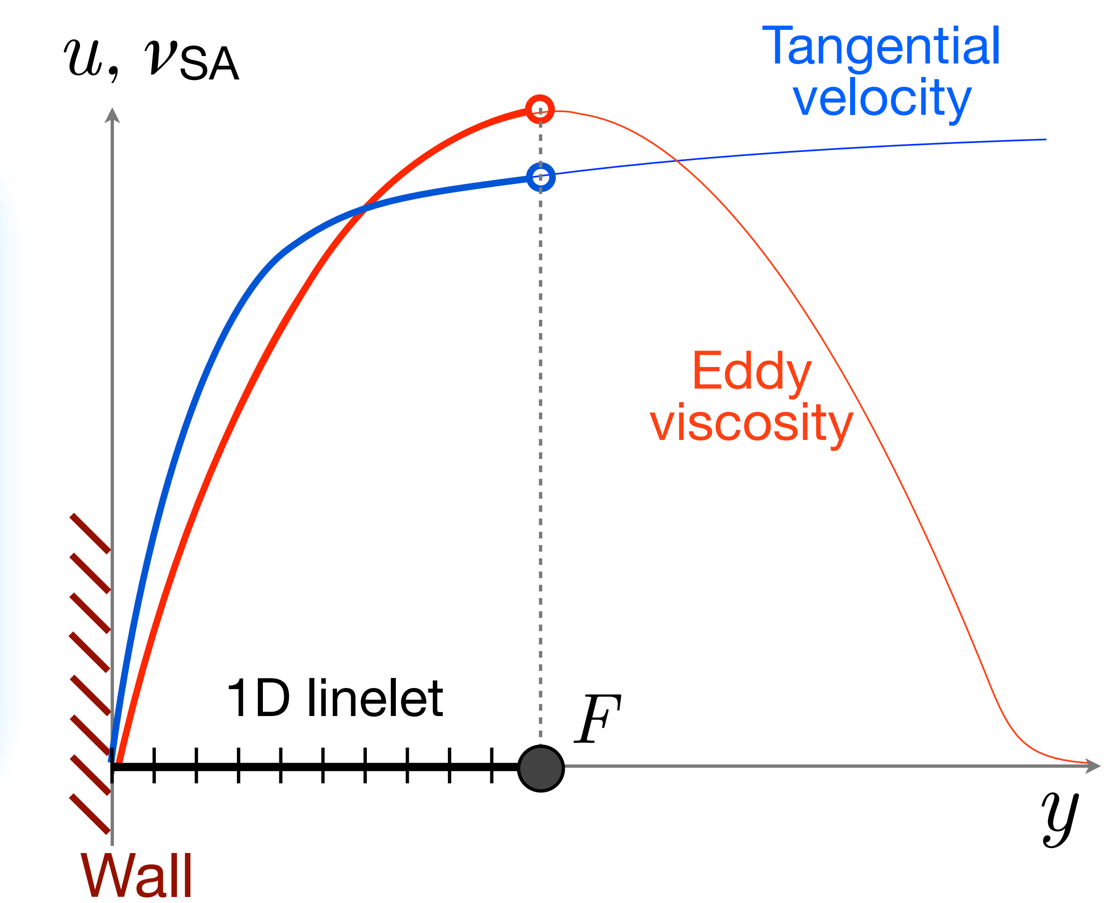
- Very good approximation of both eddy viscosity & tangential velocity
- Forcing point is *very* far from wall (~1% c), $u_F \approx$ 90% of edge velocity
- Around 10x farther from the wall than analytic wall functions
- Include pressure gradient for accurate wall shear stress when p_x is non-zero

A New ODE-based Wall Model

ODE solver for ***bvp4***

$$\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x} + \psi(y)\rho\left[u_F\frac{\partial u}{\partial x}\bigg|_F + v_F\frac{\partial u}{\partial y}\bigg|_F\right]$$

$$\frac{\partial}{\partial y}\left((\nu+\widetilde{\nu})\,\frac{\partial\widetilde{\nu}}{\partial y}\right) = \text{wall-normal diffusion} + \text{Production} - \text{Destruction}$$



- Reformulate 2nd-order equations as system of four 1st-order BVPs
- Newton solves, using 6th-order adaptive ODE solver from Shampine and Muir
- Use warm starts on each linelet after initial solve ~2x cost of analytic WF
- Other details of implementation and coupling in AIAA 2017-0528

Model Recap

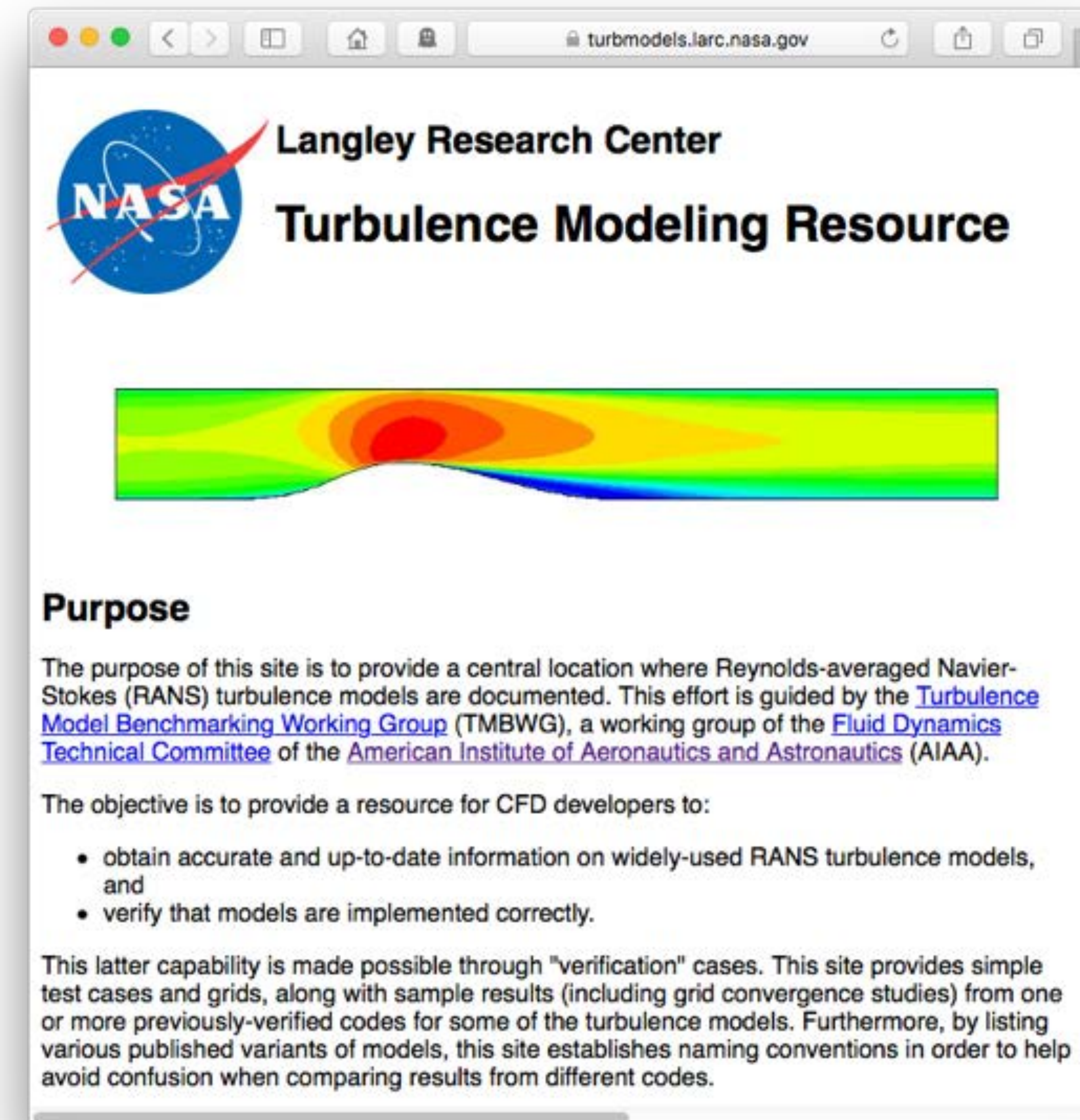
- **SA wall function:** analytic, diffusion eq + mixing length model
- **SA-BVP:** ODE solve, diffusion eq. + SA turbulence model
- *bvp4*: ODE solve, streamwise momentum + SA turbulence model

Numerical Results

Verification and Validation using examples from the NASA Turbulence Modeling Resource

Computational Examples from TMR

1. Turbulent bump in channel
2. NACA 0012
3. NACA 4412 with trailing edge separation



Goals

- Perform basic V & V
- Look at mesh convergence
- Examine model behavior and the role of various terms in real flows

Complete mesh convergence studies in the paper – just include highlights here

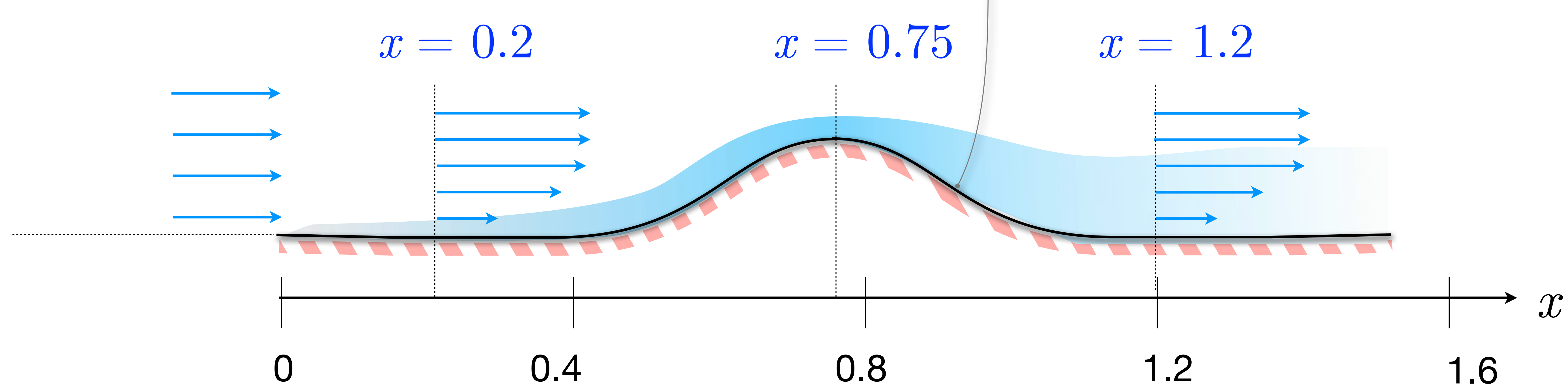
Turbulent Bump In Channel

TMR: “VERIF/2DB: 2D Bump-in-channel Verification Case”

$$y = \frac{1}{20}\left[\sin\left(\frac{\pi x}{9} - \frac{\pi}{3}\right)\right]^4$$

$$M_\infty = 0.2$$

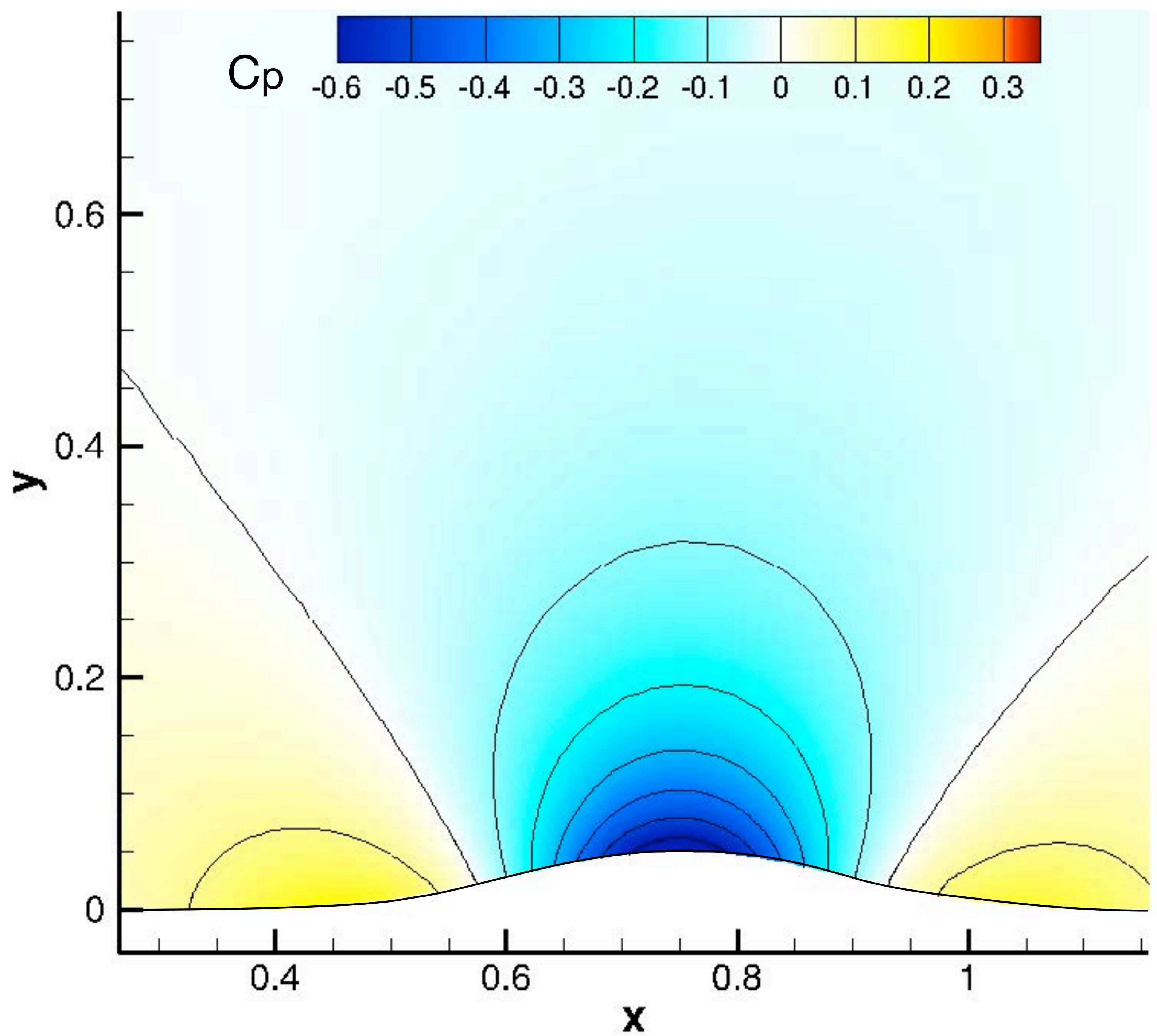
$$Re_L = 3 \times 10^6$$

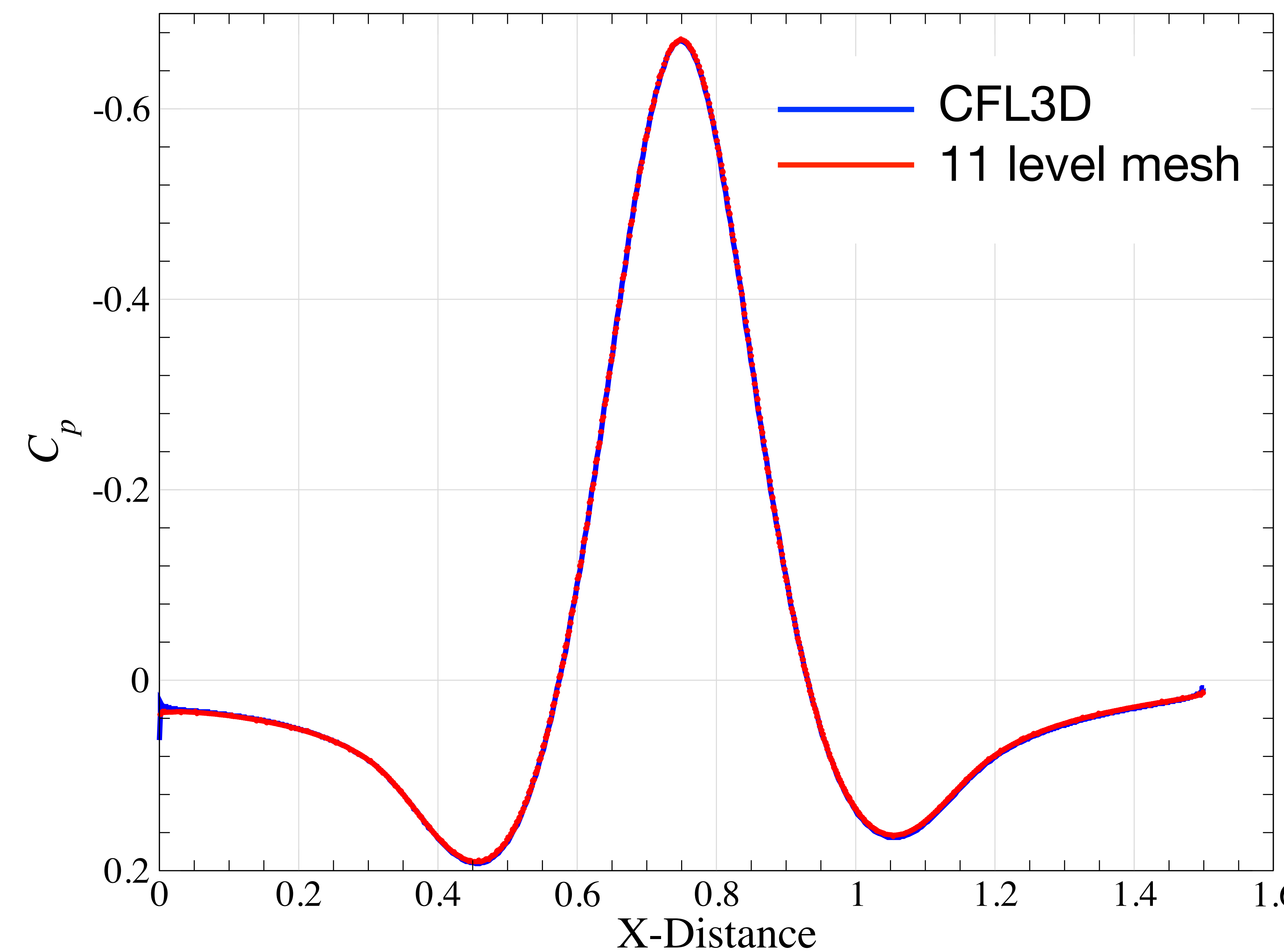


- Inlet & exit 25 units away, symmetry plane 5 units above
- Mesh-converged body-fitted results on 1409 x 641 mesh (~900 k points)
- Compare results with CFL3D reference solution with SA turbulence model on finest mesh

Turbulent Bump In Channel

Bump: Isobars and surface pressure comparison

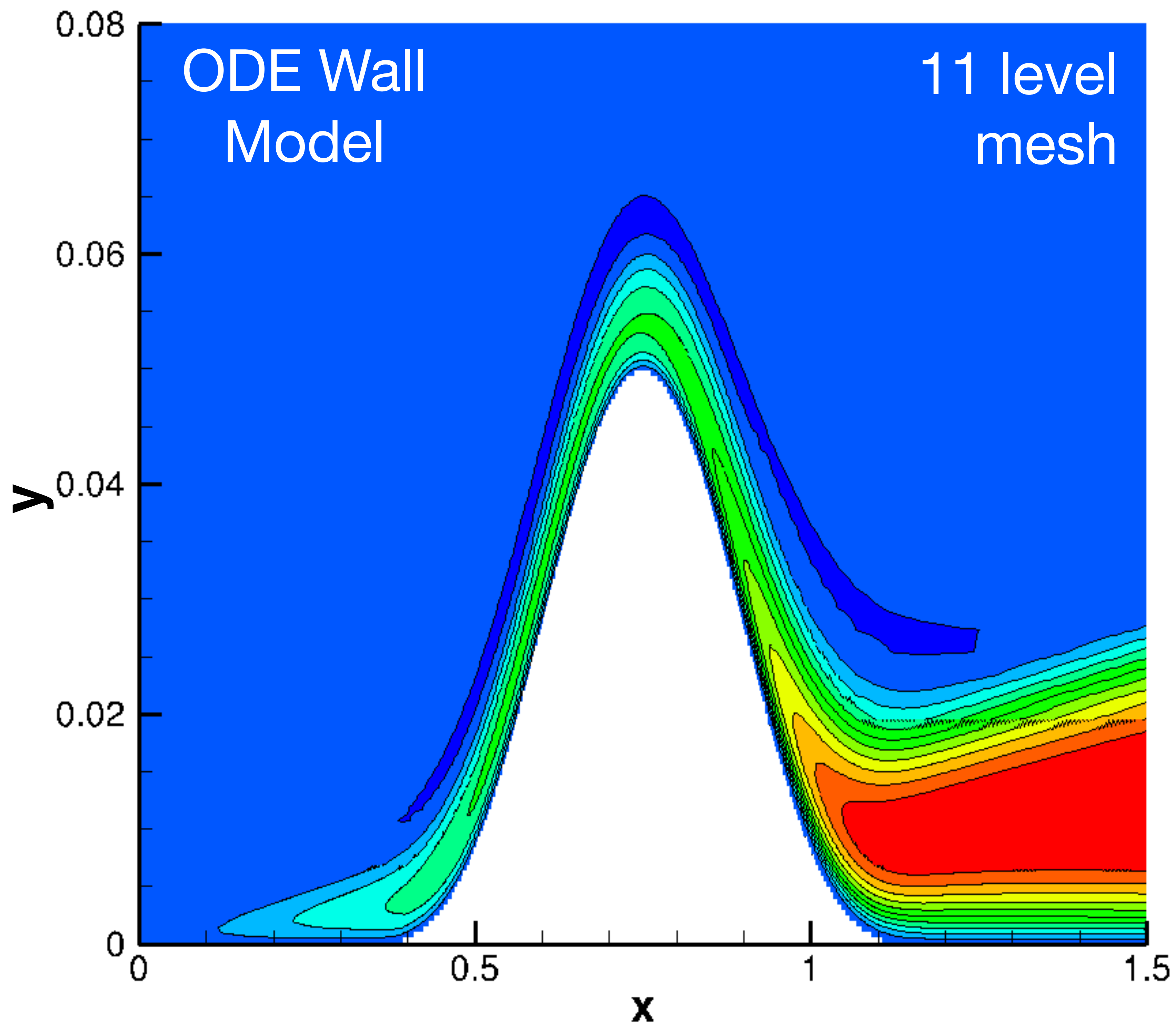


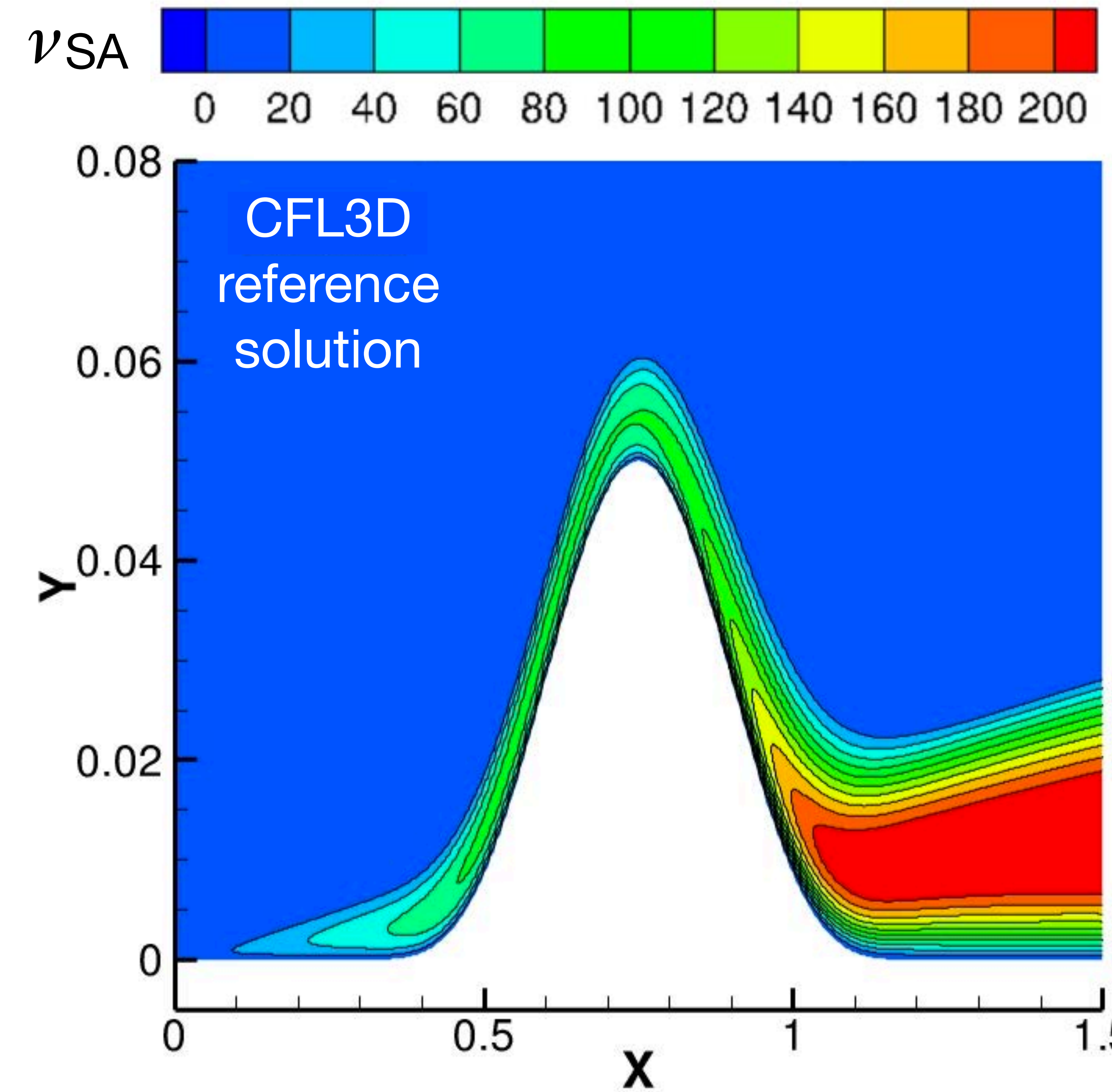


		# Cells or Points	Minimum Cell Dimensions
Coarse	10 levels	52 k	$\Delta y = 0.0012$
Medium	11 levels	70 k	$\Delta y = 0.0006$
Fine	12 levels	91 k	$\Delta y = 0.0003$
Reference solution		900 k	$\Delta y = 2.5e\text{-}7$

Turbulent Bump In Channel

Bump: Eddy viscosity, ν_{SA}

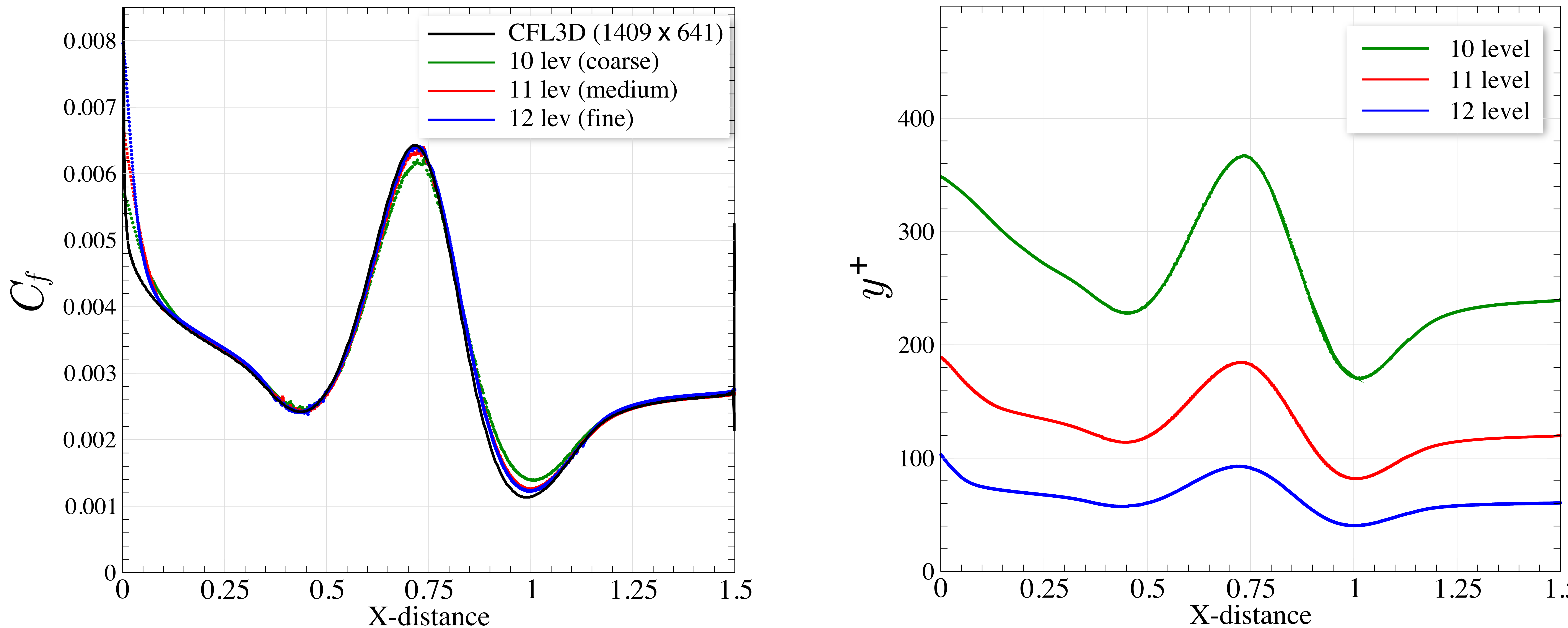




- Good agreement for evolution and peak eddy viscosity
- Slight negative values of ν_{SA} outside of boundary-layer due to 2nd-order advective terms, easily controlled by negative-SA turbulence model

Turbulent Bump In Channel

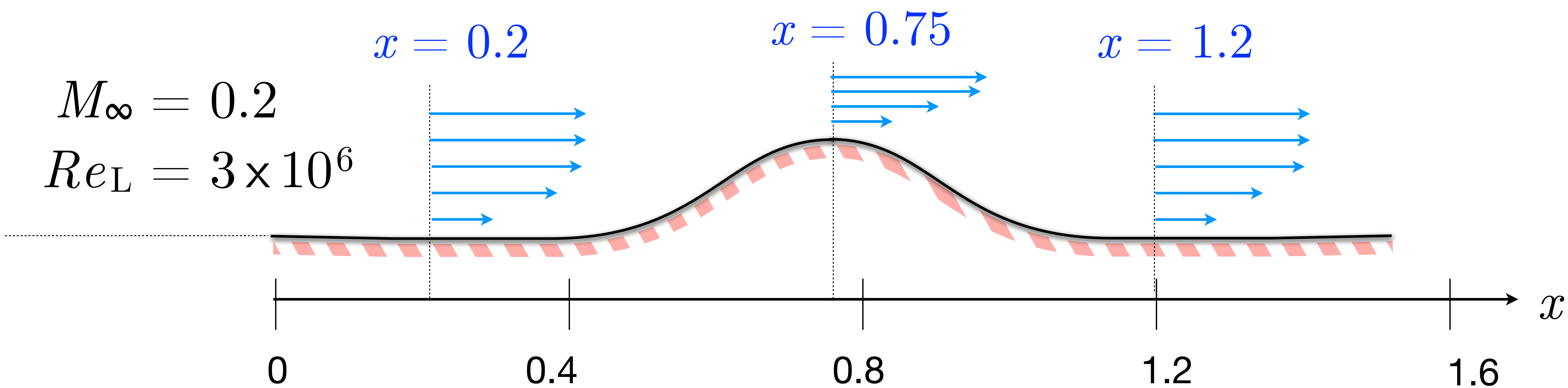
Bump: - Skin friction mesh convergence



- Smooth C_f historically challenging for cut-cell meshes, but look good here
- Slight noise from HLLC flux when face-normal velocity passes through zero
- Good agreement progressing toward mesh convergence, results ordered by dissipation

Turbulent Bump In Channel

Bump: Boundary layer velocity and eddy viscosity profiles

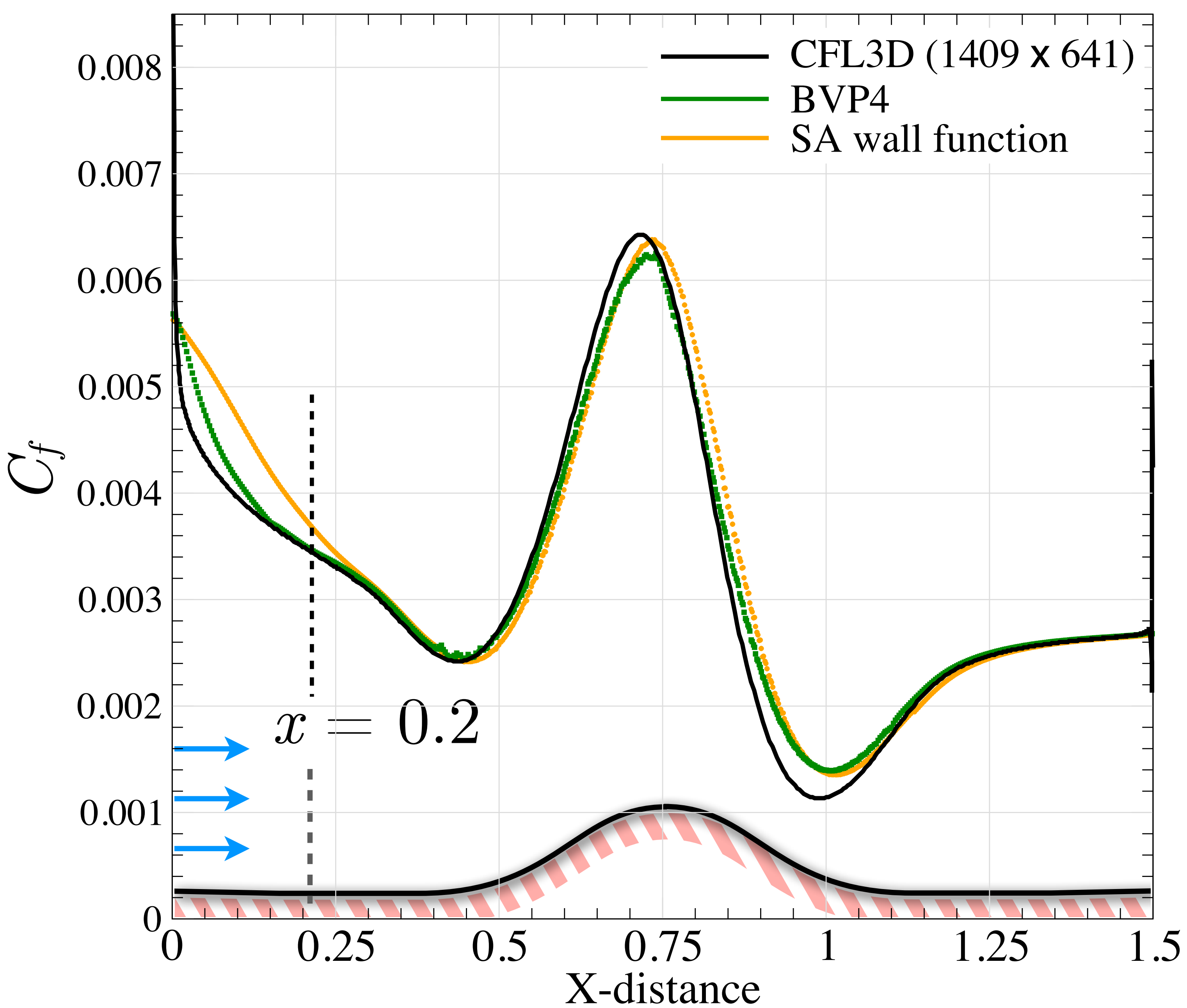


- Paper examine mesh convergence at all 3 stations
- Boundary layer thickens by approx. factor of 2 at each station
- Since resolution of Cartesian mesh is constant, resolution roughly doubles each time we move downstream

Turbulent Bump In Channel

Bump: Compare bvp4 with analytic SA wall-function

Skin friction on coarse mesh

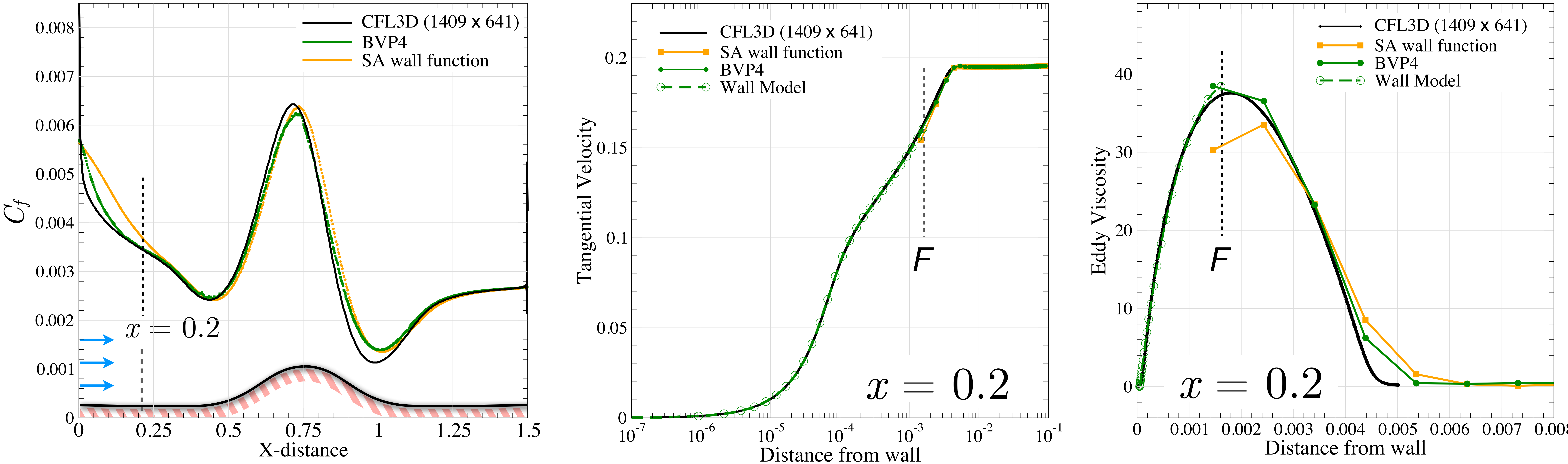


- Even on coarsest grid, SA wall function does reasonably good job
- To see differences, look up front on coarse grid, $x = 0.2$, ($y^+ \approx 280$)

Turbulent Bump In Channel

Bump: Compare bvp4 with analytic SA wall-function

Skin friction on coarse mesh



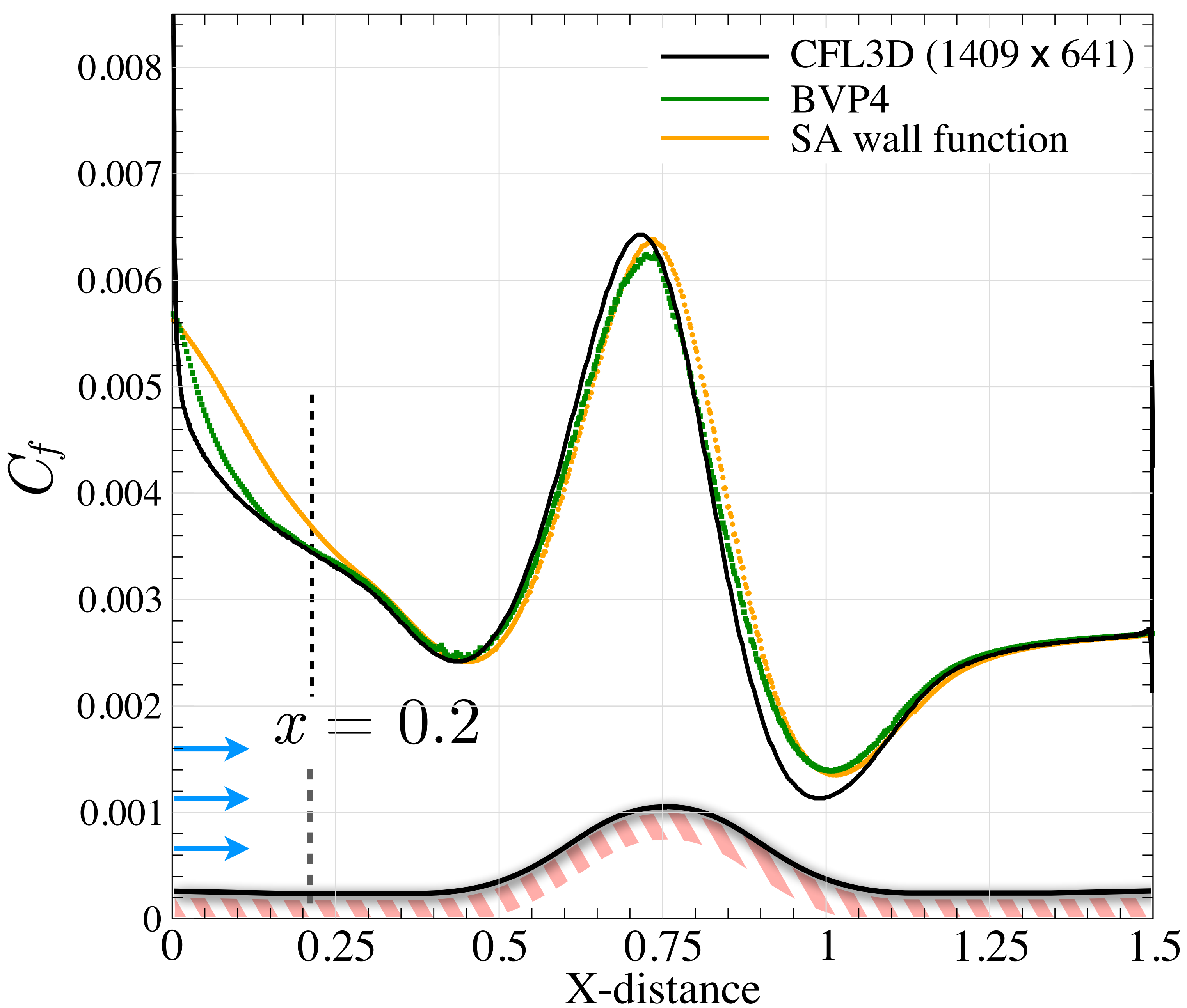
- Even on coarsest grid, SA wall function does reasonably good job
- To see differences, look up front on coarse grid, $x = 0.2$, ($y^+ \approx 280$)
- Skin friction discrepancy comes from misprediction of eddy viscosity by analytic wall function since it assumes a mixing-length model

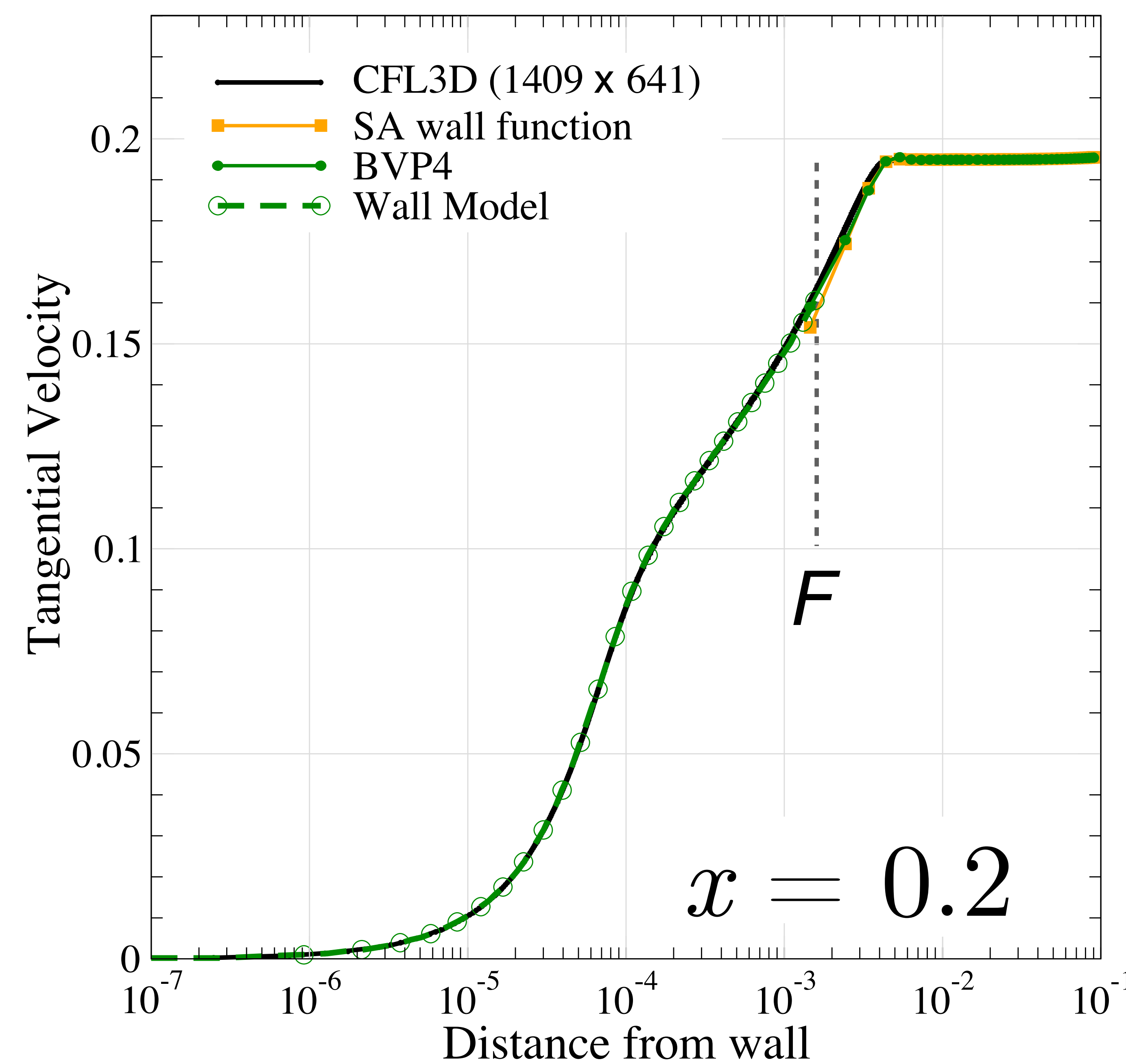
Turbulent Bump In Channel

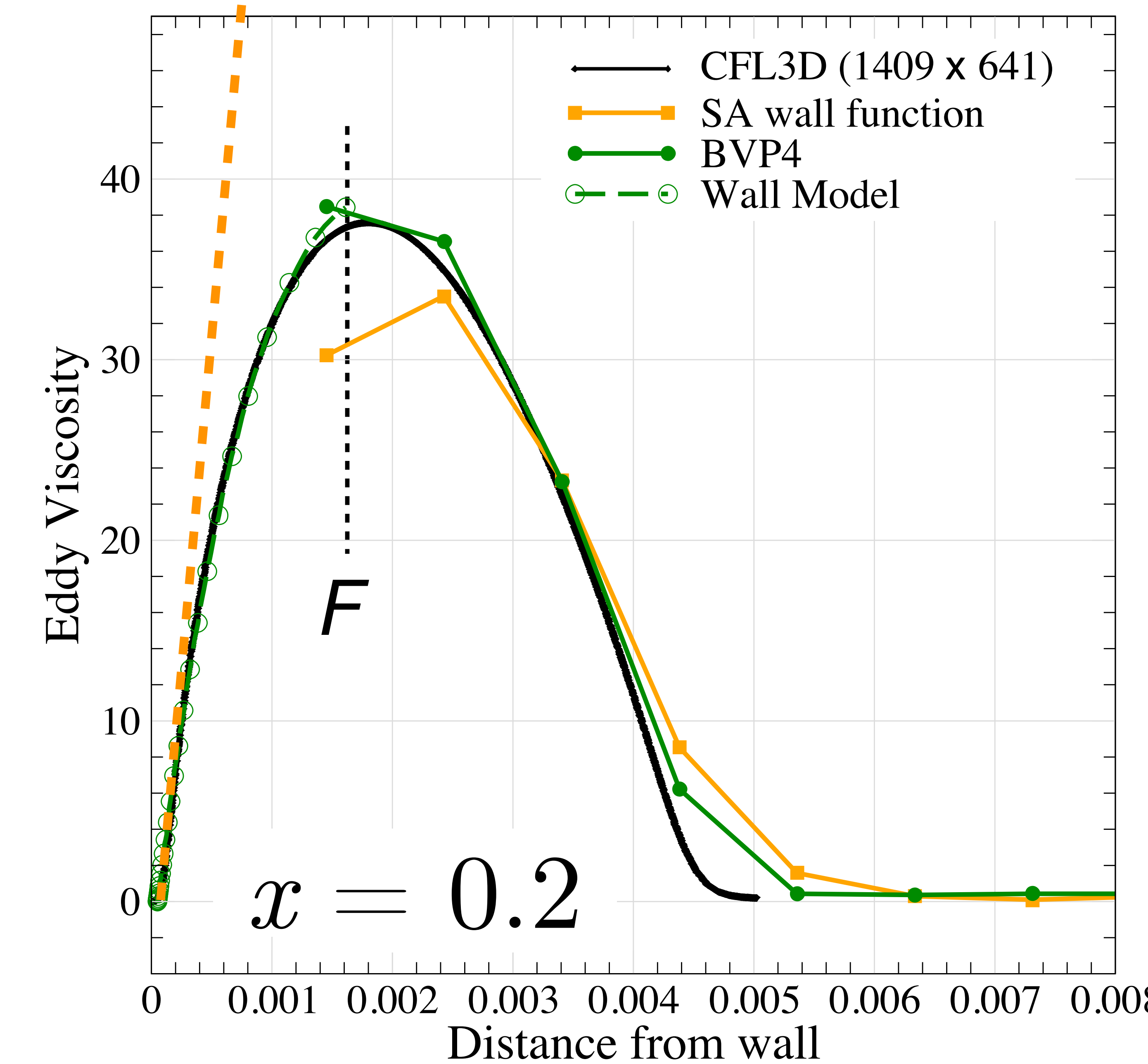
mixing-length model for eddy-viscosity

Bump: Compare bvp4 with analytic SA wall-function

Skin friction on coarse mesh



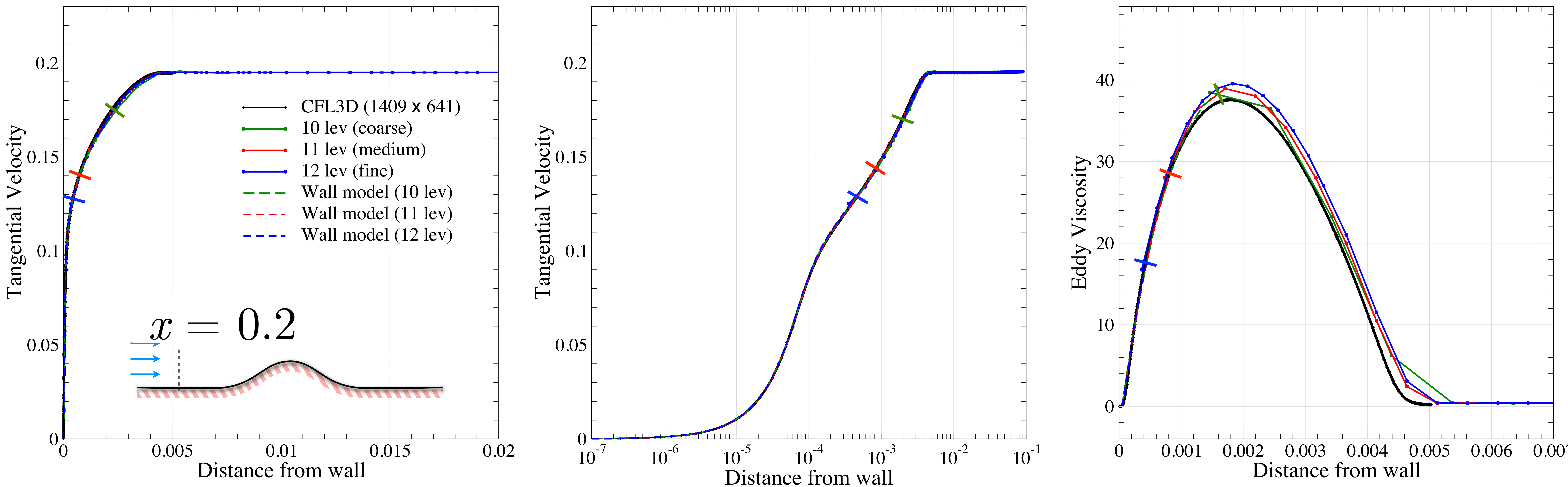




- Even on coarsest grid, SA wall function does reasonably good job
- To see differences, look up front on coarse grid, $x = 0.2$, ($y^+ \approx 280$)
- Analytic wall function overpredicts eddy viscosity by about factor of 3,
  – is inconsistent with outer solution

Turbulent Bump In Channel

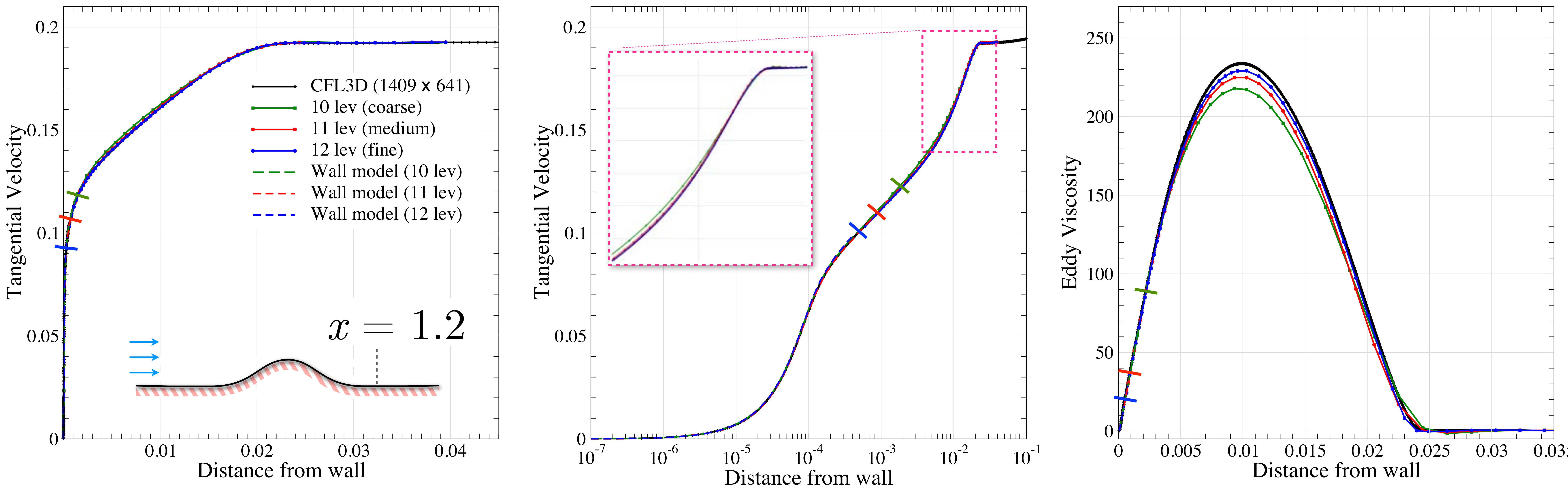
Bump: Mesh convergence of velocity and eddy viscosity profiles



- 7 curves on each plot, (wall model & field solution) x (coarse, med, fine) + CFL3D
- Very good agreement for velocity, good agreement for eddy viscosity
- $x = 0.2$ is the most under resolved station, ~4-5 Cartesian cells in boundary layer

Turbulent Bump In Channel

Bump: Mesh convergence of velocity and eddy viscosity profiles

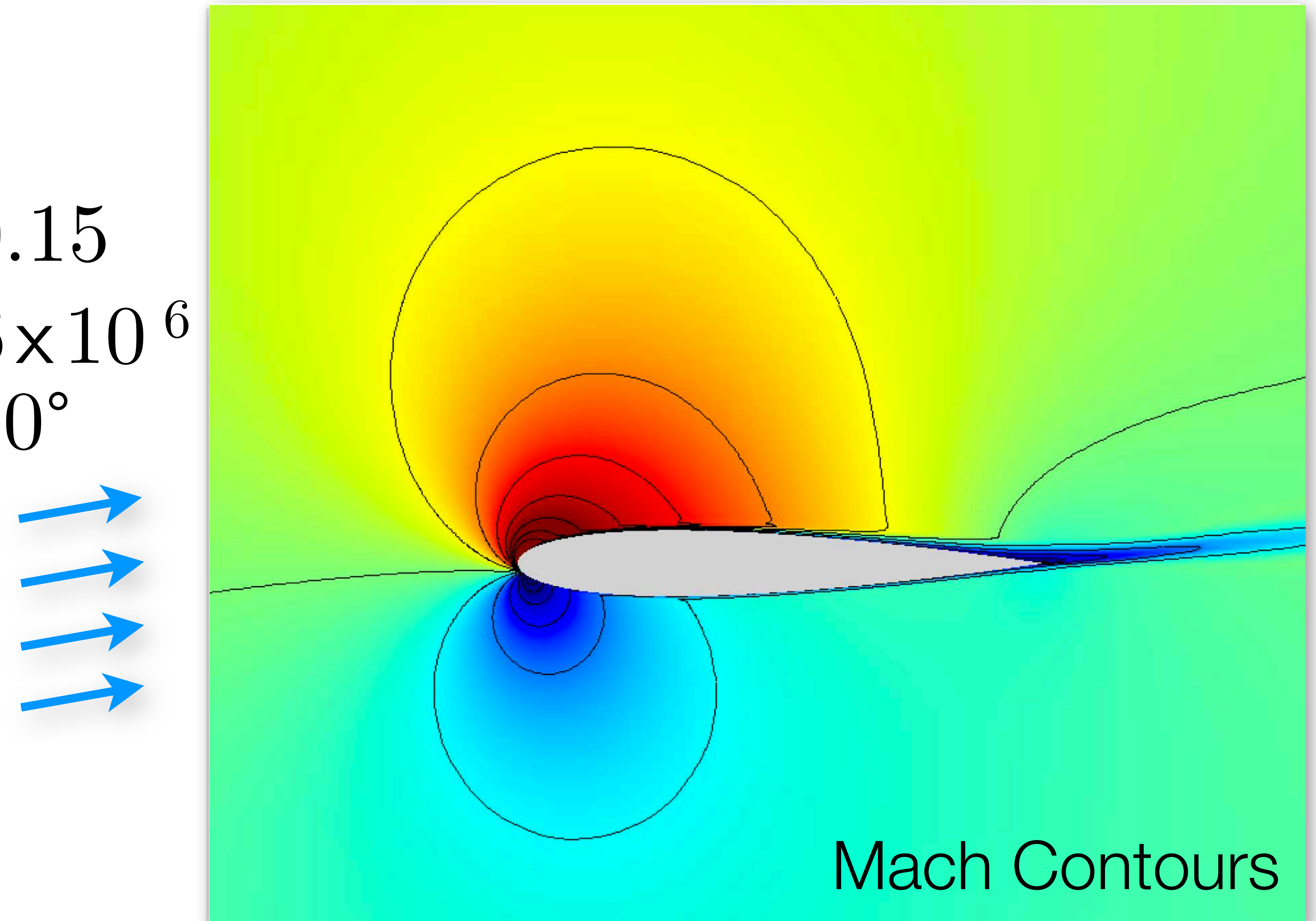


- B-L about 4x thicker by x=1.2
- Aft of bump, slight adverse pressure gradient, thick boundary layer
- Velocity profiles show very good agreement -- even on semi log scale
- Eddy viscosity peak being eroded slightly by dissipation on outer mesh

NACA 0012

Modified NACA 0012 geometry with sharp trailing edge

$$M_\infty = 0.15$$
$$Re_L = 6\times10^6$$
$$\alpha_\infty = 10°$$

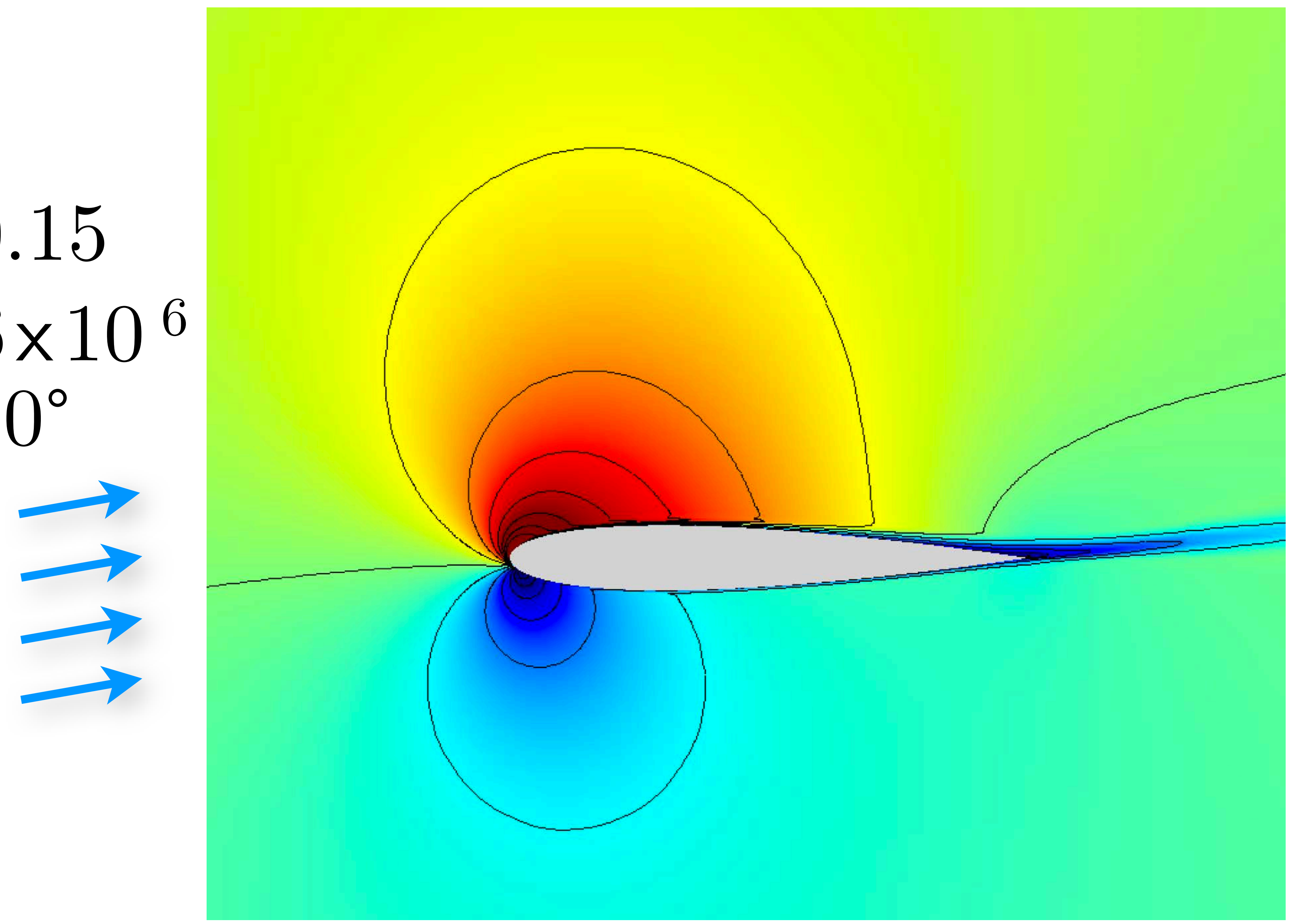


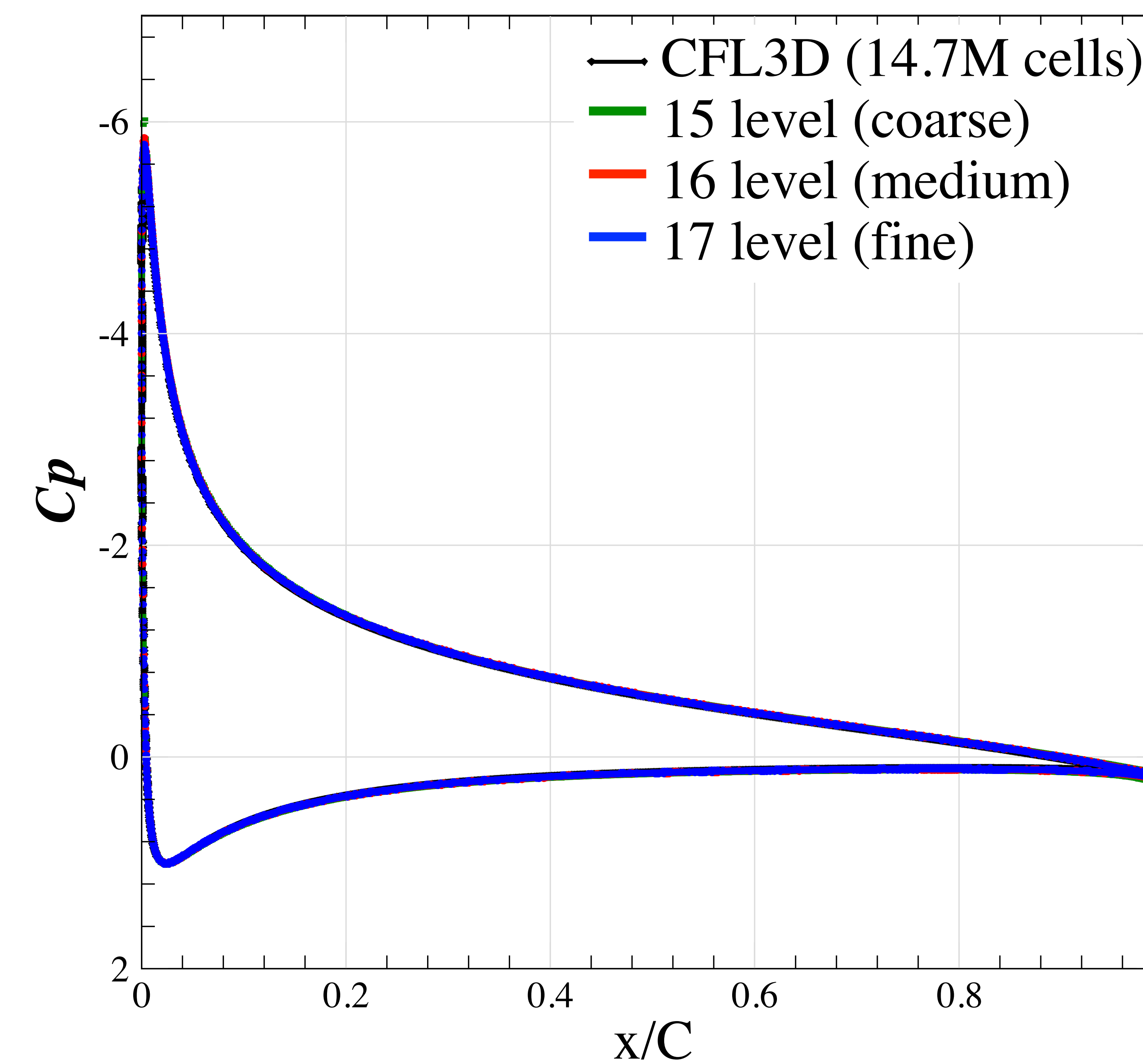
- Validation example "*2DN00: 2D NACA 0012 Airfoil Validation Case*" of TMR website
- Refinement studies on grids up to 14.7M points
- Compare with CFL3D, SA model with no circulation correction
- Use this case to dissect the role of various terms in the bvp4 model

NACA 0012

Modified NACA 0012 - Surface pressure mesh convergence

$$M_\infty = 0.15$$
$$Re_L = 6\times10^6$$
$$\alpha_\infty = 10^\circ$$

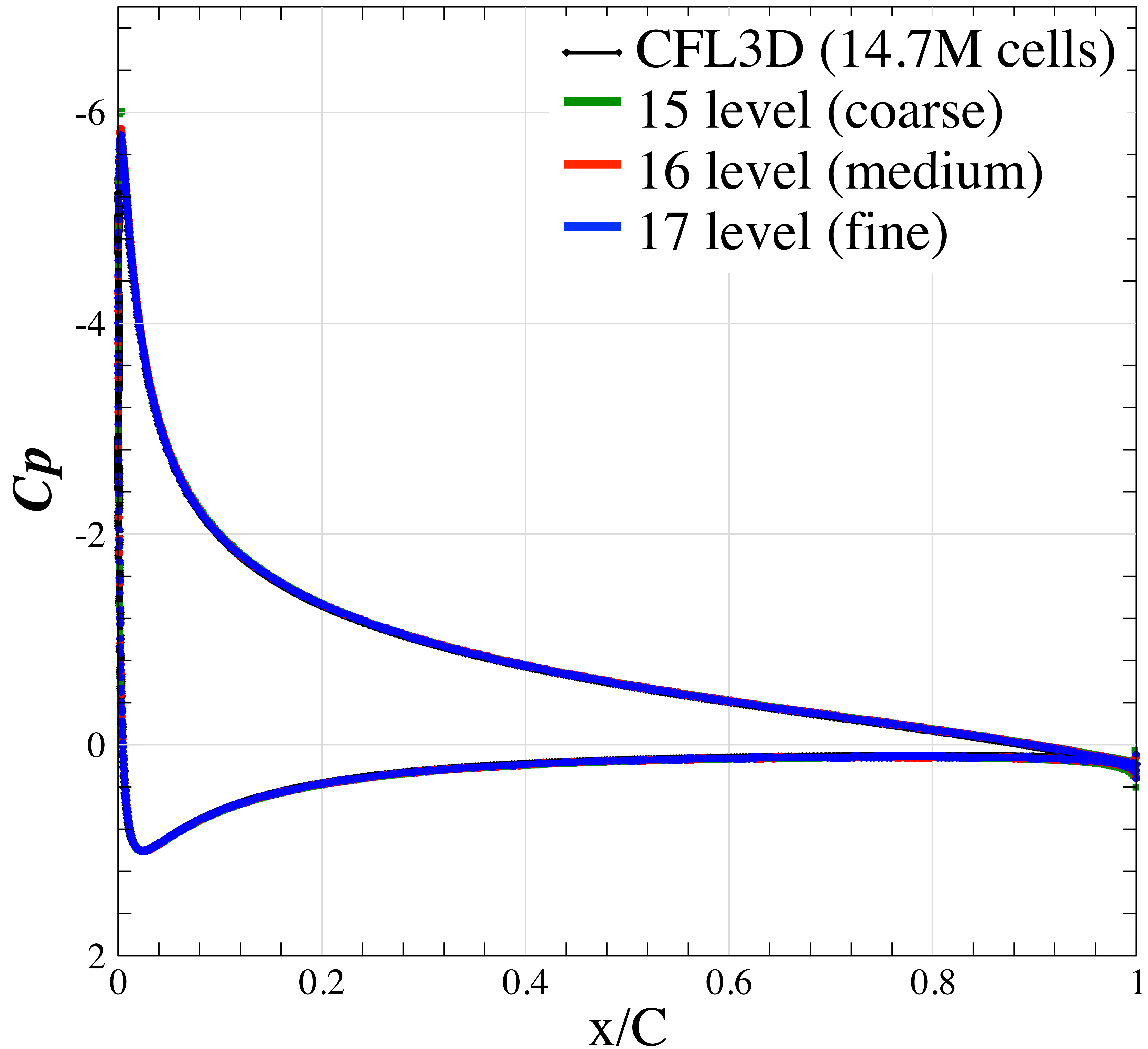




		# Cells or Points	Minimum Cell Dimensions
Coarse	15 levels	58 k	$\Delta x = \Delta y = 1.7\text{e-}3\,C$
Medium	16 levels	80 k	$\Delta x = \Delta y = 8.4\text{e-}4\,C$
Fine	17 levels	133 k	$\Delta x = \Delta y = 4.2\text{e-}4\,C$
Reference solution		14.7M	$1.\text{e-}7\,C \times 1.25\text{e-}5\,C$

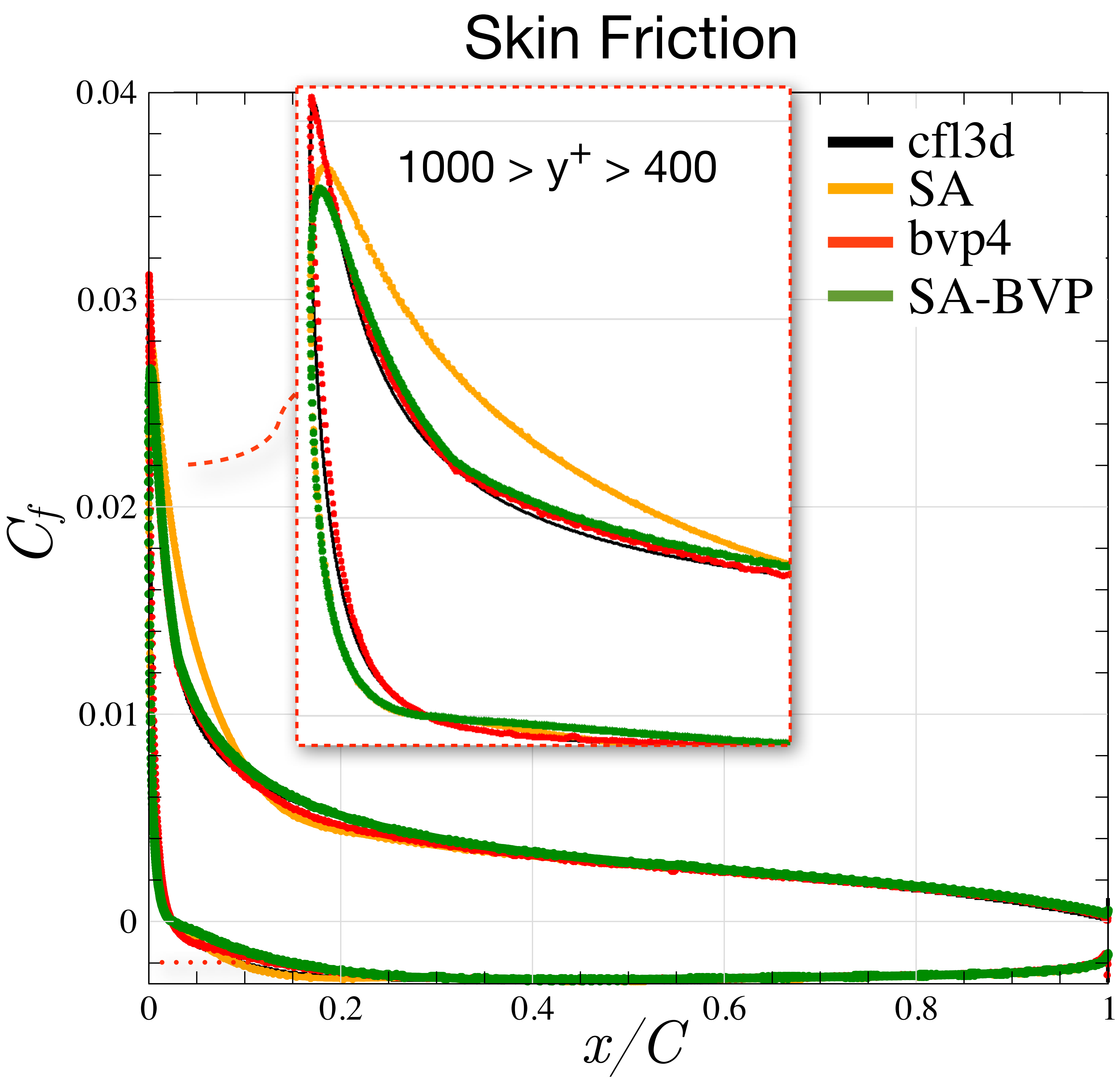
NACA 0012

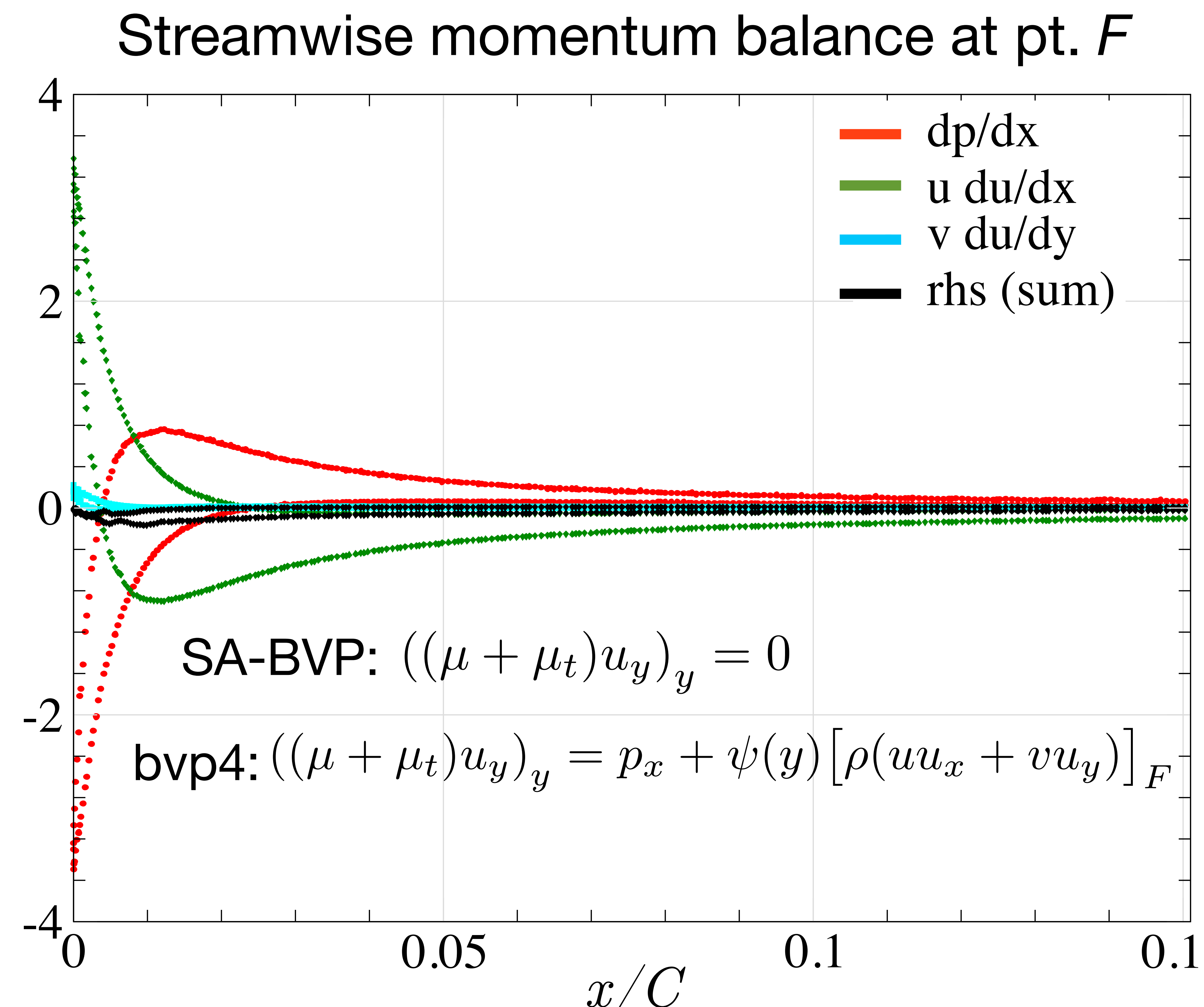
Modified NACA 0012 - Surface pressure mesh convergence



NACA 0012

Modified NACA 0012 - Compare bvp4 with analytic SA-BVP & SA wall function

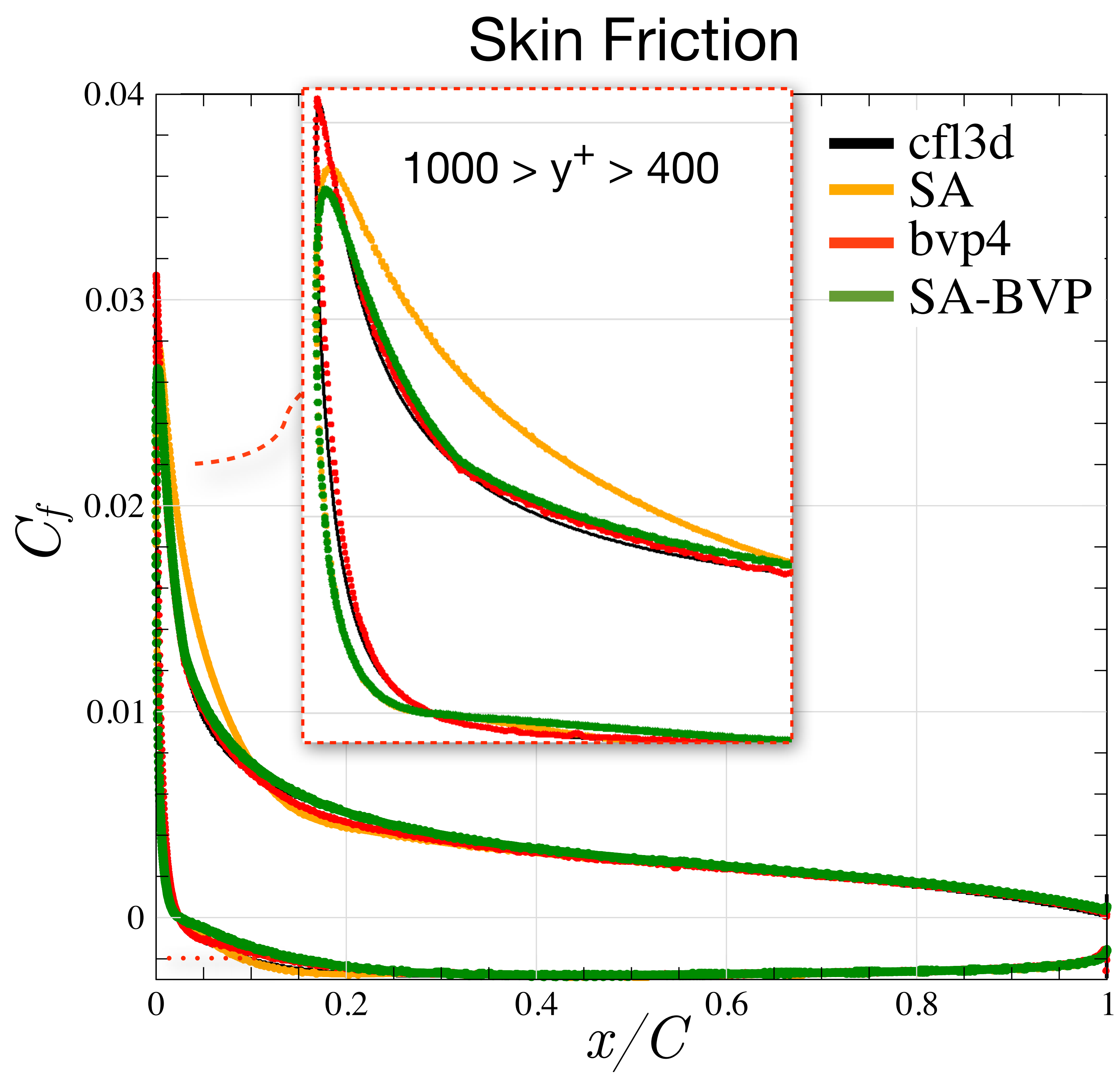


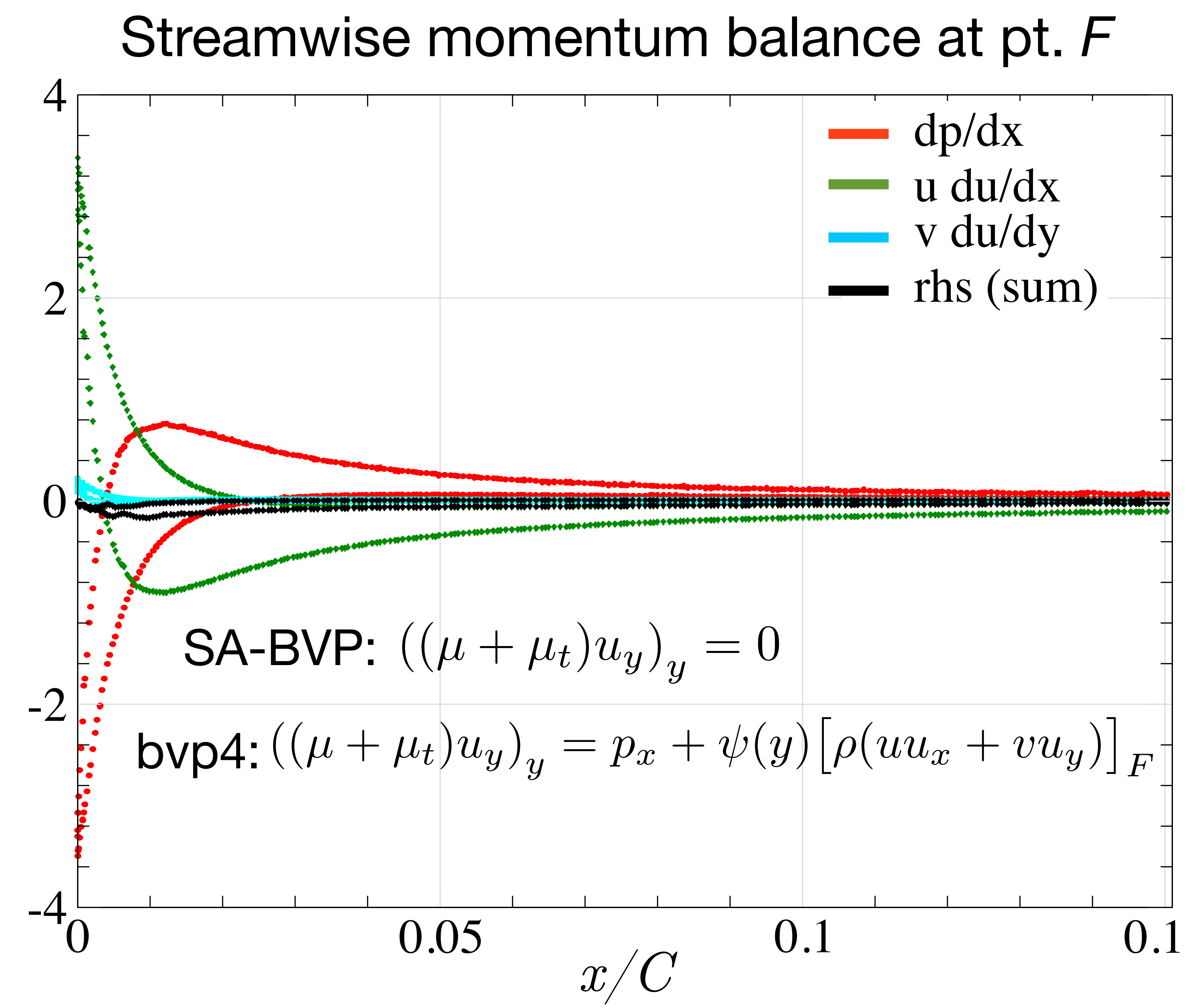


(Terms measured at forcing point)

NACA 0012

Modified NACA 0012 - Compare bvp4 with analytic SA-BVP & SA wall function

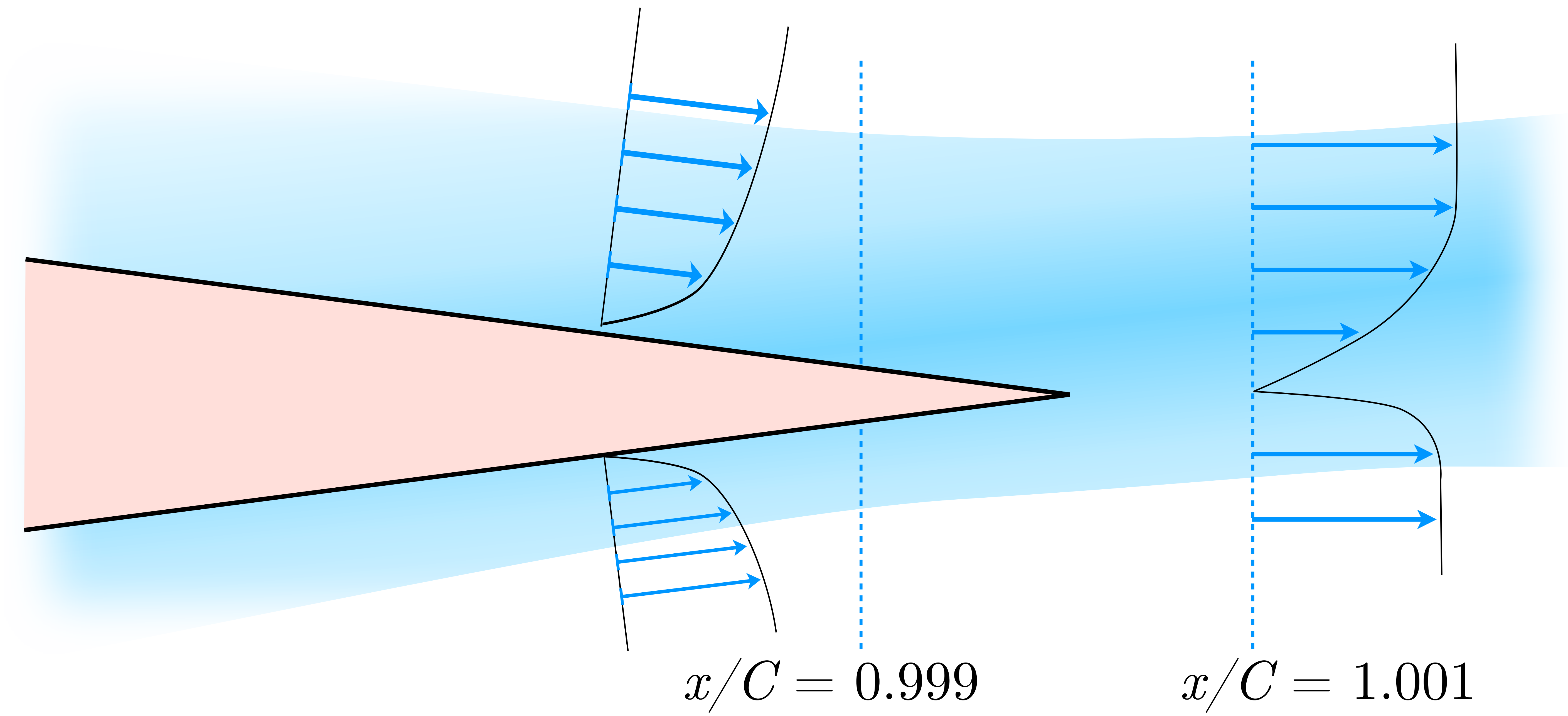




- SA-wall function lags both bvp4 model and SA-BVP skin friction, good around $y^+ = 300$
- SA-BVP does a good job, but misses peak and pressure-side near leading edge
- bvp4 sees p_x at the wall, but by the time you reach *F*, p_x is largely shutoff by the convective balance $(u\,u_x + v\,u_y)$ and RHS nearly vanishes

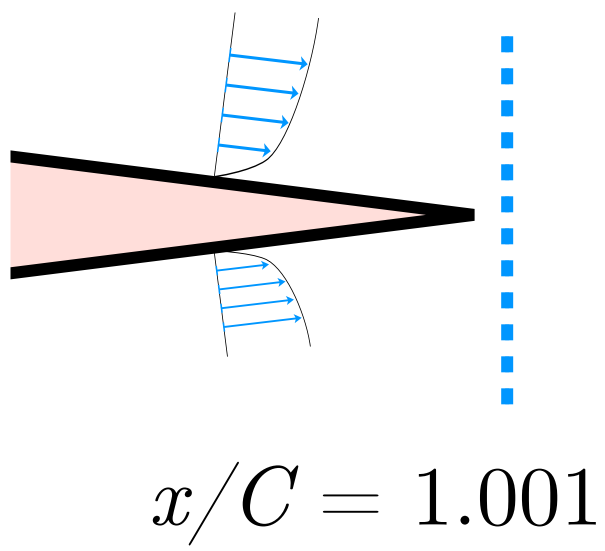
NACA 0012

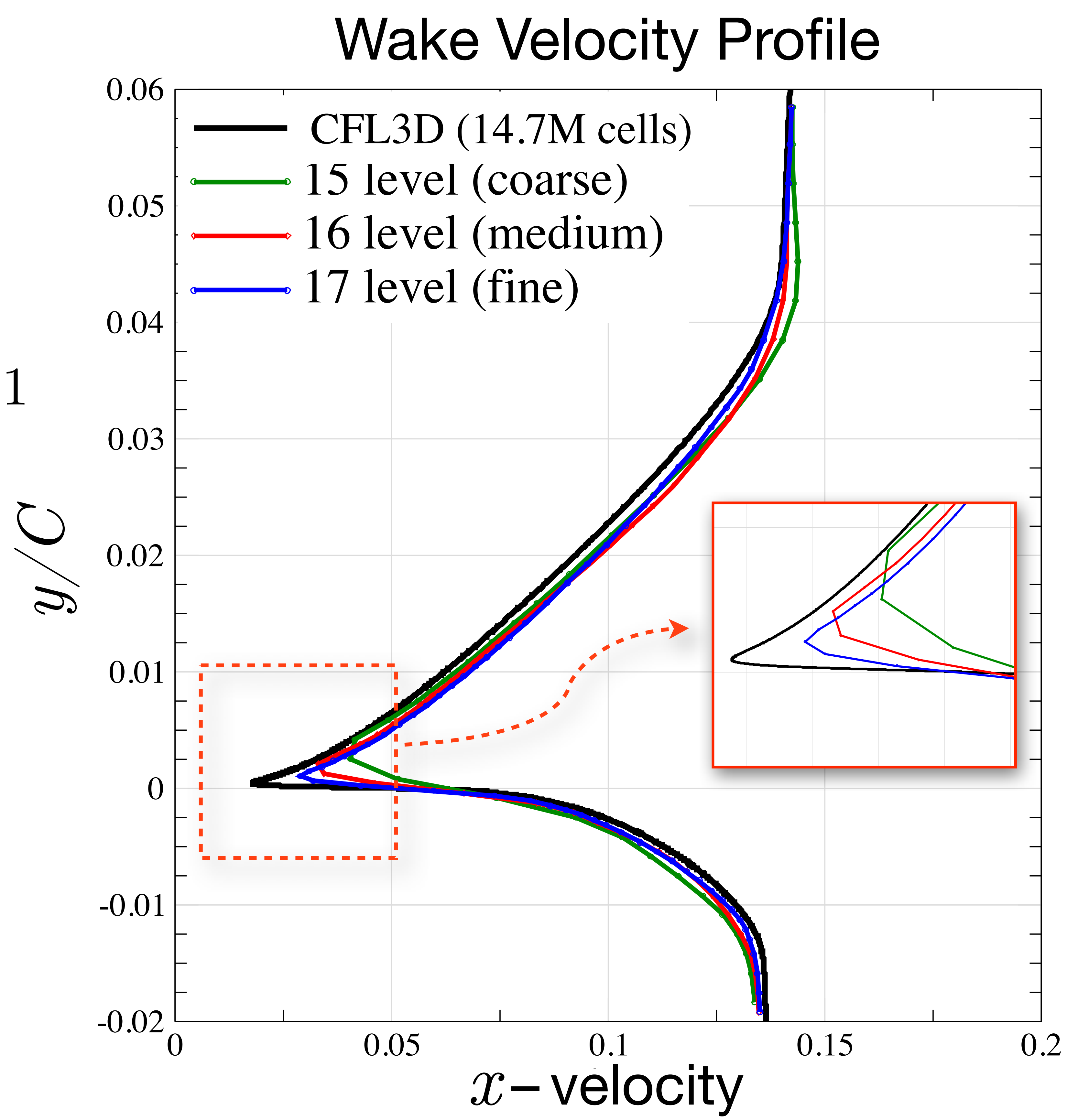
Modified NACA 0012 - Wake surveys of velocity and eddy viscosity

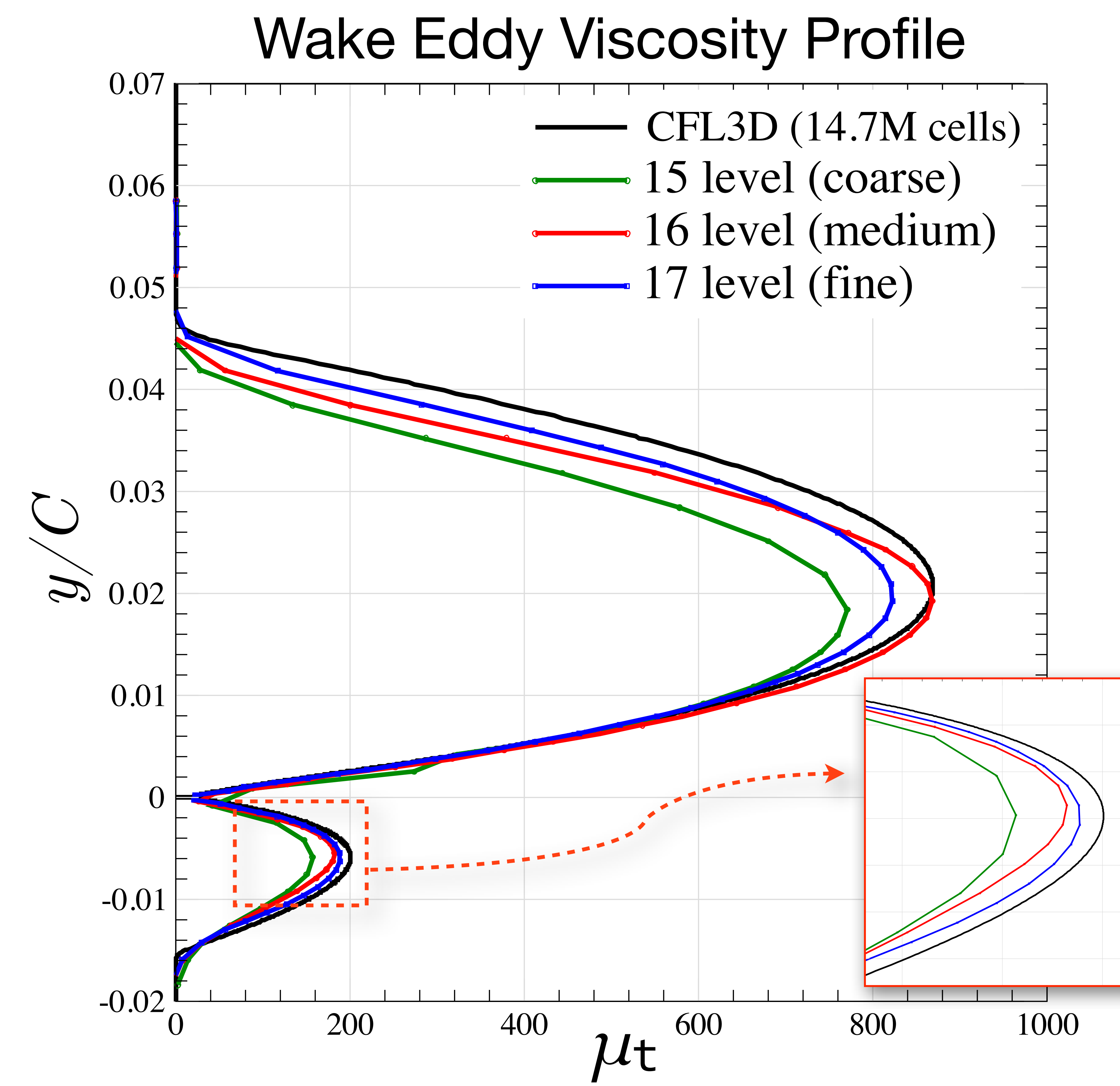


- Comparison data exist for profiles at x/C = 0.999 and 1.001
- Examine data at x/C = 1.001, (similar results at x/C = 0.999)

NACA 0012



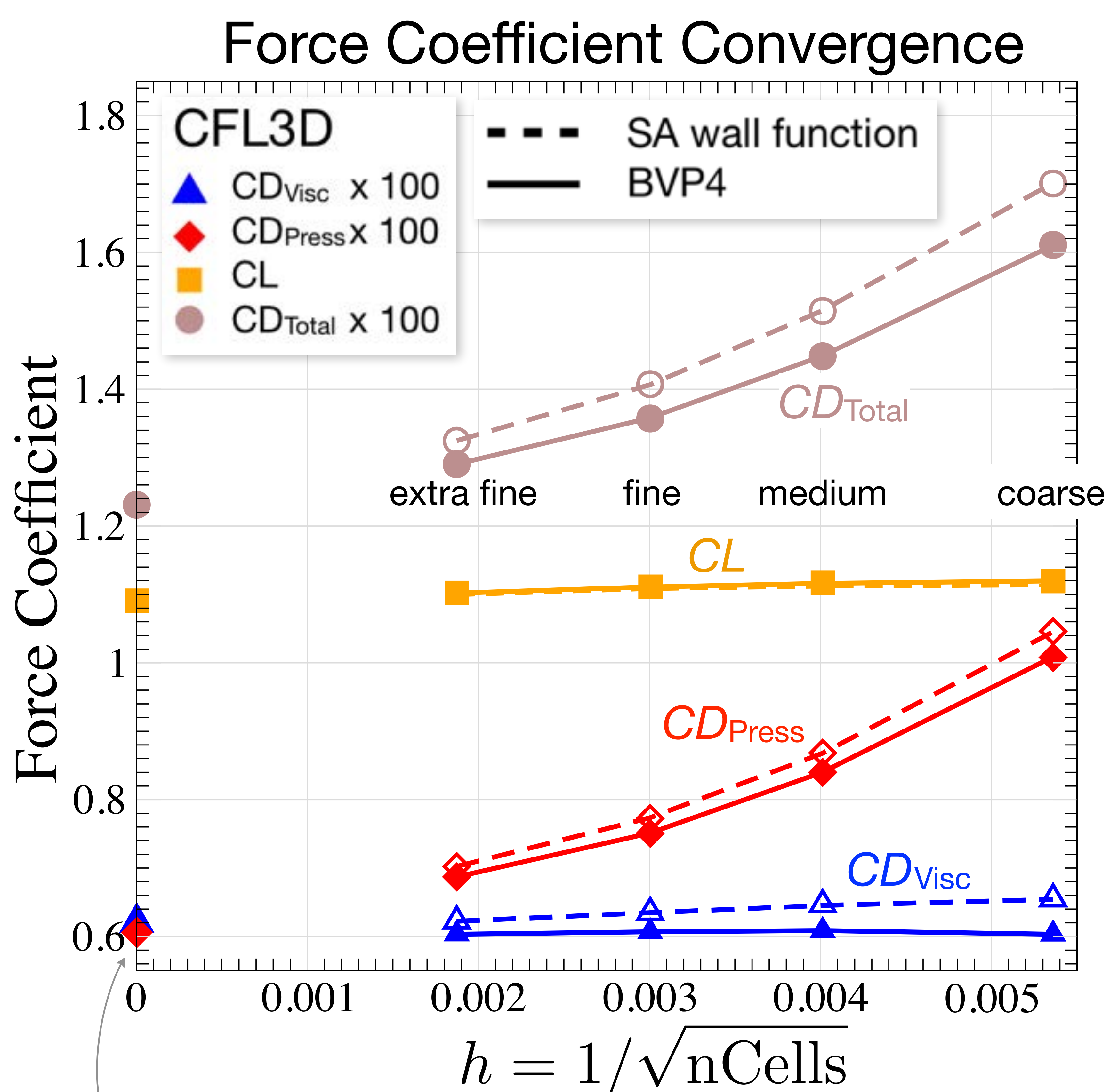




- Low dissipation inviscid flux helps resolution in cusp – good mesh convergence behavior
- Lower surface eddy viscosity shows good mesh convergence
- Upper surface eddy viscosity not yet mesh converged – literature shows slow convergence

NACA 0012

Mesh convergence of integrated force coefficients



mesh converged extrapolated values

- NASA TMR has mesh convergence data*
- Show extrapolated "infinite grid" values of force coefs.
- Analysis by Diskin *et al.*† shows challenges obtaining mesh converged results,
- Pressure drag converges slowest
- $C_{D,\text{visc}}$ is within 2 counts on the coarsest mesh
- **Full disclosure:** *Re-ran using mesh family with outer boundary located at 500C, added "extra fine" mesh*

	Coarse	*Medium*	*Fine*	*extra fine*	*Reference*
nCells	35 k	62 k	111 k	285 k	14.7 M
LE & TE spacing	$0.086\%C$	$0.05\%C$	$0.032\%C$	$0.016\%C$	$0.00125\%C$

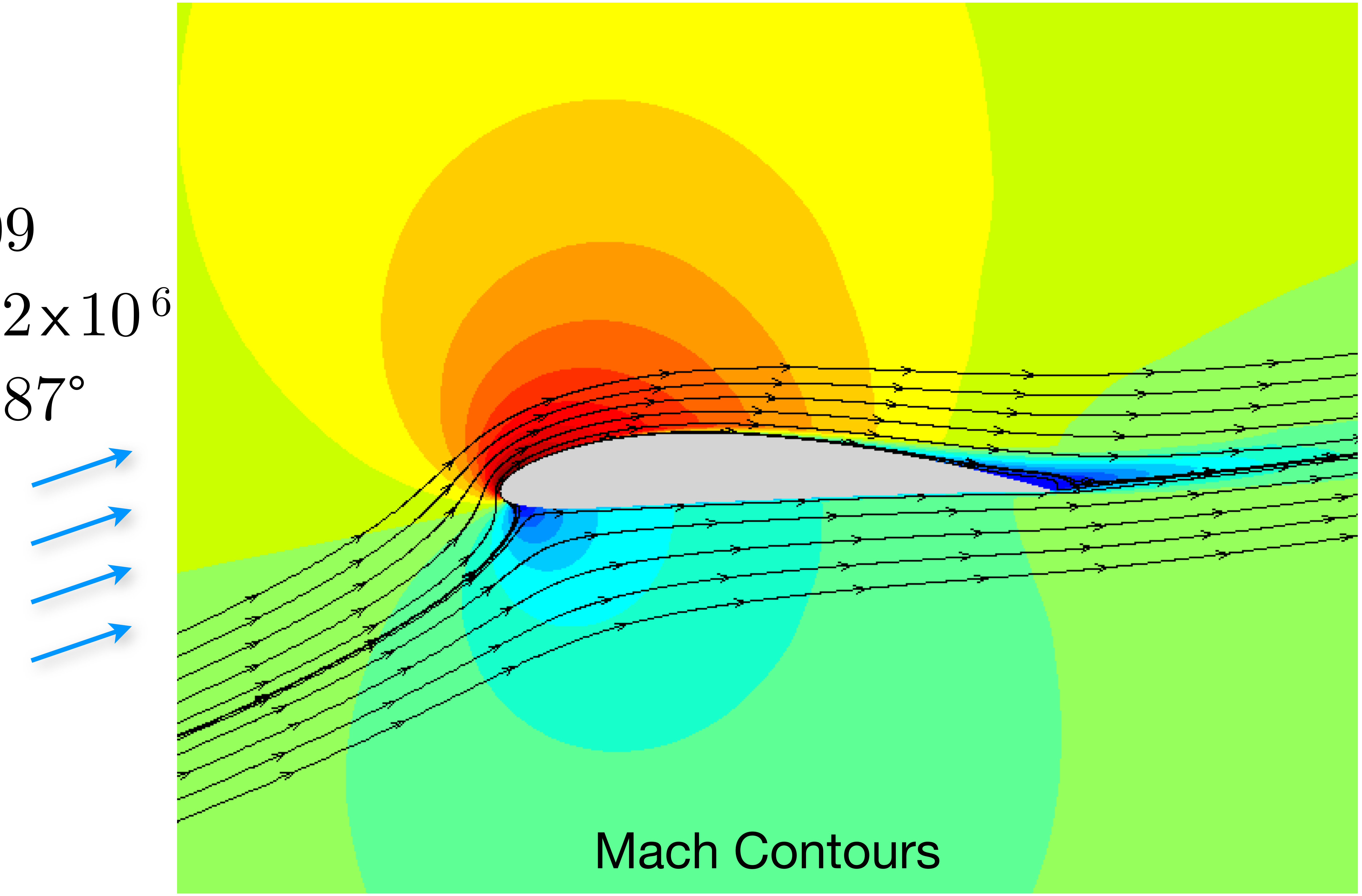
** Data from CFL3D with SA model on "family II" grid, no point vortex correction & 2nd-order turbulent advection*

† AIAA 2015-1746 "Grid Convergence for Turbulent Flow", Disken,Thomas, Rumsey & Schwöppe

NACA 4412

Modified NACA 4412 with trailing edge separation bubble

$$M_\infty = 0.09$$
$$Re_L = 1.52\times10^6$$
$$\alpha_\infty = 13.87°$$



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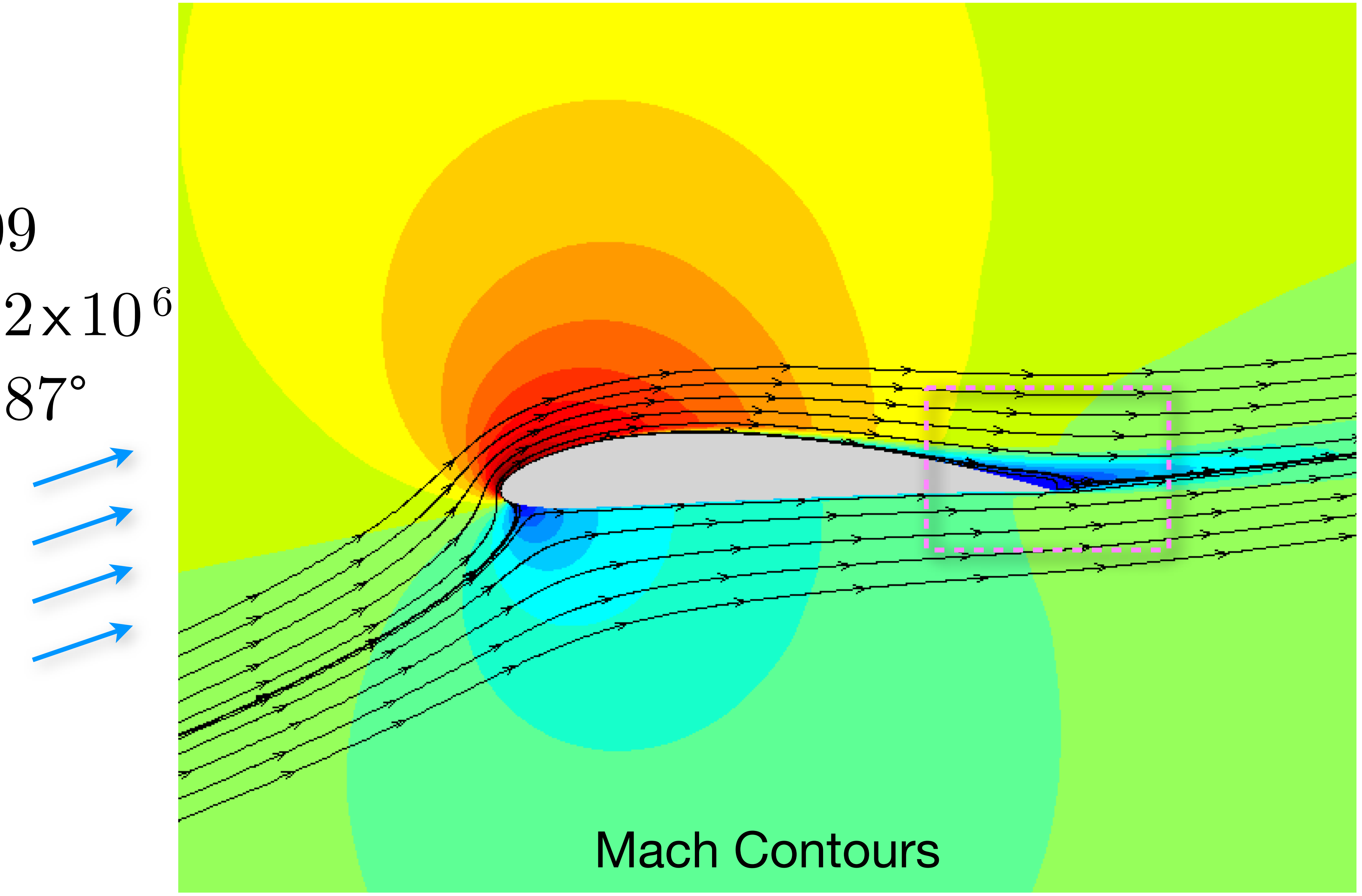
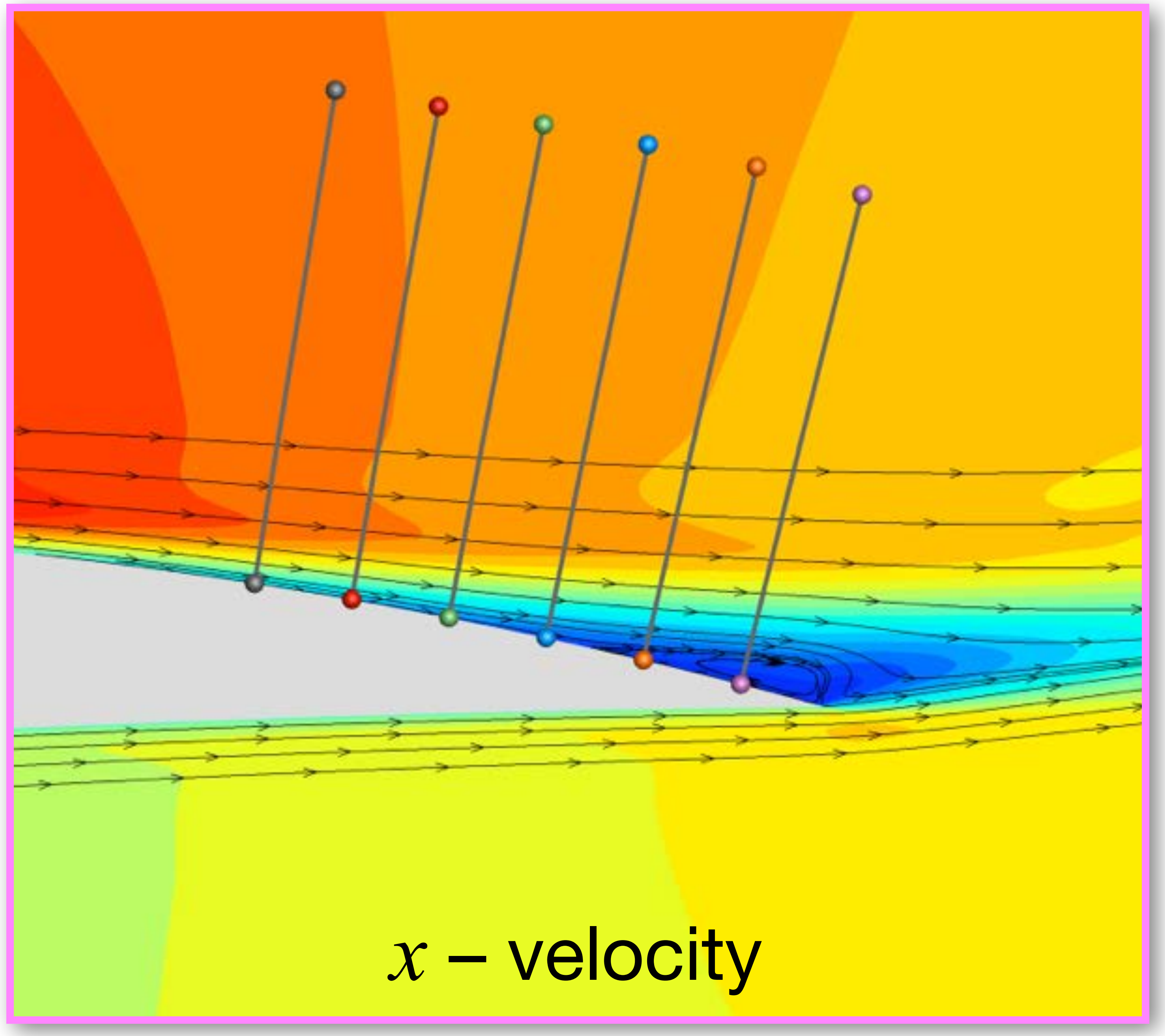
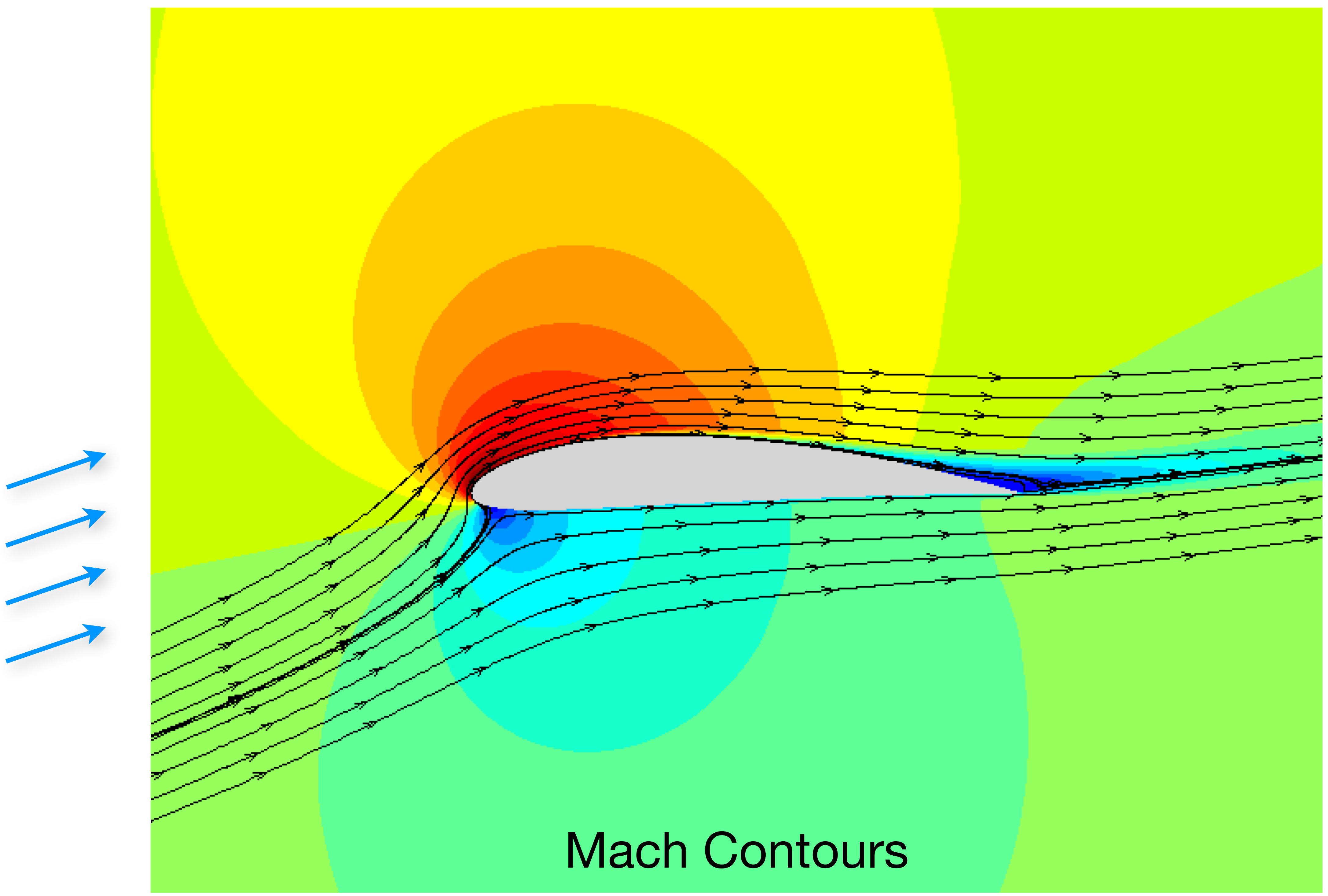


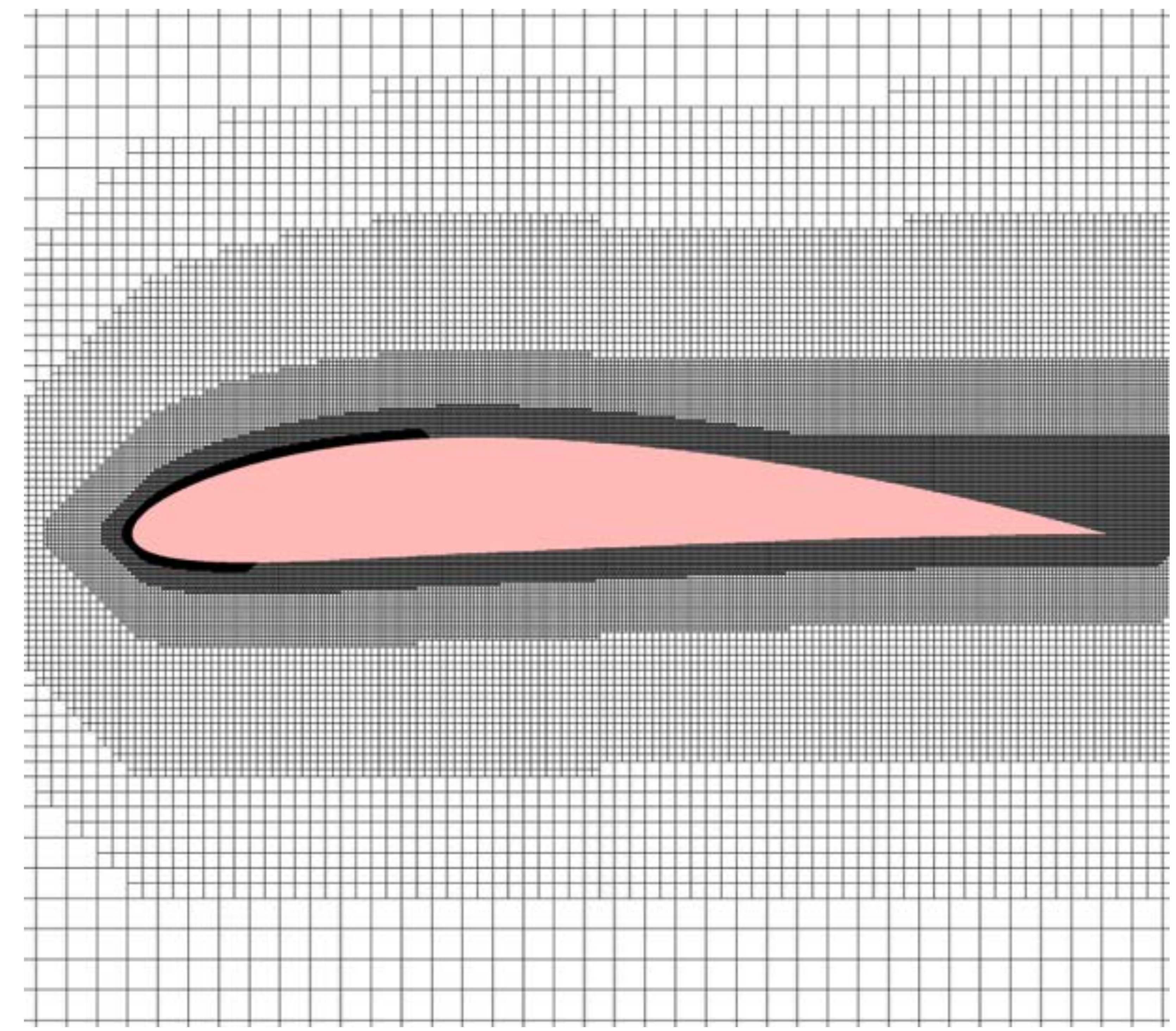


- Validation example in the “Extended cases” section of NASA TMR
- Near maximum lift conditions with a smooth-body separation bubble
- Experiment by Coles & Wadcock (1979) with hot-wire velocity profiles
- Reference data form CFL3D on 897 x 257 grid (≈230k points)

NACA 4412

Modified NACA 4412 with trailing edge separation bubble

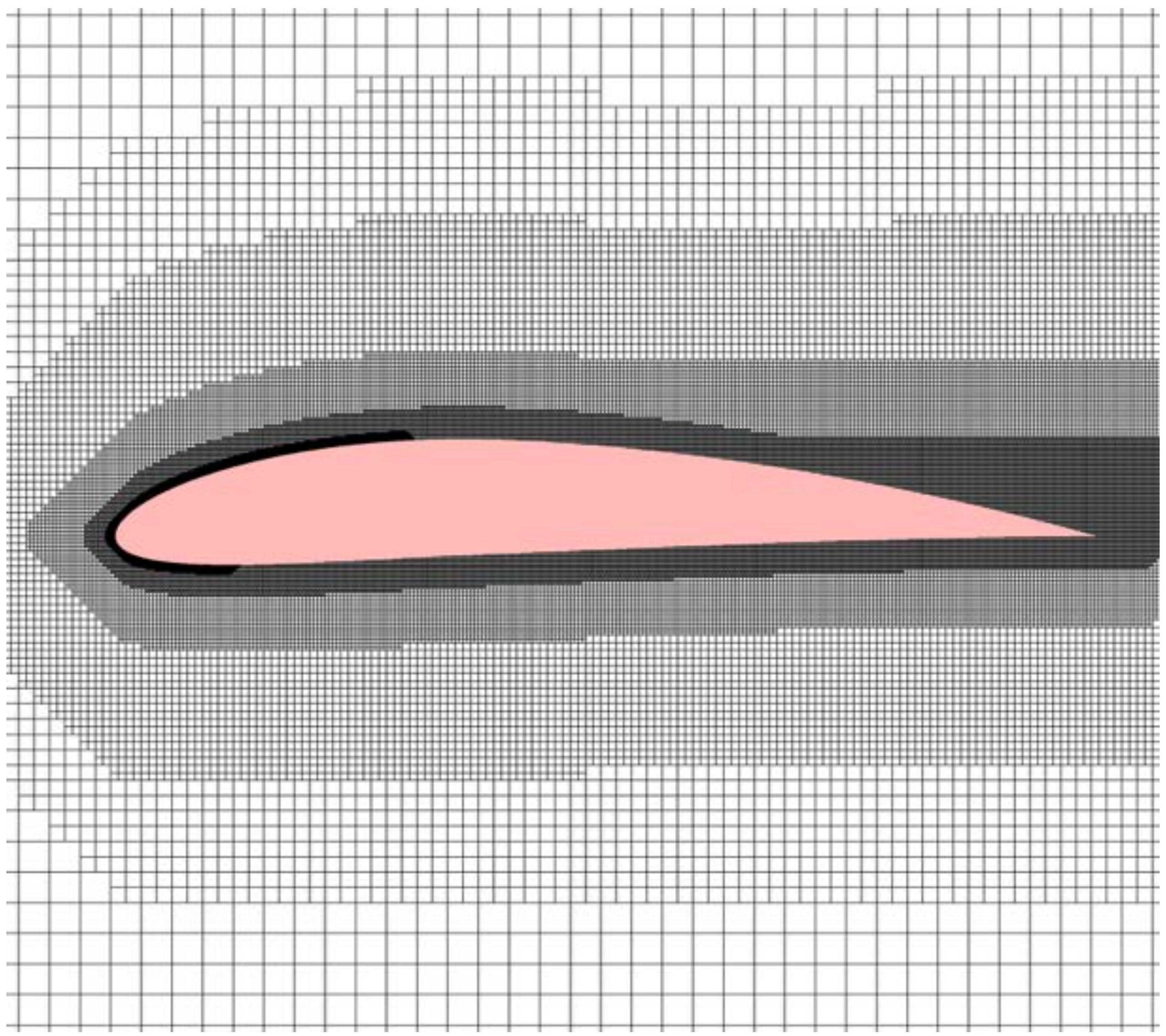


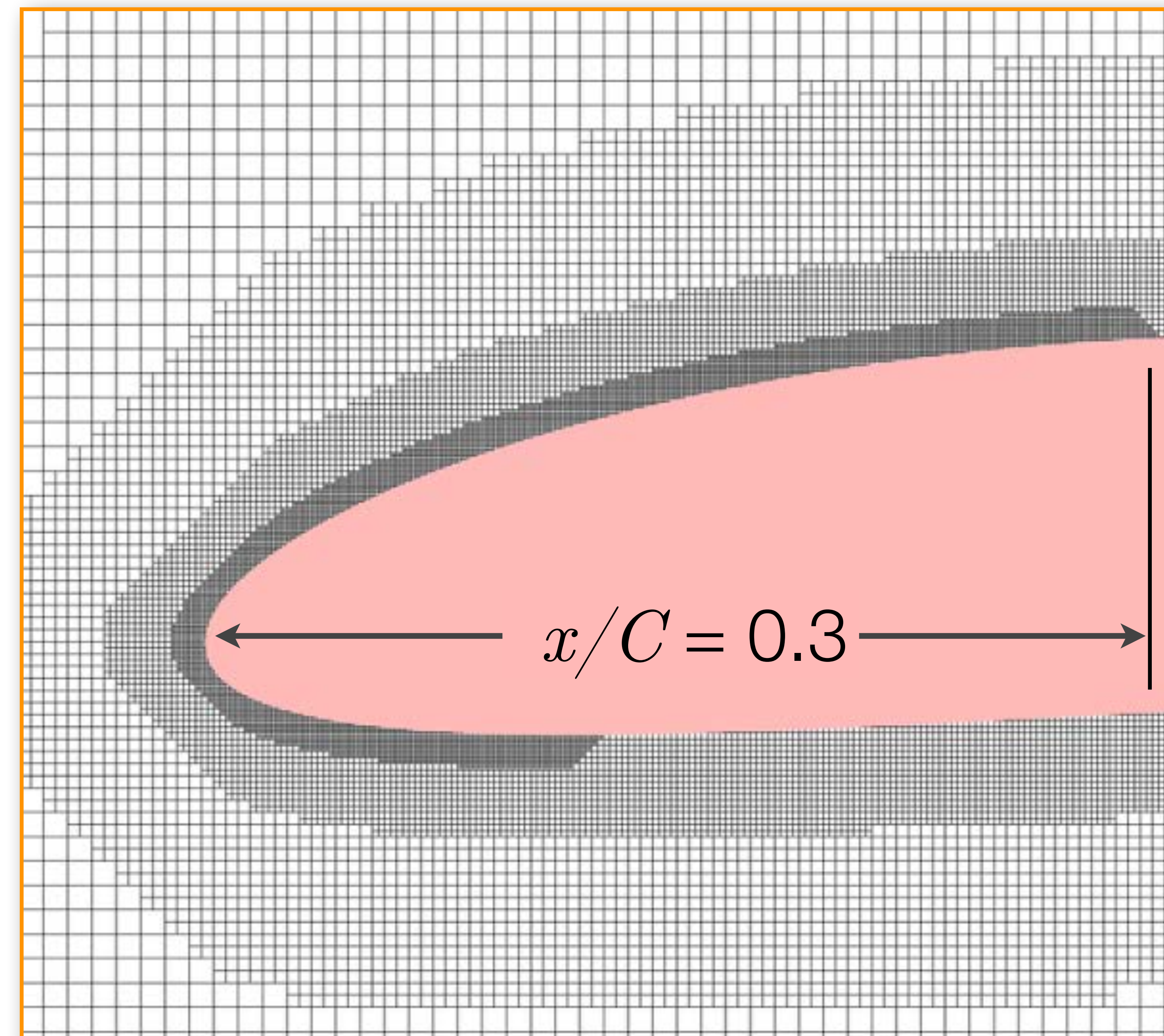


- Multilevel Cartesian mesh with ~59 k cells
- 1-level of mesh refinement near leading edge
- Leading edge, $\Delta x = 0.1\% C$, trailing edge $\Delta x = 0.2\% C$, ~1200 cut cells

NACA 4412

Modified NACA 4412 with trailing edge separation bubble





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- Leading edge, $\Delta x = 0.1\%C$, trailing edge $\Delta x = 0.2\%C$, ~1200 cut cells

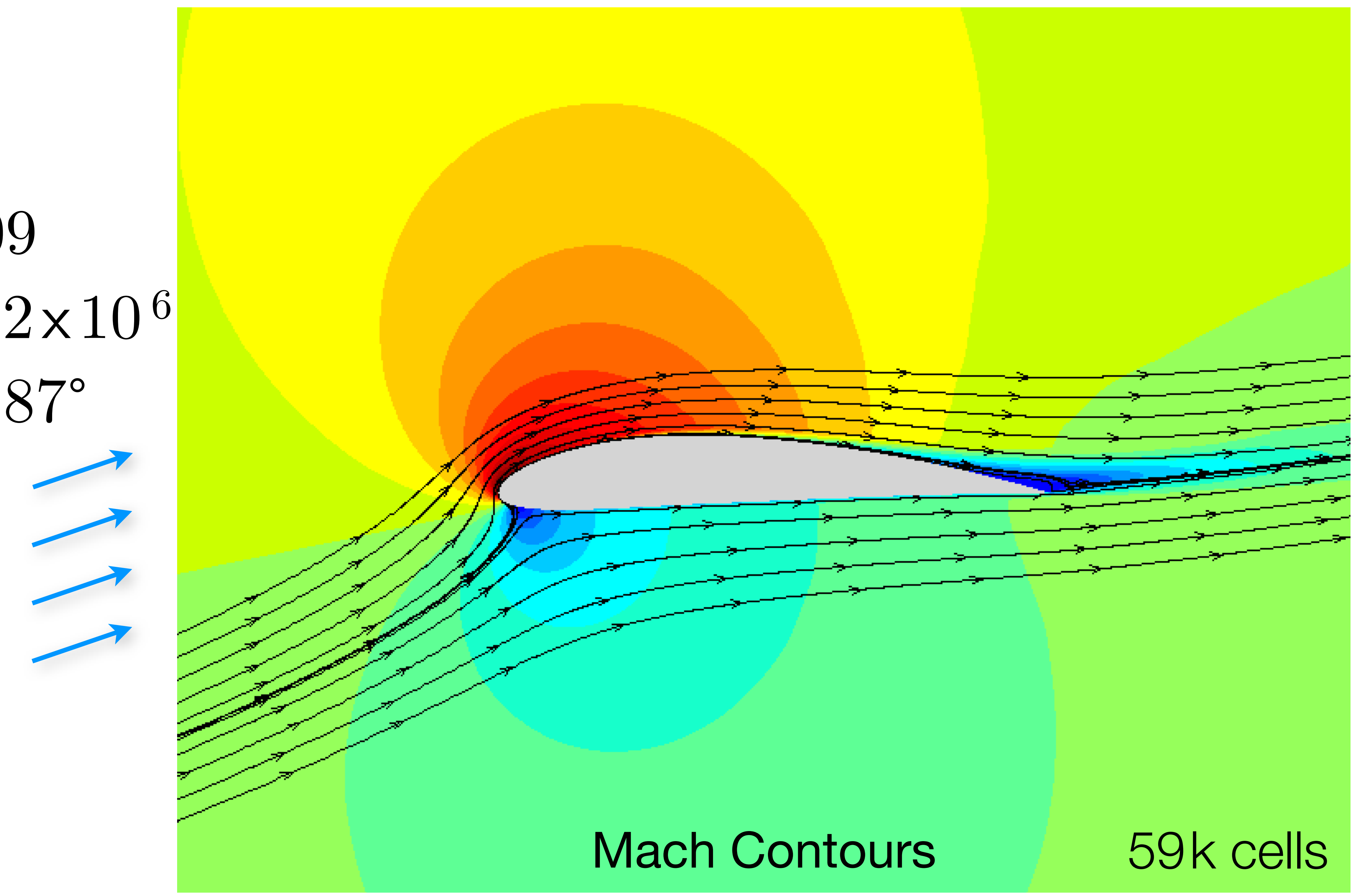
NACA 4412

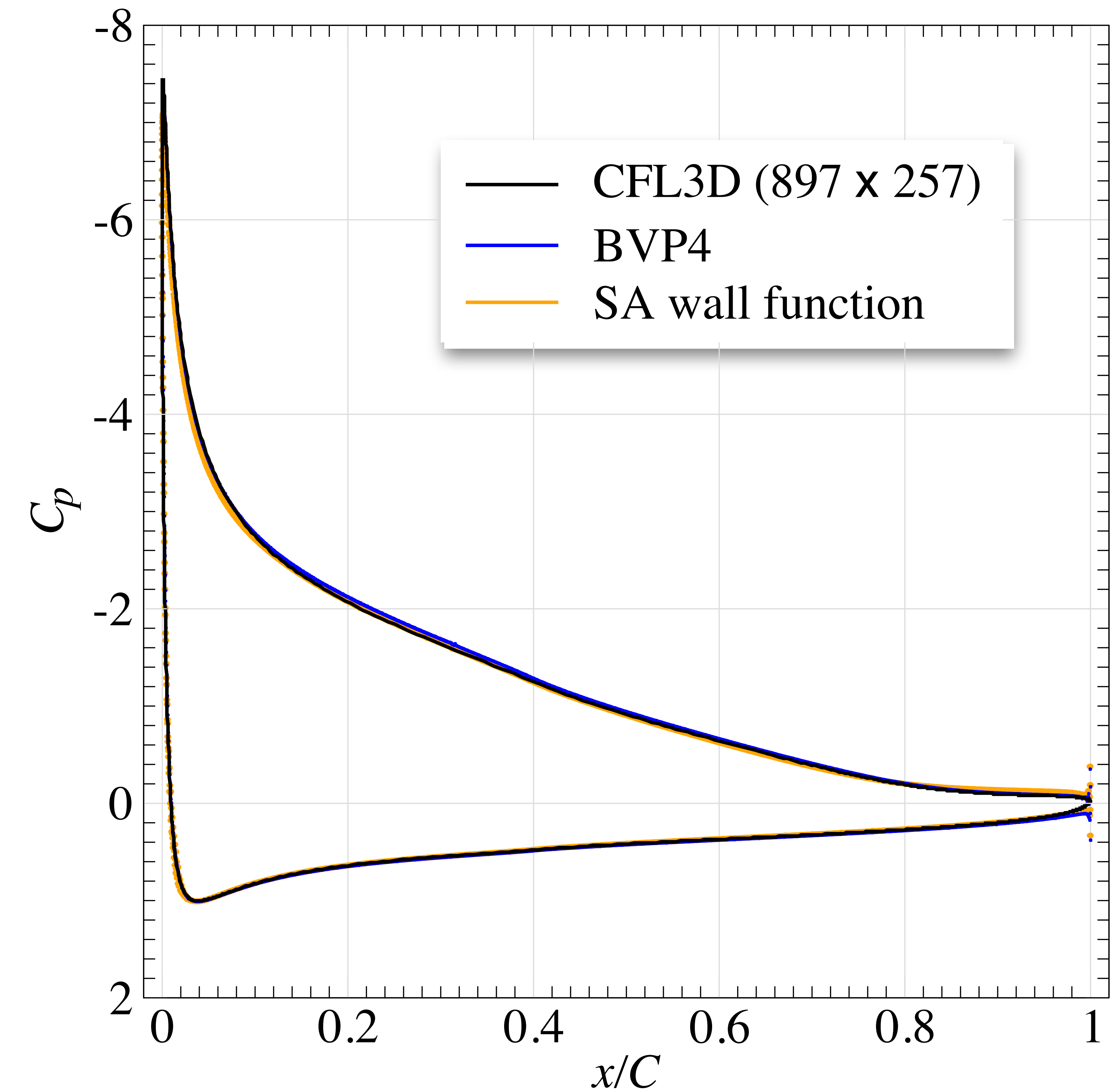
Modified NACA 4412 – Surface pressure comparison

$M_\infty = 0.09$

$Re_L = 1.52 \times 10^6$

$\alpha_\infty = 13.87°$

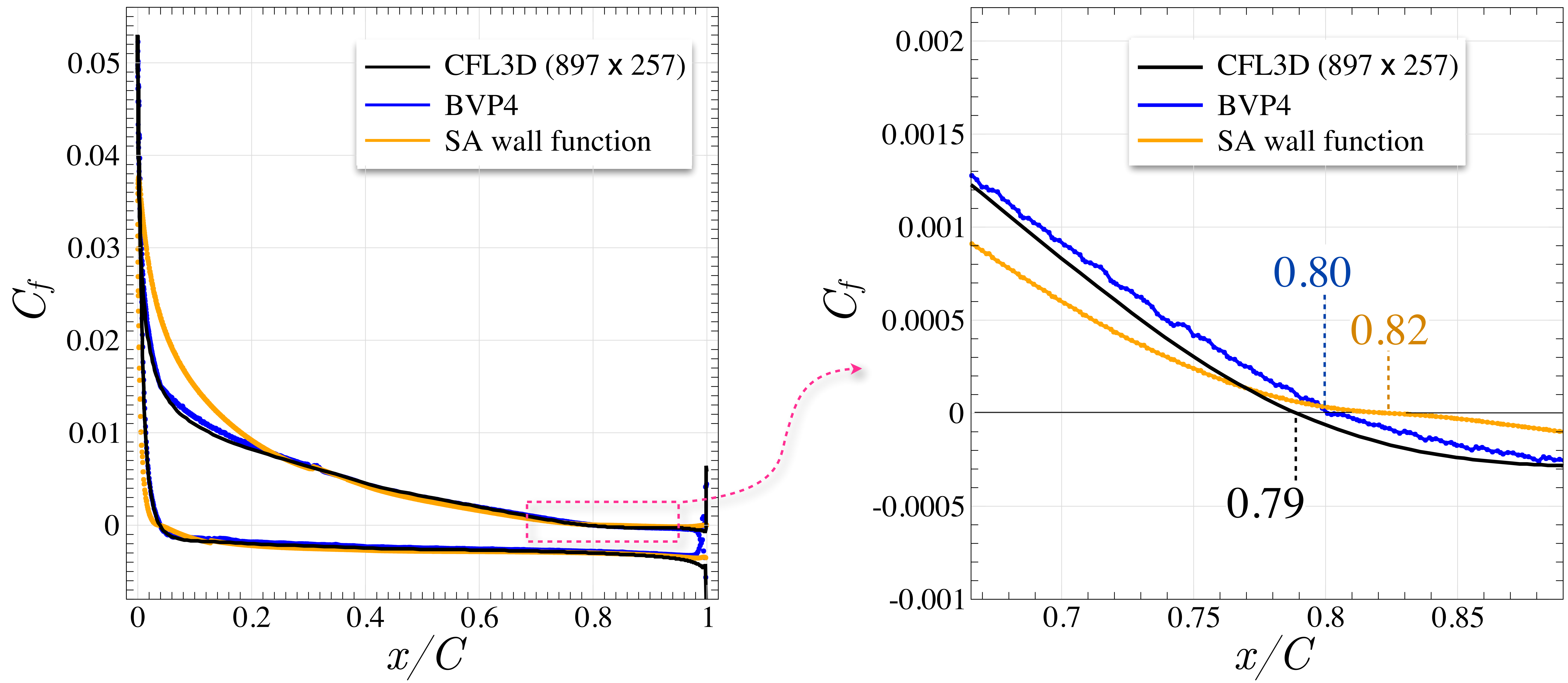




- Good comparison of surface pressure coefficient with both models
- SA-wall function & bvp4 nearly indistinguishable from CFL3D results

NACA 4412

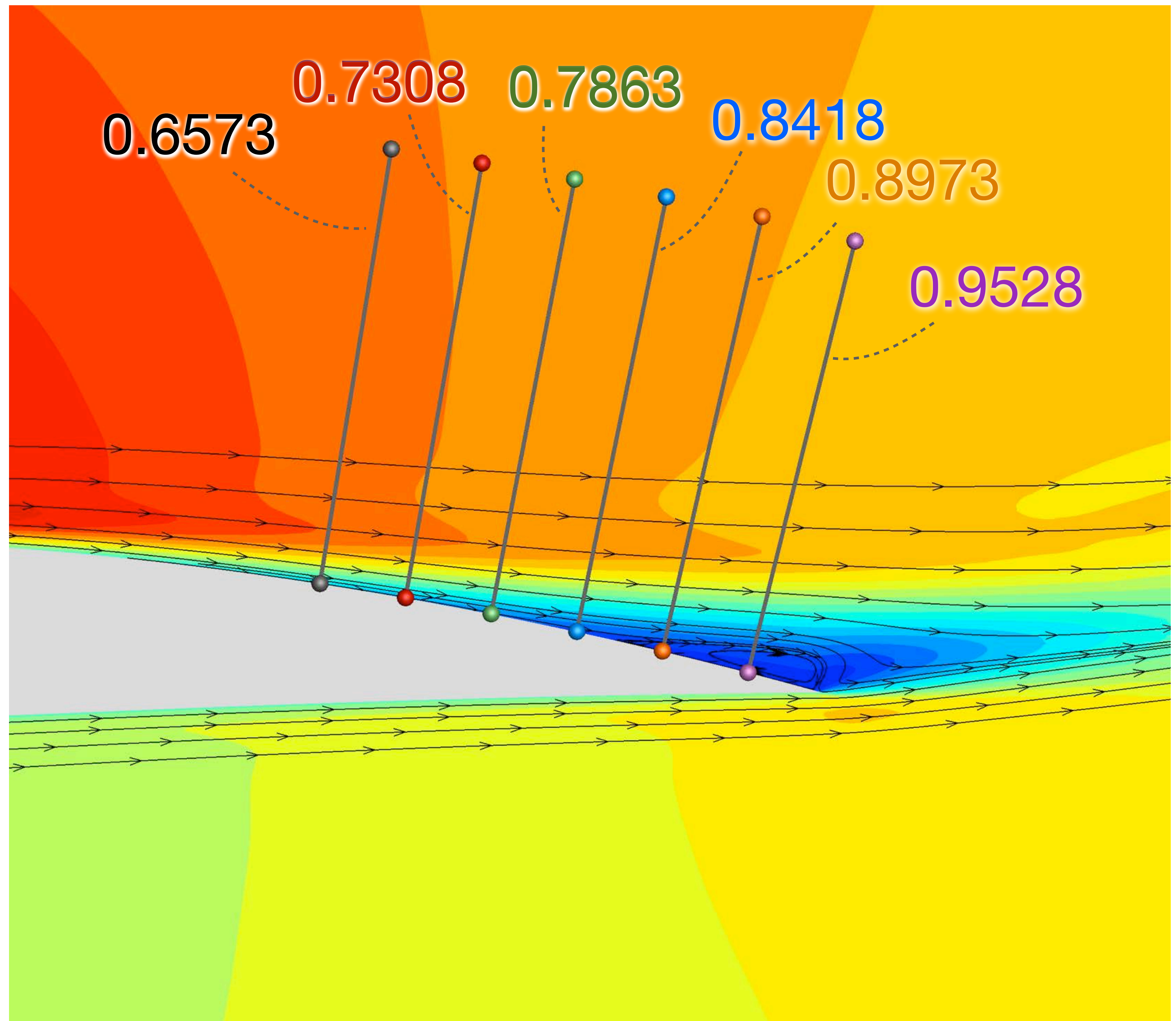
Modified NACA 4412 – Skin friction comparison



- bvp4 substantially outperforms wall function near leading edge with thin boundary-layer & steep pressure gradient
- bvp4 predicts separation location within 1% of mesh resolved CFL3D result
- Noise in bvp4 due to interpolation of ($u\, u_x$ & $v\, u_y$) at forcing point

NACA 4412

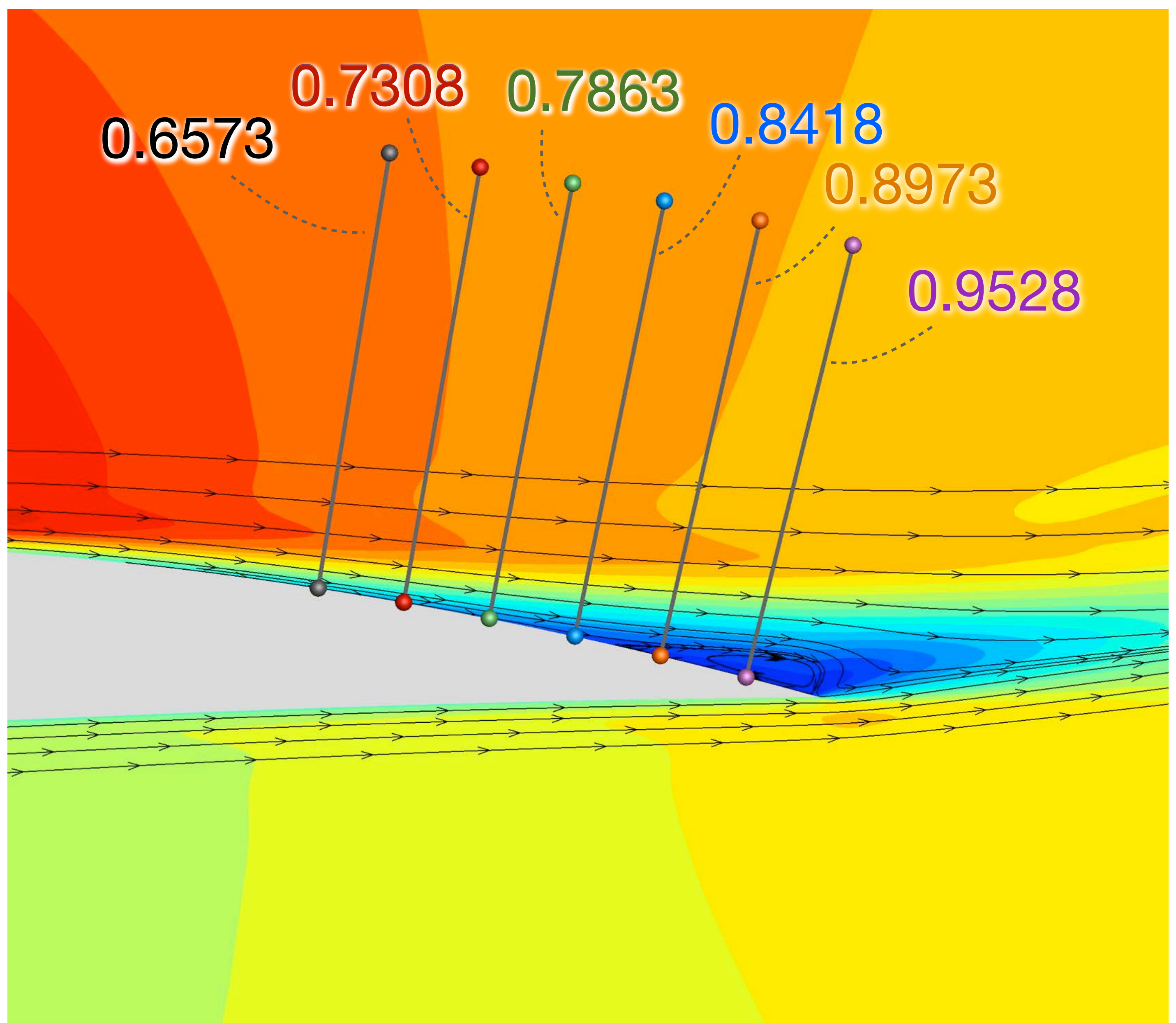
Modified NACA 4412 – Velocity comparison near separation

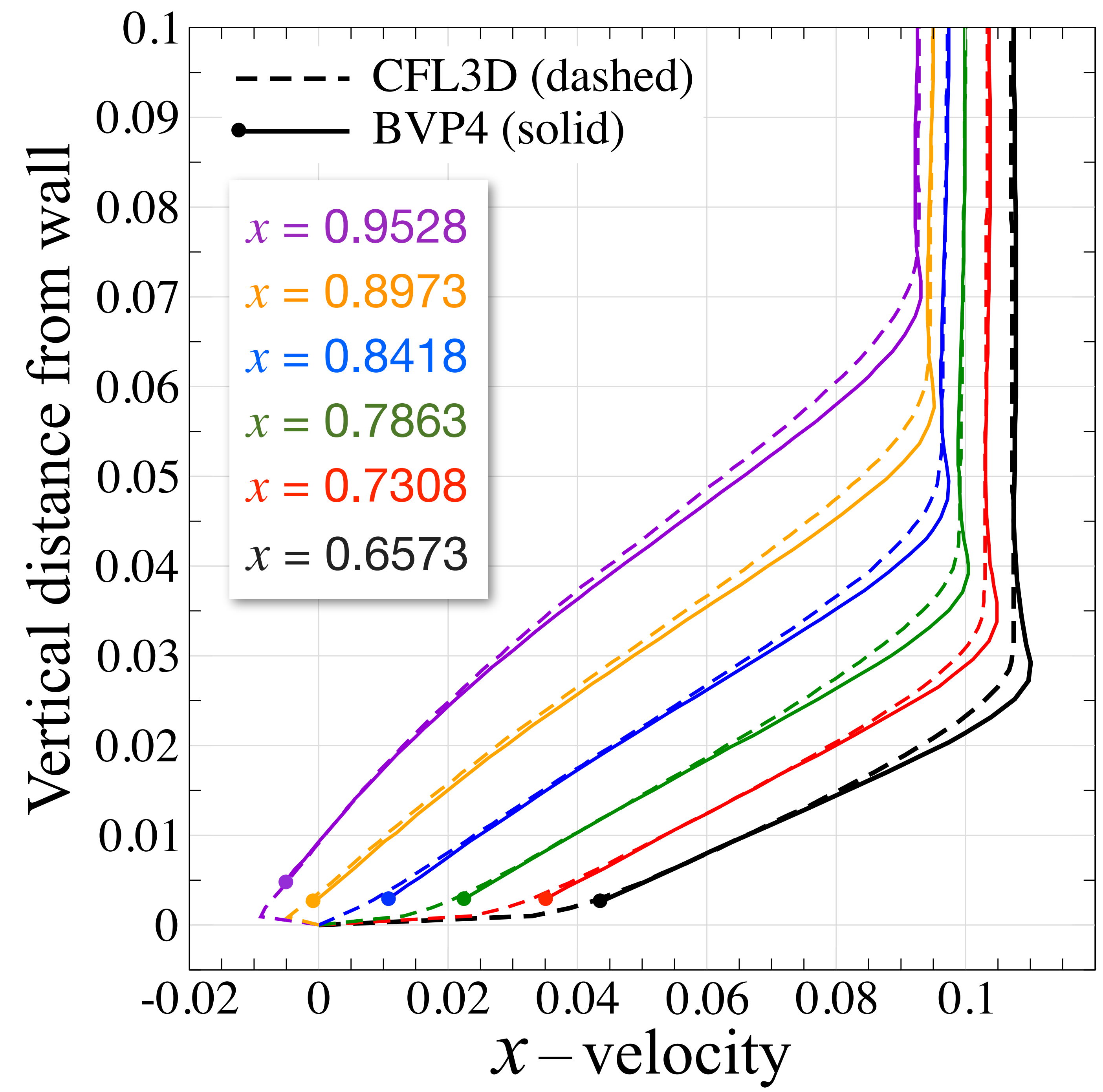


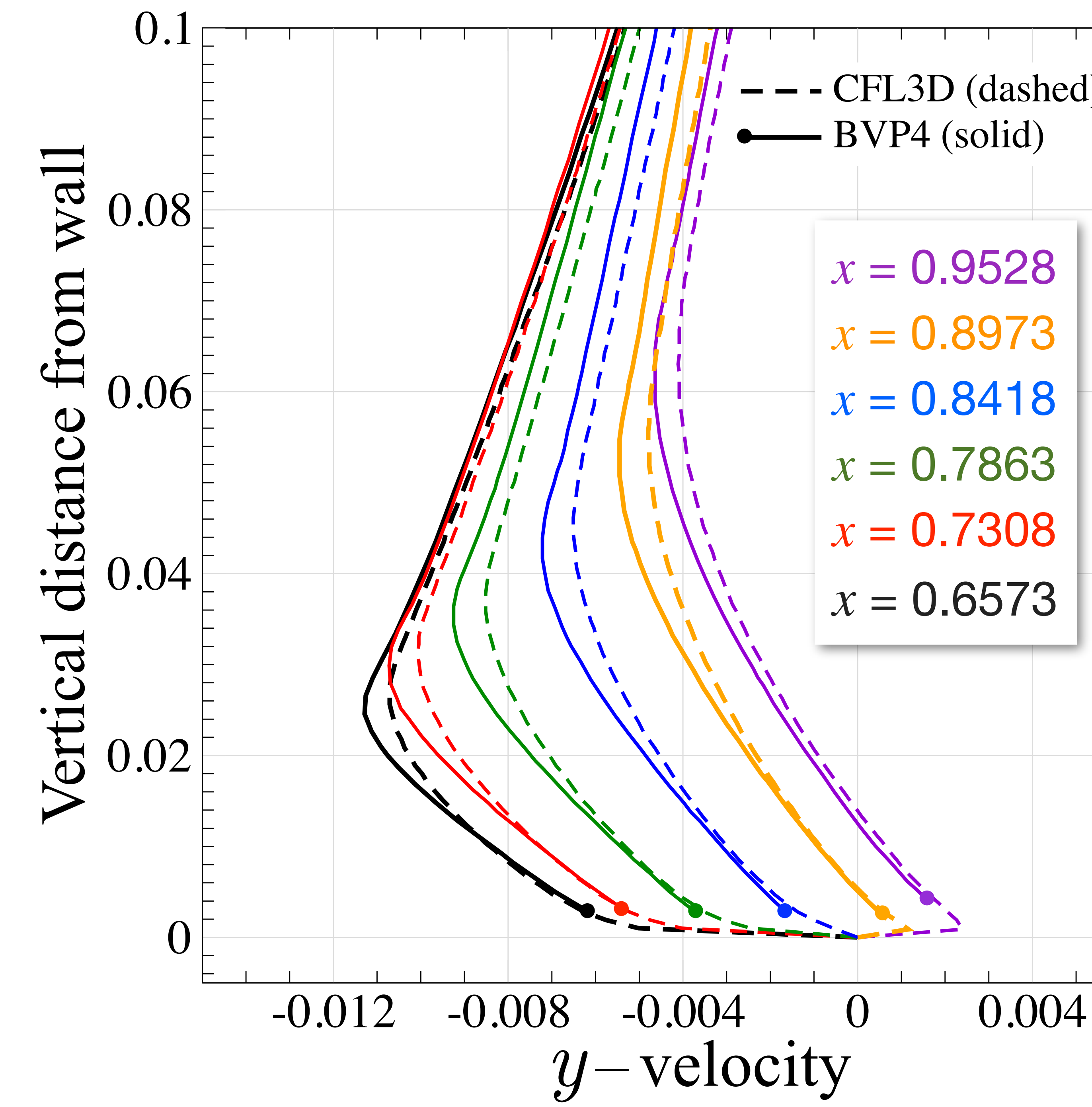
Locations of hot-wire surveys in experiment (Coles & Wadcock, 1979)

NACA 4412

Modified NACA 4412 – Velocity comparison near separation







- Good prediction of both x and y components of velocity through separation bubble
- Vertical velocity about an order of magnitude smaller than horizontal
- Slight “viscous overshoot” due to coarseness of Cartesian mesh, $\Delta x = \Delta y = 0.2\%C$

Summary

- Developed a new ODE-based wall model for RANS equations & presented initial V&V studies. Demonstrated on several well-studied flows including one with smooth body separation

- $\boldsymbol{bvp4}$ model:
  - Solves a coupled set of ODEs posed as two-point boundary value problems for the streamwise velocity and the turbulent viscosity
  - Includes both the streamwise pressure gradient and the convective balance and is valid at significantly larger values of y^+ than analytic wall functions
  - All the ODE solves are completely local and are driven by data at a single field point
  - Computational cost is about 2-3x that of analytic wf's on same mesh

✓ Permits wall spacing on the outer mesh that is 4 to 8 times coarser than is possible with analytic wall functions

✓ Can be used on both body-fitted & non-body fitted meshes

✓ Shows promise for non-body-fitted RANS

ToDo List

- Driving source of error in most of the simulations to date has been discretization error on Cartesian mesh - Do we need to improve the Cartesian mesh scheme?
- Issues with interpolation stencil for forcing points:
  - Smoother interpolation from background mesh to forcing points
  - Adaptation interfaces in streamwise direction against body
- Examine issues with re-laminarization near the leading edge and when the boundary layer is extremely thin near stag point - look at standard recommendations on background levels of ν_{SA}
- Mesh generation module for enriching an existing mesh for RANS simulation

Acknowledgements

Many many thanks!

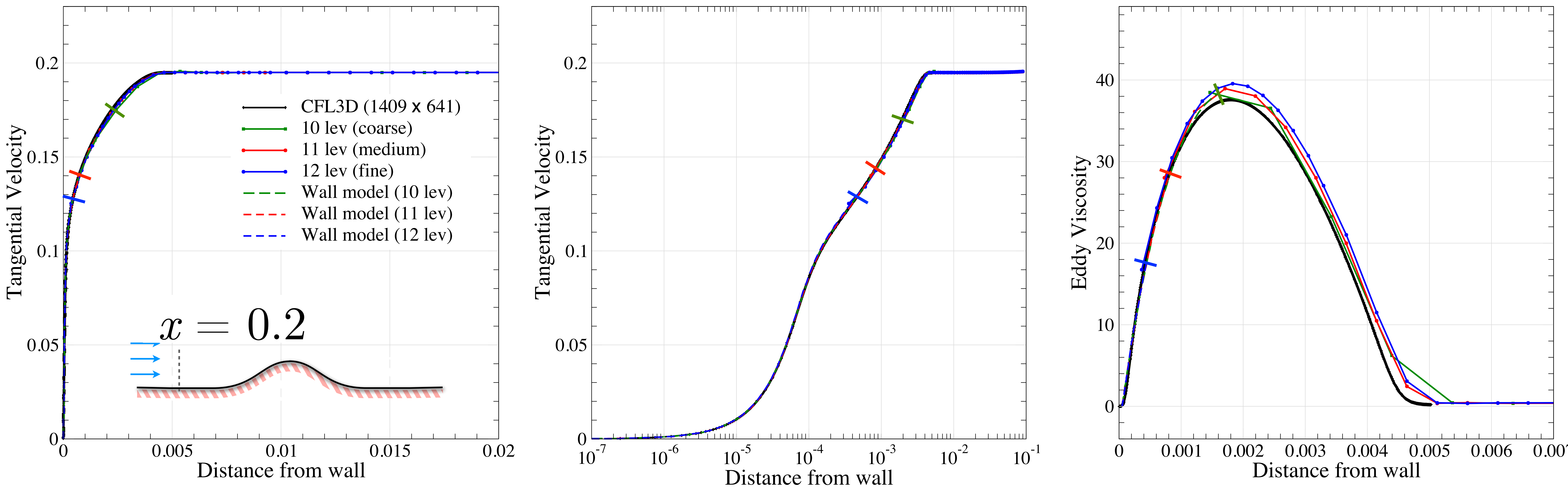
- Marsha Berger was supported by grants through the DOE and AFOSR
- NASA Turbulence Modeling Resource *https://turbmodels.larc.nasa.gov*
- Chris Rumsey (NASA LaRC)
- Marian Nemec (NASA ARC)
- Thomas Pulliam (NASA ARC)
- Steven Allmaras (MIT)
- Mike Olsen (NASA ARC)

Questions?

Backup

Turbulent Bump In Channel

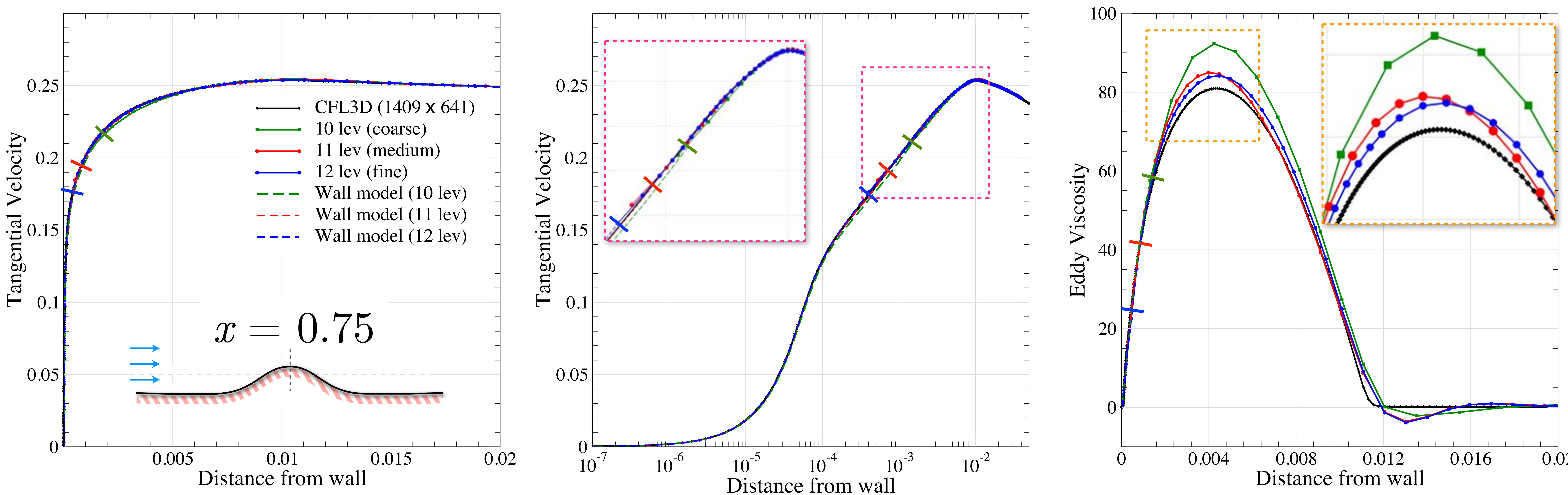
Bump: Boundary layer velocity and eddy viscosity profiles



- 7 curves on each plot, (wall model & field solution) x (coarse, med, fine) + CFL3D
- Very good agreement for velocity, good agreement for eddy viscosity
- $x = 0.2$ is the most under resolved station, ~4-5 Cartesian cells in boundary layer

Turbulent Bump In Channel

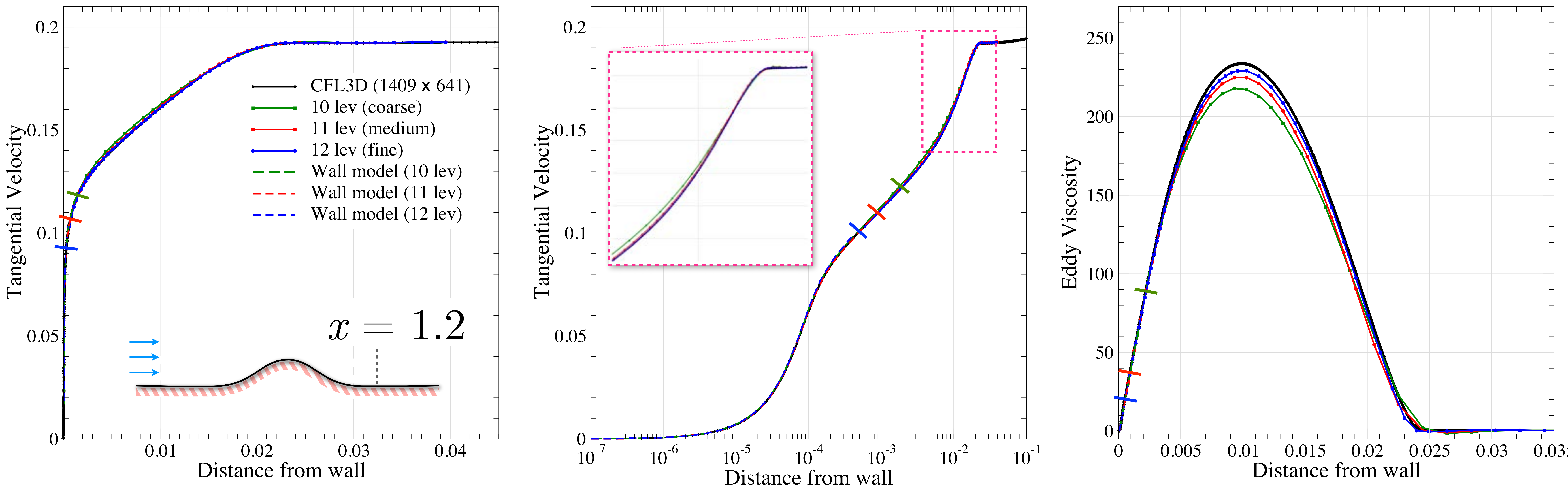
Bump: Boundary layer velocity and eddy viscosity profiles



- About twice as much Cartesian resolution at $x = 0.75$
- Profile shows effects of moderate favorable pressure gradient on the front of the bump
- Data from the wall model collapses to reference solution regardless of outer resolution
- Eddy viscosity slightly overpredicted due to lack of resolution in outer mesh

Turbulent Bump In Channel

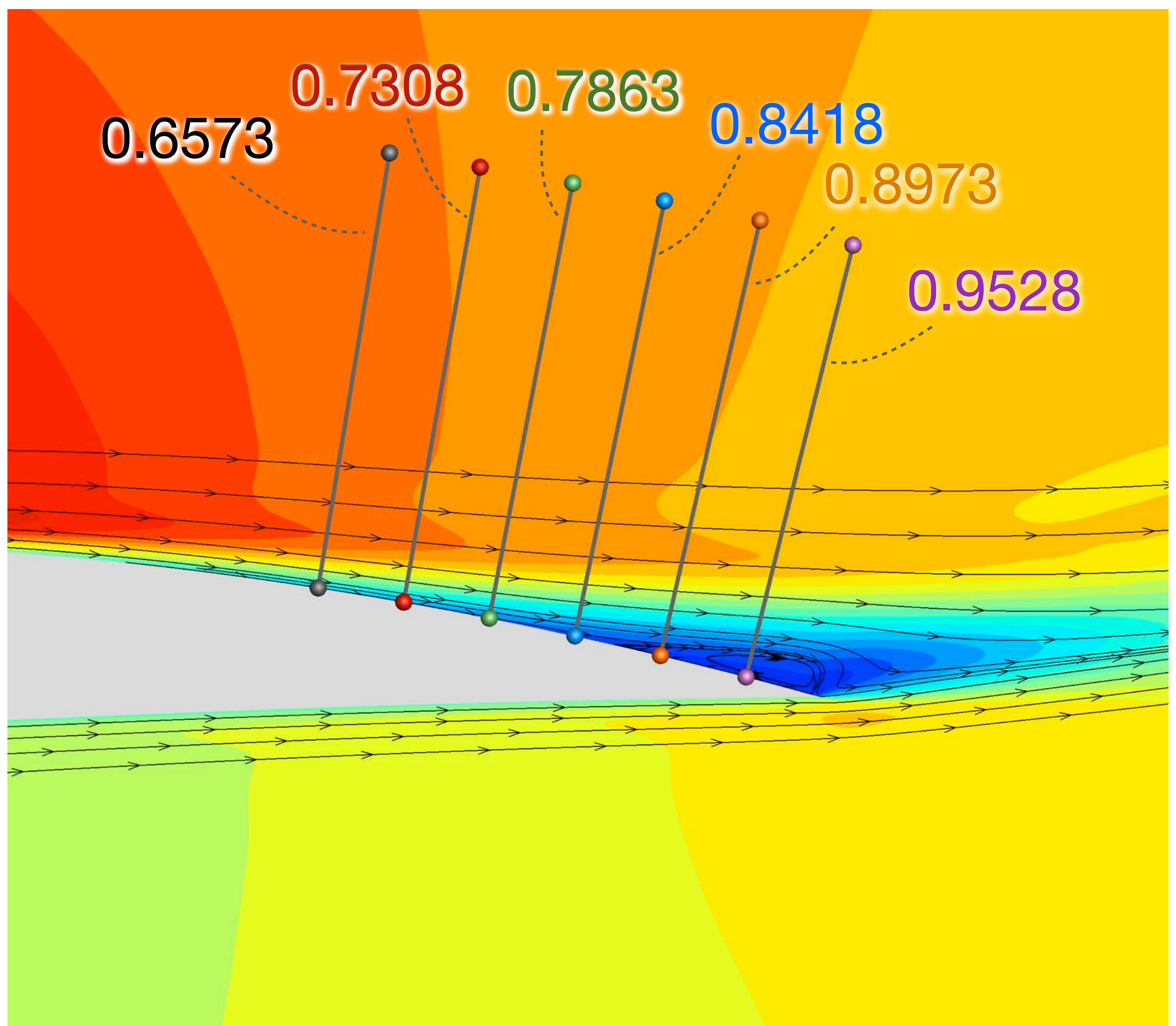
Bump: Boundary layer velocity and eddy viscosity profiles

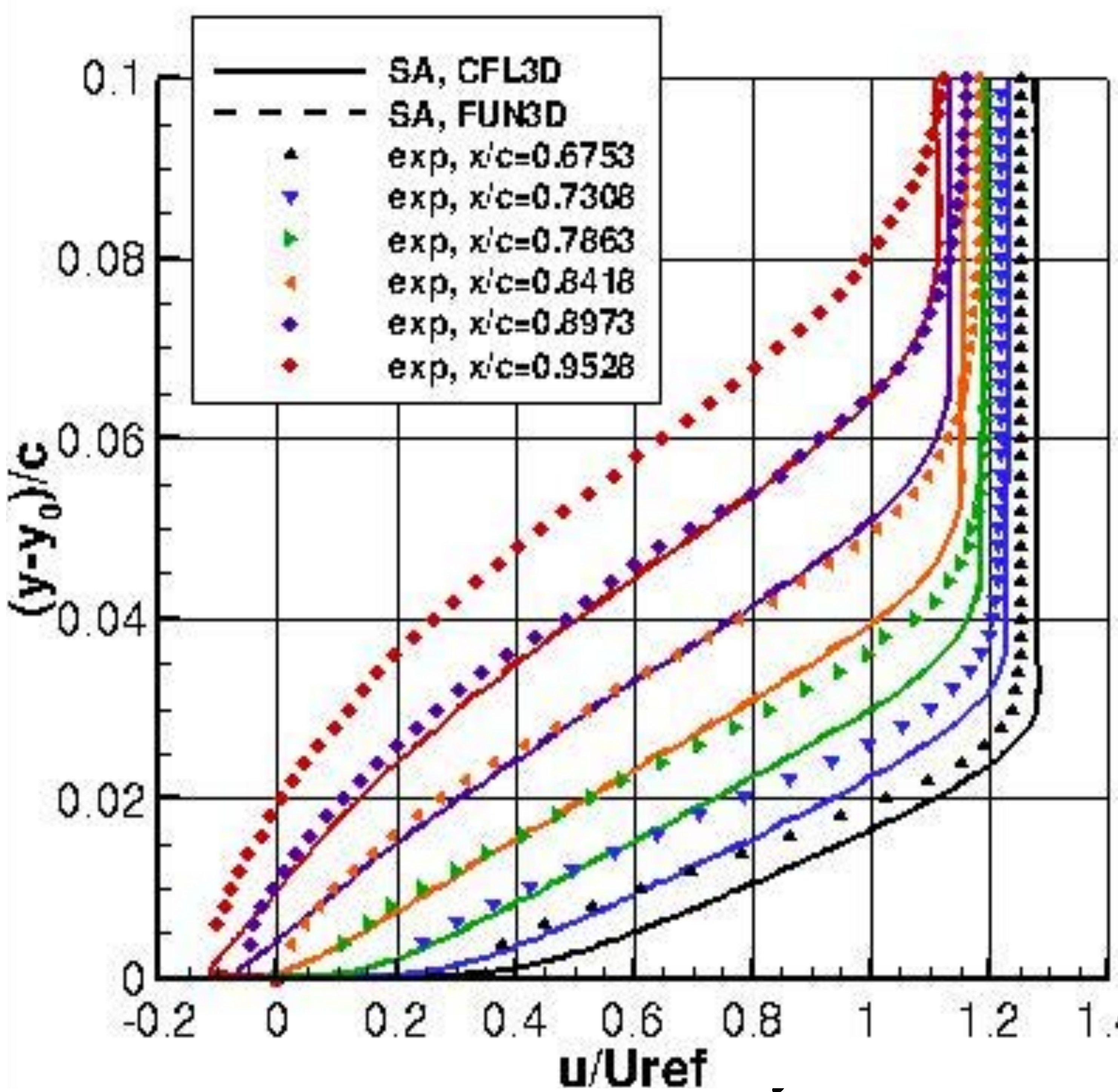


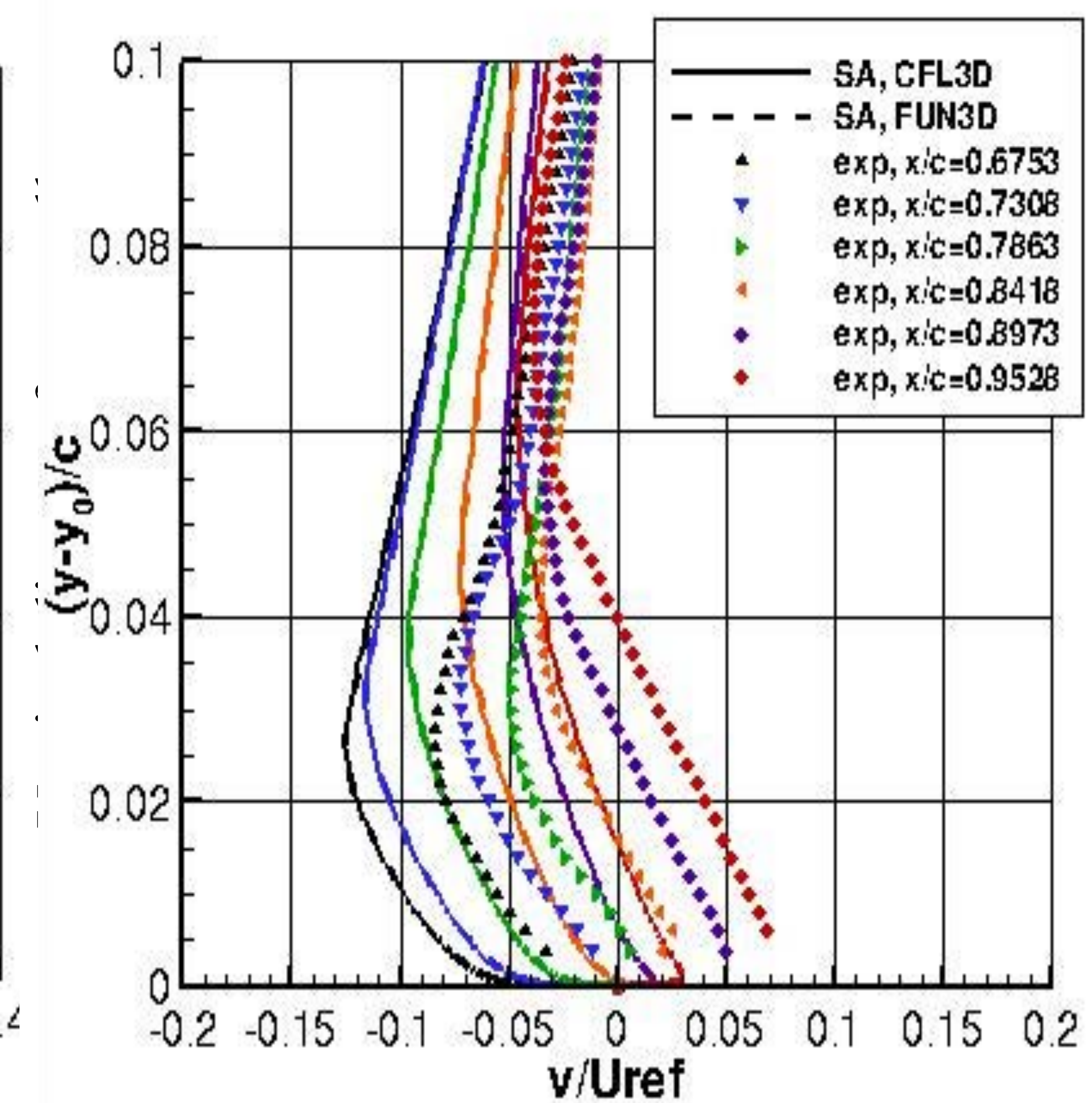
- Aft of bump, slight adverse pressure gradient, thick boundary layer
- Velocity profiles show very good agreement -- even on semi log scale
- Eddy viscosity peak being eroded slightly by dissipation on outer mesh

NACA 4412

Modified NACA 4412 – SA model vs experiment data near separation





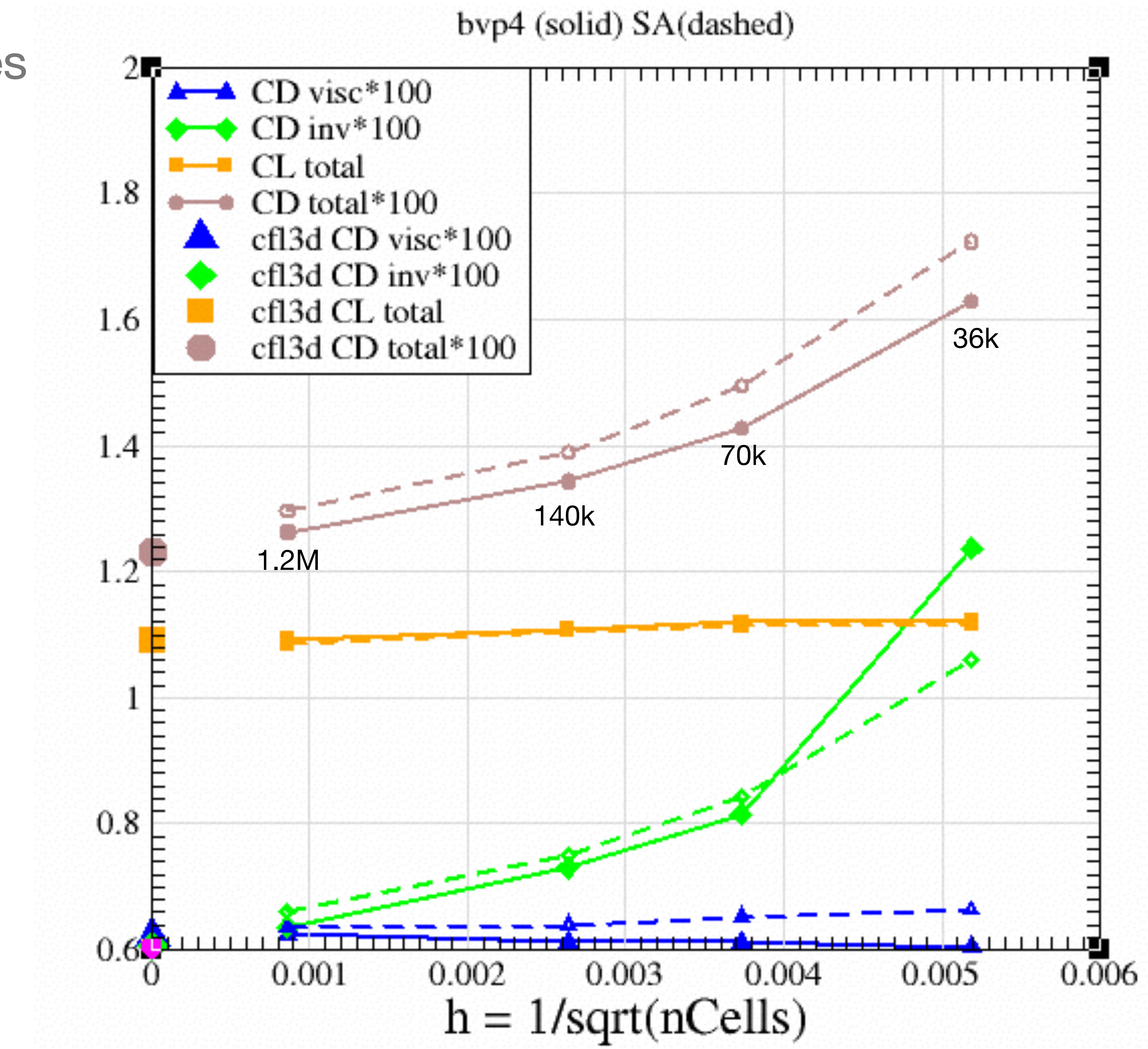


- Very sensitive example - near CL max
- SA makes some error in velocity profiles due to mis-prediction of separation location
  - Different turbulence models show up to 10%C variation in separation location
- Vertical velocity even more sensitive, since magnitudes ~10 x smaller

NACA 0012

Force Convergence, adapted meshes

Based grids on adaptive-mesh inviscid simulations with adjoint-based error control for Lift and Drag outputs



† **Data from CFL3D with SA model on “family II” grid, no point vortex correction & 2nd-order turbulent advection.**