A Perspective on Transition Modeling in Computational Fluid Dynamics

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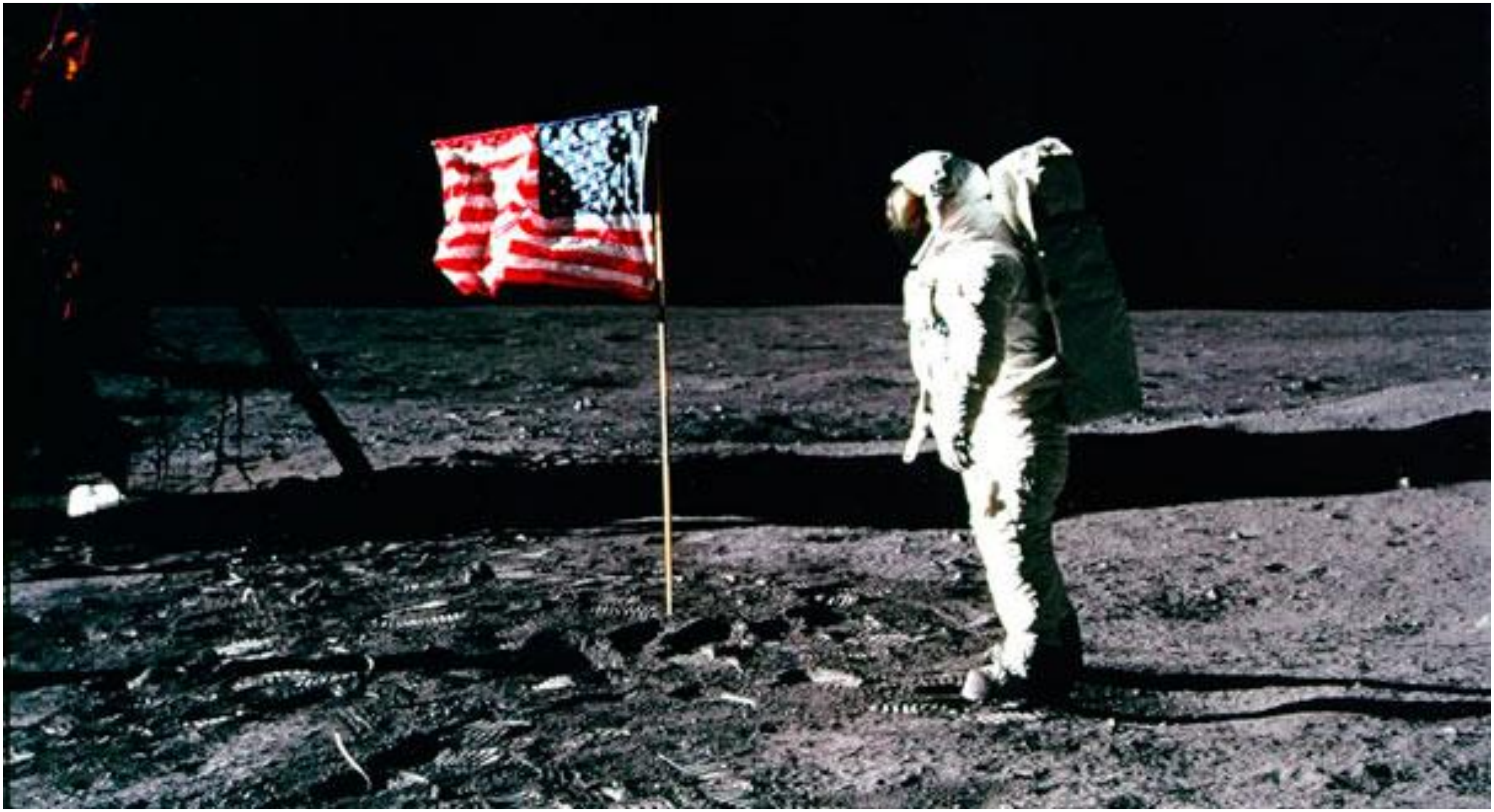
Because today is July 20th



A little bit about me

- Assistant Professor, Mechanical, Aerospace & Biomedical Engineering (MABE), 2016 – Present
  - Focus: Computational Aerodynamics

- Previously: Penn State University
  - Applied Research Laboratory, Computational Mechanics (2014– 2016)
  - PhD ('14), MS ('10), and BS ('08), all in Aerospace Engineering
  - Dissertation: "Development of a CFD-Compatible Transition Model Based on Linear Stability Theory"

- Senior Member, AIAA
  - Member, Applied Aerodynamics Technical Committee
  - Chair, APATC Education Subcommittee
  - Lead, CFD Transition Modeling Discussion Group

Outline

- Introduction
- Survey of Classical Transition Methods
- Survey of RANS-based Transition Methods
- Amplification Factor Transport Equation
- New Frontiers in CFD Transition Modeling
- Conclusion

Introduction

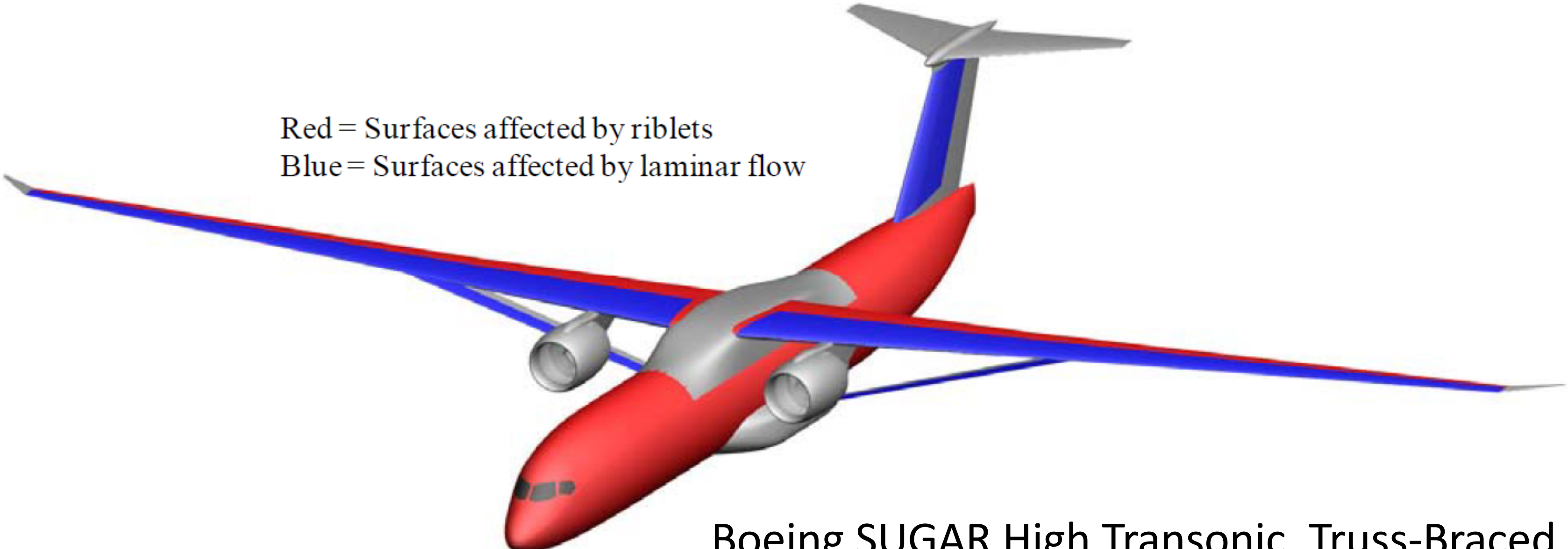
- For a commercial transport,

$$Range = aM\left(\frac{L}{D}\right)\left(\frac{1}{TSFC}\right)\ln\left(1+\frac{W_{fuel}}{W_{landing}}\right)$$

  - Propulsion → *TSFC*
  - Structures → $W_{landing}$
  - **Aerodynamics → *M*(*L*/*D*)**

Ultra-Efficient Commercial Air Vehicle Concepts

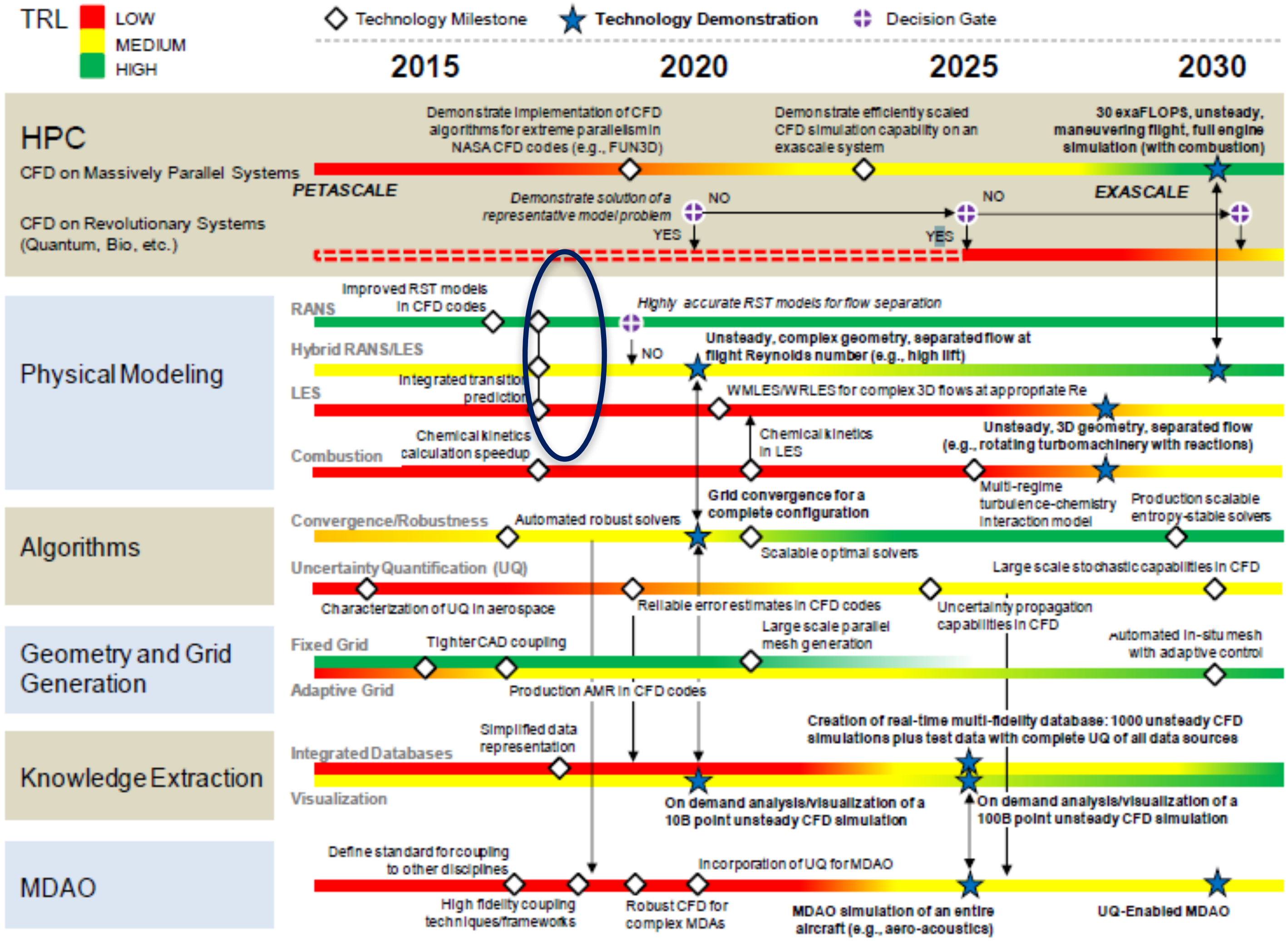
- Natural and hybrid laminar flow are an enabling technology
  - CFD is a necessity for design and analysis



Boeing SUGAR High Transonic, Truss-Braced Wing
Source: Bradley and Droney, 2011

CFD Vision 2030 Roadmap

NASA CFD Vision 2030 Technology Development Roadmap [Slotnick et al.]

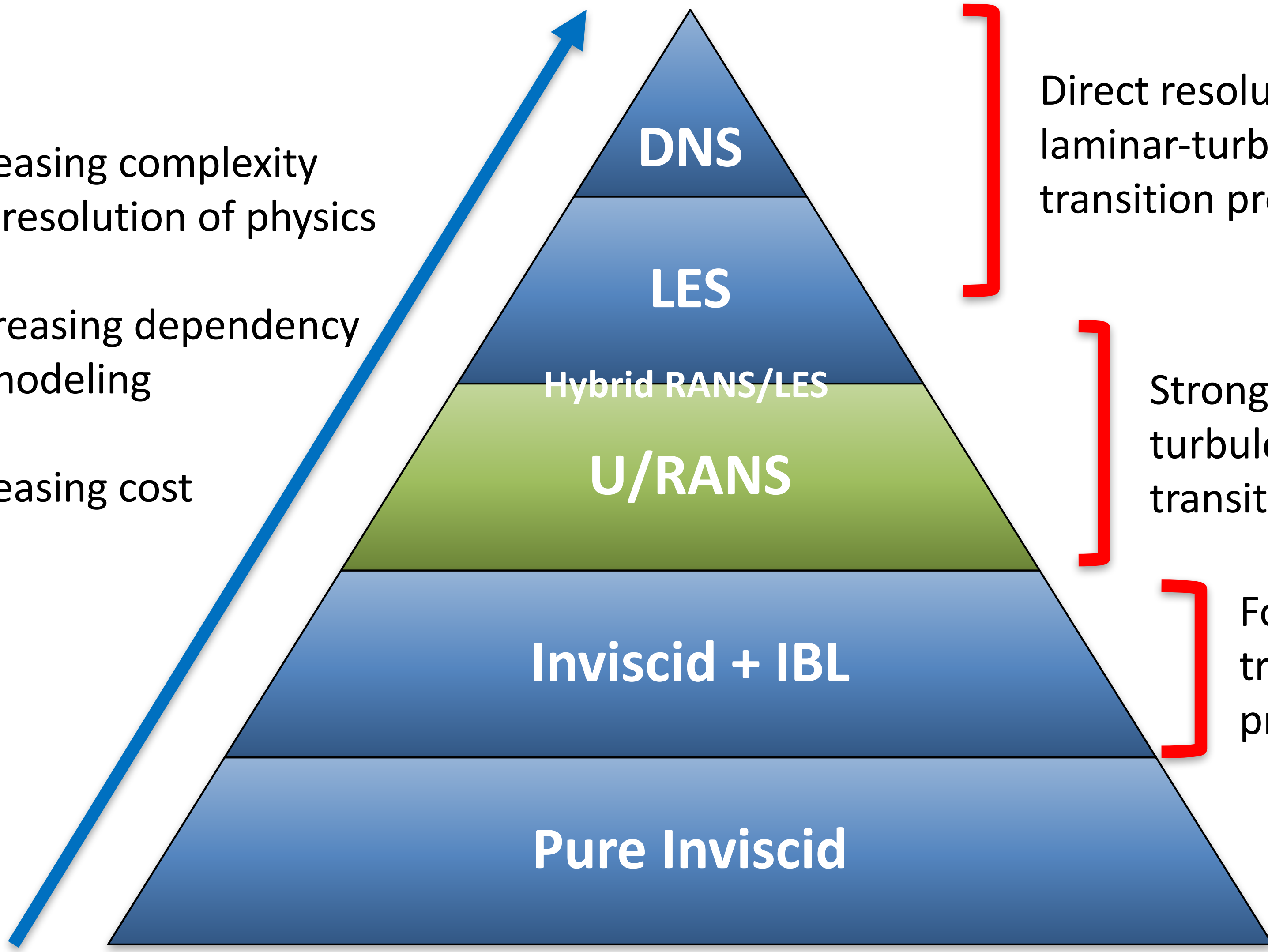


(Crude) Hierarchy of Computational Aerodynamics

Increasing complexity and resolution of physics

Decreasing dependency on modeling

Increasing cost



Direct resolution of laminar-turbulent transition process

Strong emphasis on turbulence modeling, transition neglected

Focus of classic transition-prediction methods

To CFD or not to CFD....

Aerodynamics CFD development driven by transonic/supersonic aircraft

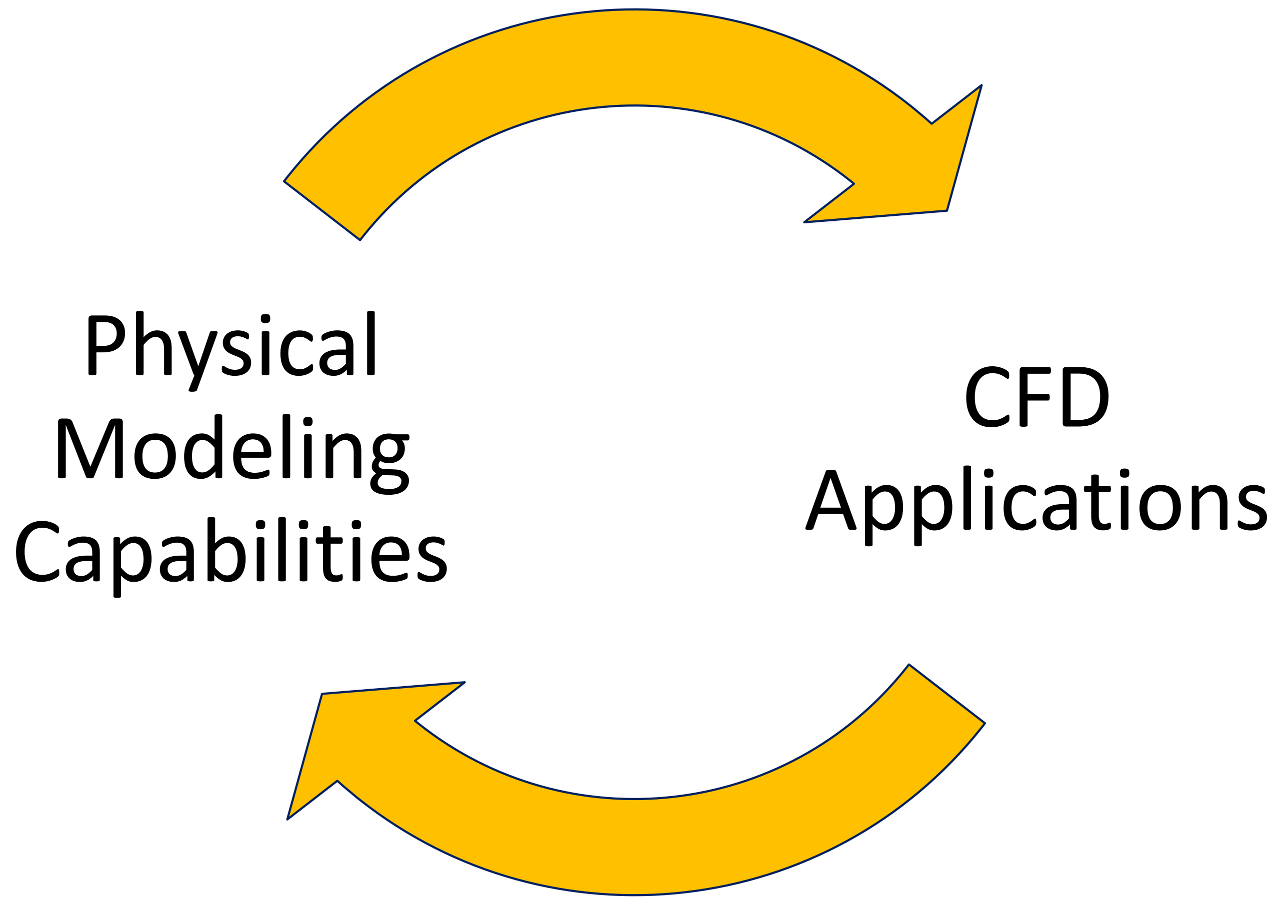
Traditional laminar-flow applications are low speed

- "Full" CFD not needed
- Inviscid + IBL are more trusted

Ventus 3 sailplane
[schempp-hirth.com]

The result: an analysis schism based on flight regimes

A Vicious (Viscous?) Circle



Technology Barriers

Inviscid + IBL Methods	RANS Methods
Segregation of viscous and inviscid flow regimes	Full coupling of viscous and inviscid regimes
Integral boundary-layer methods are a cornerstone of understanding	Much attention placed on turbulence closure – locally defined transport PDEs
Extensive development of transition models based on IBL properties	Highly specialized, scalable, parallelized solution algorithms

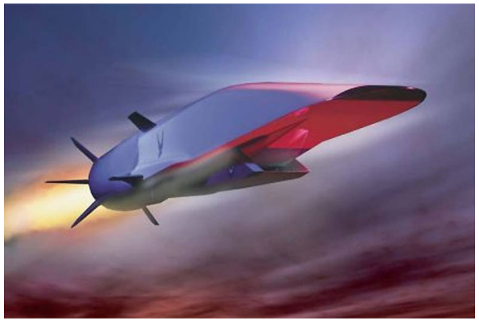
Where do we need to go?

More than just natural transition on fixed-wing aircraft

LFC systems on existing aircraft

Rotorcraft

Hypersonics



Turbomachinery

Transition modeling capabilities need to keep up with the CFD state of the art ("CFD Compatibility")

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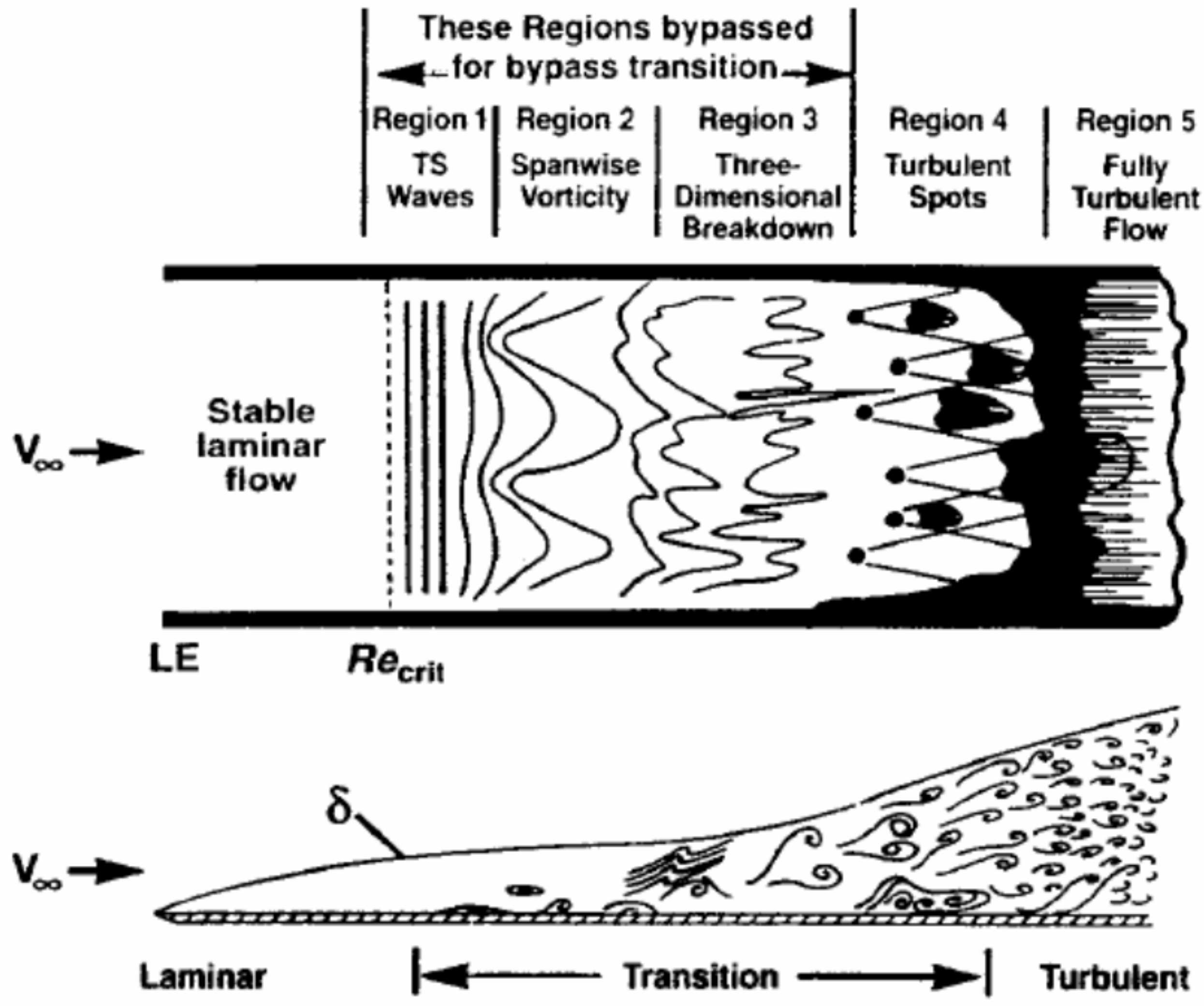
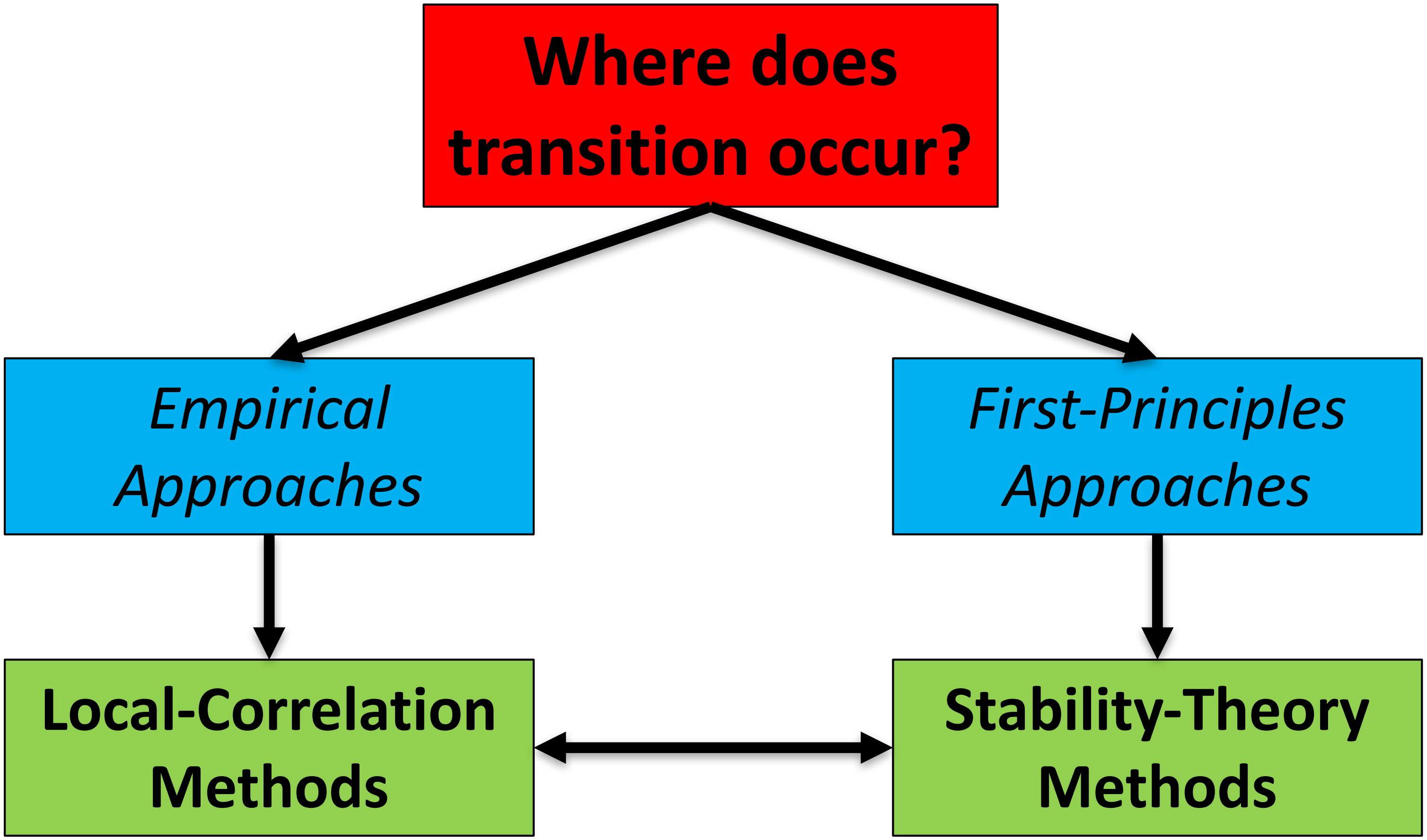
The Transition Process



Source: H. Schlichting, *Boundary Layer Theory*, 1975.

"Classical" Modeling Approaches



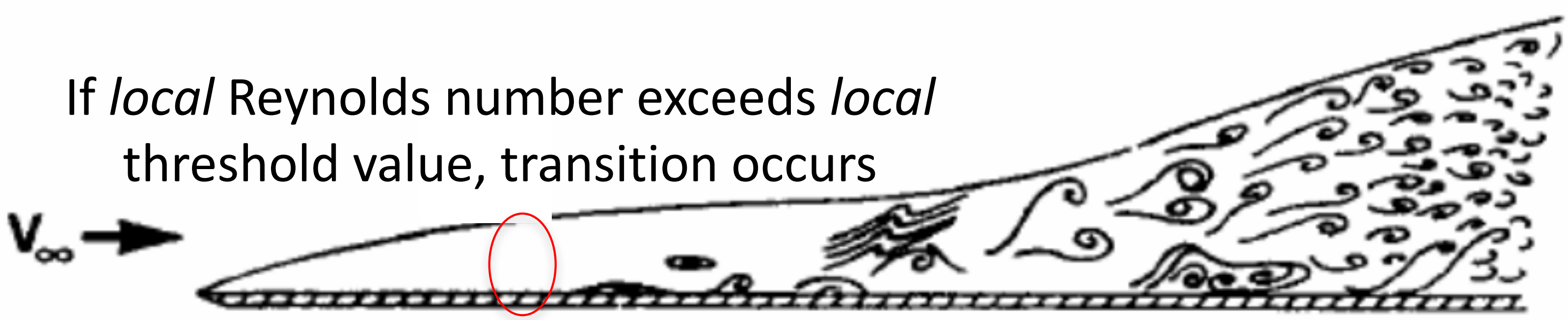
Local-Correlation Methods – An Overview

Pressure-gradient parameter
"Health"

Empirical correlation →

Transitional Reynolds number
"Age"

If *local* Reynolds number exceeds *local* threshold value, transition occurs



Example:

- Abu-Ghannam and Shaw correlation (1980)

$$Re_{\delta 2,transition} = 163 + \exp\left[F\left(\Lambda_2\right)\left(1 - \frac{Tu\left(\%\right)}{6.91}\right)\right]$$

$$F\left(\Lambda_2\right) = \begin{cases} 6.91 + 12.75\Lambda_2 + 63.34\Lambda_2{}^2 & \left(\Lambda_2 \leq 0\right) \\ 6.91 + 2.48\Lambda_2 - 12.27\Lambda_2{}^2 & \left(\Lambda_2 > 0\right) \end{cases}$$

  - Developed primarily for turbomachinery flows
  - Well-suited for bypass transition

Stability-Theory-Based Methods

Linear-Stability Theory

- Small velocity and pressure perturbations
- Perturbation streamfunction assigned a spectral form

$$\psi' = \phi(y)\exp\left[i\left(\alpha x + \beta z - \omega t\right)\right]$$

- For fully developed, parallel flow,

$$\left[\left(\alpha U + \beta W - \omega\right)\left(\frac{d^2}{dy^2} - \alpha^2 - \beta^2\right) - \left(\alpha\frac{d^2U}{dy^2} + \beta\frac{d^2W}{dy^2}\right)\right]\phi = \frac{\nu}{i}\left(\frac{d^2}{dy^2} - \alpha^2 - \beta^2\right)^2\phi$$

Orr-Sommerfeld Equation

(a non-local eigenvalue problem)

Predicting Transition with LST

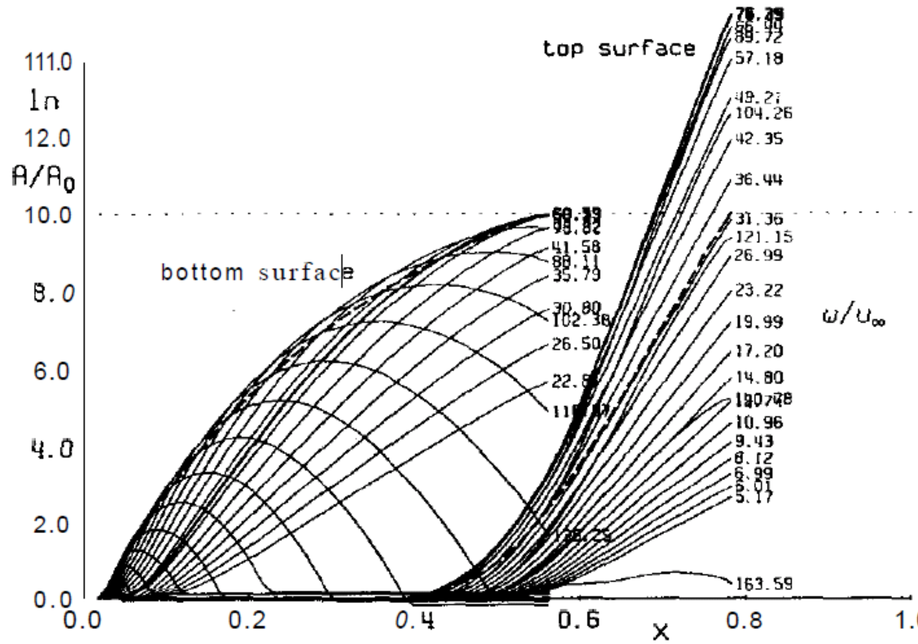
Instability growth based on boundary-layer development

$$N = \int_{s_0}^{s} -\mathrm{Im}\left[\alpha\left(\omega, \beta, Re_{\delta 2}, H_{12}\right)\right] ds$$

- Databases of pre-calculated α-β-ω solutions

e^N method (1956)

- Smith and Gamberoni (Douglas Aircraft),van Ingen (TU Delft)
- Critical amplification factor
  - $N_{crit} \approx 9$



Source: Drela (2003)

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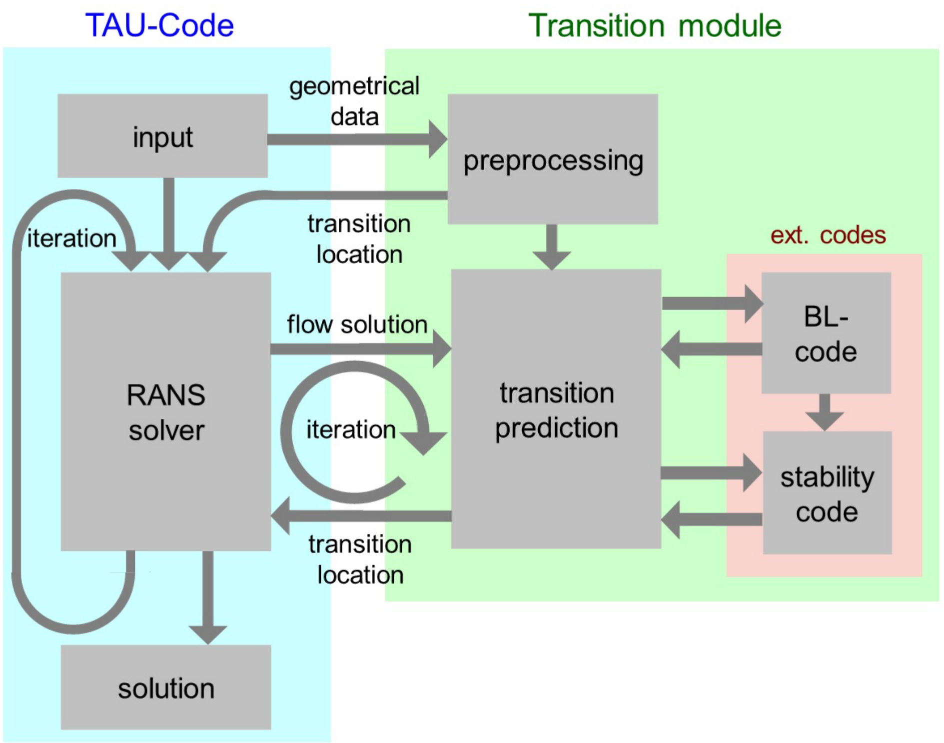
In-Situ Post-Processing

Non-CFD transition models can be coupled with a CFD solution

- RANS CFD provides boundary-layer properties and/or surface pressures
- Notable examples
  - INS2D
  - DLR TAU



Source: Krumbein et al., 2015

Can be limited to simple geometries

PDE-Based Methods

Advection-diffusion-type PDEs

- Solvable on complex geometries
- Compatible with Navier-Stokes solution algorithms
- Amenable to massive parallelization

Can be placed into two somewhat broad categories

- Direct modeling of underlying physics (“physics-based”)
- Phenomenological models (“phenomenological”)

Example "Physics-Based" Model

Walters-Leylek Transitional Turbulence Model

$$\frac{\partial(\rho k_T)}{\partial t} + \frac{\partial(\rho u_j k_T)}{\partial x_j} = P_T + R + R_{NAT} - \rho\varepsilon - D_T + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\alpha_t}{\sigma_k}\right)\frac{\partial k_T}{\partial x_j}\right]$$

$$\frac{\partial(\rho k_L)}{\partial t} + \frac{\partial(\rho u_j k_L)}{\partial x_j} = P_L - R - R_{NAT} - \rho\varepsilon - D_L + \frac{\partial}{\partial x_j}\left[\mu\frac{\partial k_L}{\partial x_j}\right]$$

$$\frac{\partial(\rho\varepsilon)}{\partial t} + \frac{\partial(\rho u_j \varepsilon)}{\partial x_j} = C_{\varepsilon 1}\frac{\varepsilon}{k_T}\left(P_T + R_{NAT}\right) + C_{\varepsilon R} R \frac{\varepsilon}{\sqrt{k_L k_T}} - C_{\varepsilon 2}\frac{\varepsilon^2}{k_T} - \frac{\varepsilon}{k_T} D_T + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\alpha_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]$$

- Based on "laminar kinetic energy"
- Successful for some applications (e.g. bypass transition)
- Emphasis on non-linear growth

Problem with First-Principles Approaches

RANS is incompatible with Linear Stability Theory

- Reynolds-averaging

$$\left| -\overline{u_i' u_j'} \right| \gg 0$$

$$\overline{U_i u_j'} = 0$$

- Linear Stability Theory

$$u_i' u_j' \to 0$$

$$U_i u_j' \neq 0$$

Phenomenological models have shown more promise

Example of Phenomenological Model

Langtry-Menter Local-Correlation Transition Model (2009)

$$\frac{\partial(\rho\gamma)}{\partial t}+\frac{\partial(\rho u_j\gamma)}{\partial x_j}=P_\gamma-E_\gamma+\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_f}\right)\frac{\partial\gamma}{\partial x_j}\right]$$

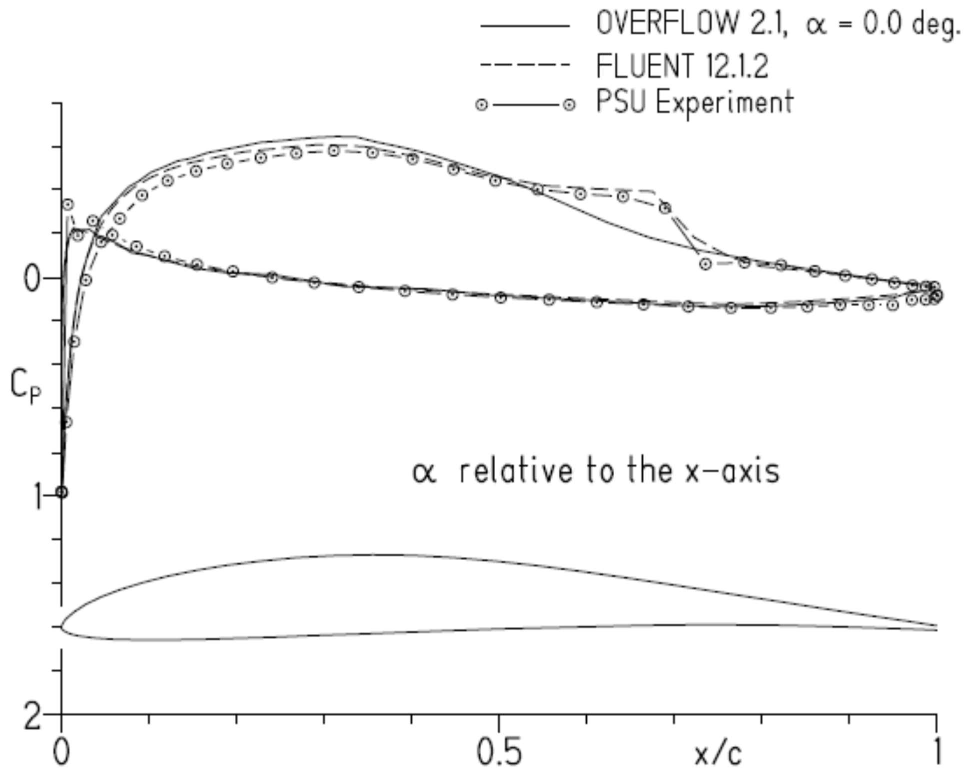
$$\frac{\partial\left(\rho\overline{Re_{\theta t}}\right)}{\partial t}+\frac{\partial\left(\rho u_j\overline{Re_{\theta t}}\right)}{\partial x_j}=P_{\theta t}+\frac{\partial}{\partial x_j}\left[\sigma_{\theta t}(\mu+\mu_t)\frac{\partial\overline{Re_{\theta t}}}{\partial x_j}\right]$$

- Coupled to popular k-ω SST eddy-viscosity model
- Effectively a PDE version of a Λ_2-$Re_{\delta 2}$ model

Publication of this model was a watershed moment in CFD transition modeling

Impact of Langtry-Menter Model

Eppler E 387, Re = 3 x 10^5



(a) α = 0.0 deg.

Source: Maughmer and Coder, U.S. Army RDECOM TR 10-D-106

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Motivation

Author’s contribution to CFD transition modeling: the amplification factor transport (“AFT”) equation

$$\frac{\partial(\rho\tilde{n})}{\partial t}+\frac{\partial(\rho u_j\tilde{n})}{\partial x_j}=\rho\Omega F_{crit}F_{growth}\frac{d\tilde{n}}{dRe_{\delta 2}}+\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_n}\right)\frac{\partial\tilde{n}}{\partial x_j}\right]$$

- Inspired by the success of the Langtry-Menter model
- PDE implementation of the approximate-envelope transition model
  - Based on linear stability theory

AFT Development

Streamwise growth of amplification factor implies an advection equation

$$u_j \frac{\partial \tilde{n}}{\partial x_j} = U \frac{d\tilde{n}}{ds} = U \frac{d\tilde{n}}{dRe_{\delta 2}} \frac{dRe_{\delta 2}}{ds}$$

- The catch: valid only on the inviscid surface streamline with IBL

RANS solvers don't "think" in terms of IBL properties

- Flow solution is just state variables at points in space
- Integral quantities not readily available, and counter to use of PDE methods

AFT Development

Integral properties estimated and correlated using a local shape factor

AFT2014:

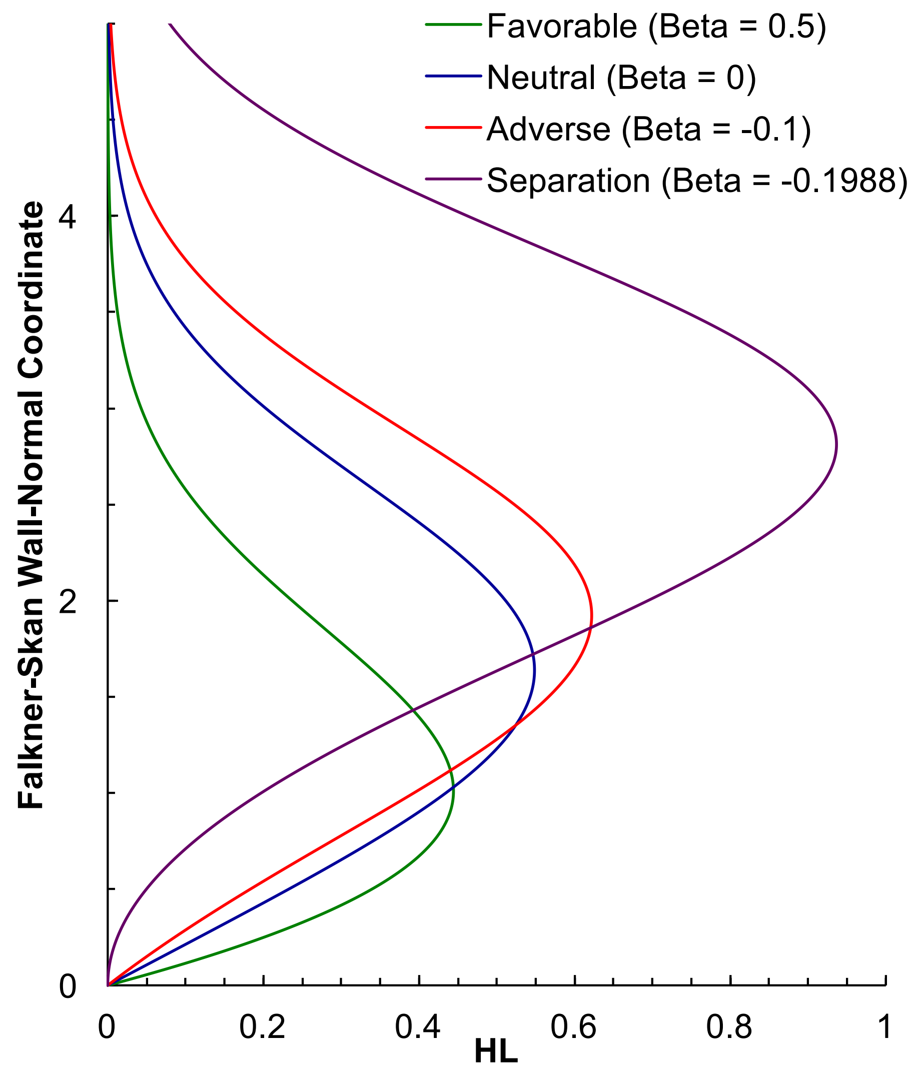
$$H_L = \frac{Sd}{U_e}$$

- U_e determined based on isentropic flow assumption

AFT2017:

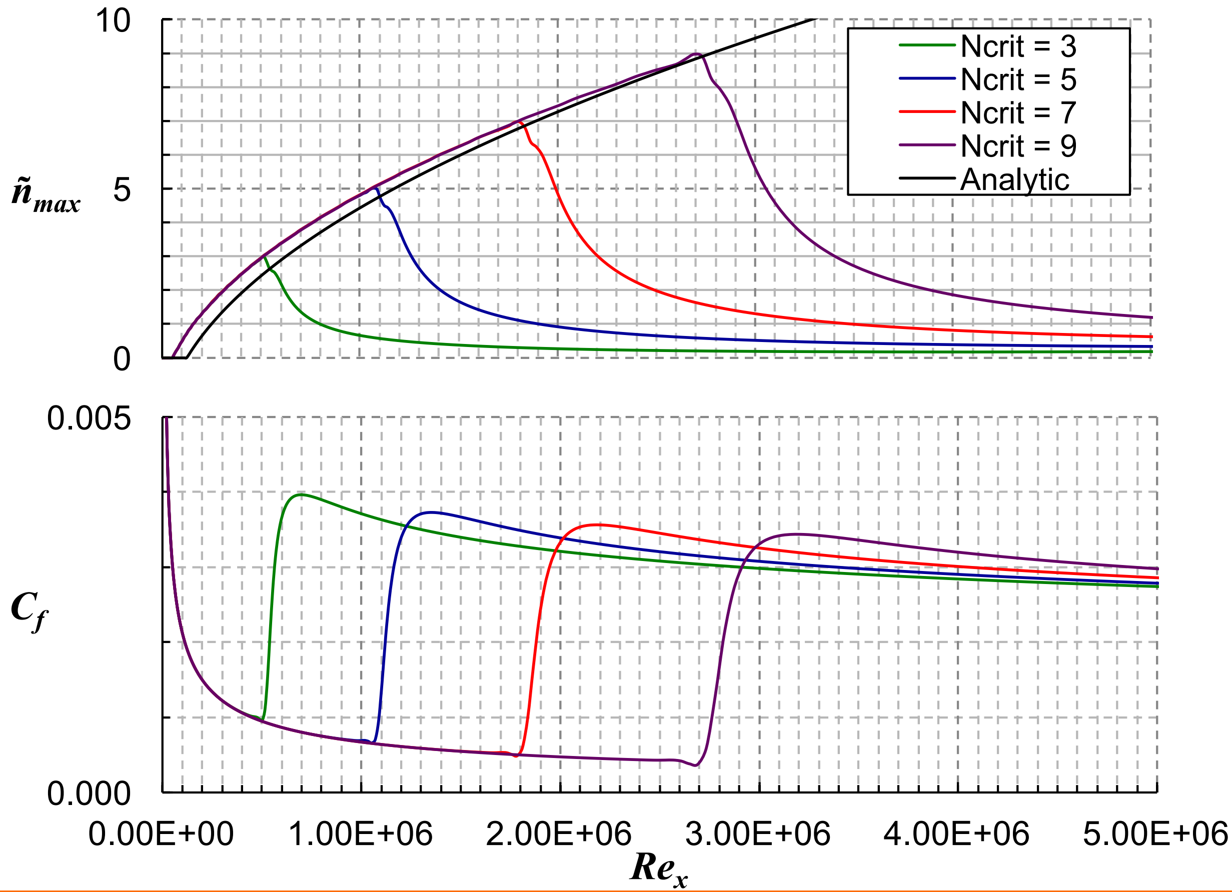
$$H_L = \frac{d^2}{\mu}\left[\nabla\left(\rho\vec{u}\cdot\nabla d\right)\cdot\nabla d\right]$$

- Wall-normal gradient of wall-normal momentum



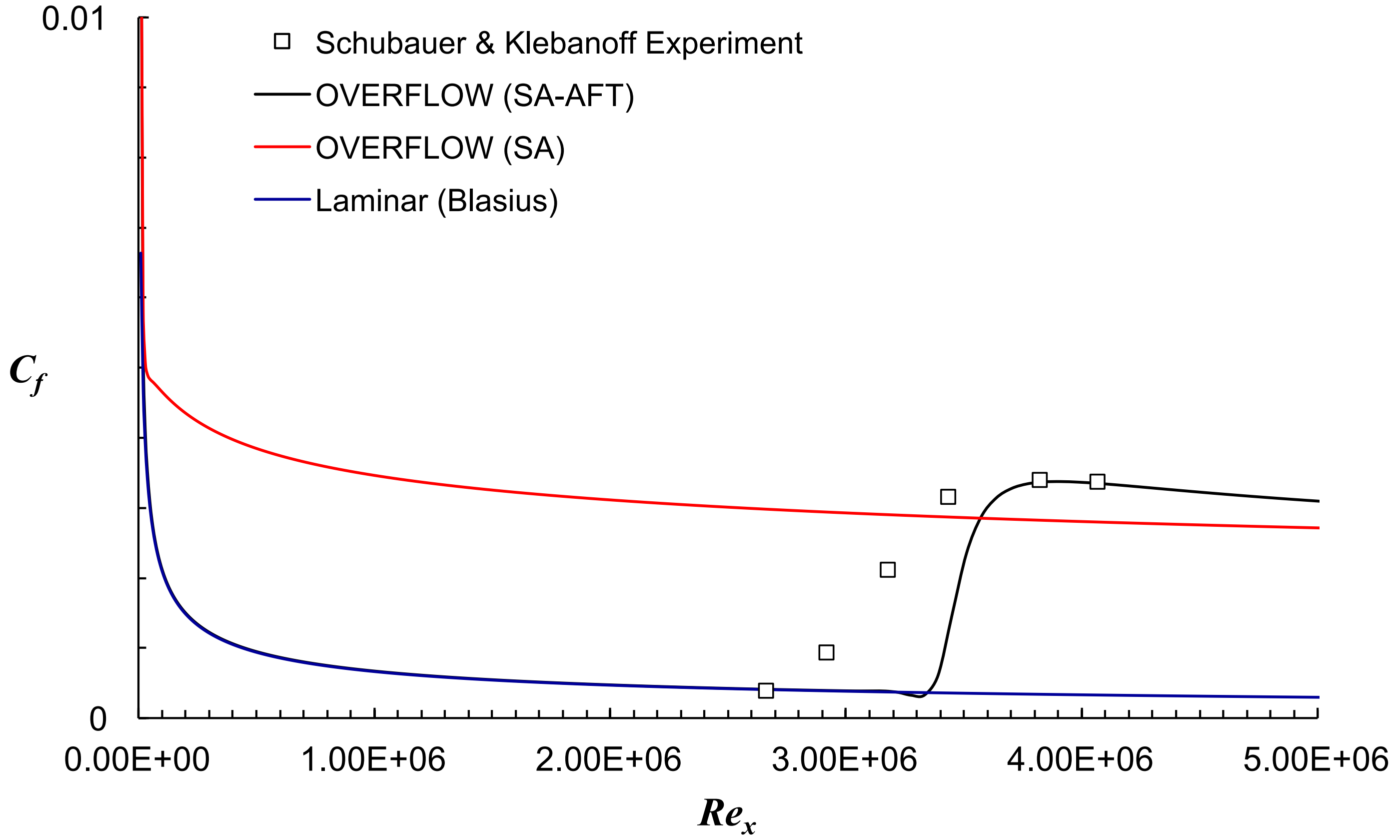
Zero-Pressure-Gradient Flat Plate

Growth of transported amplification factor, SA-AFT2014



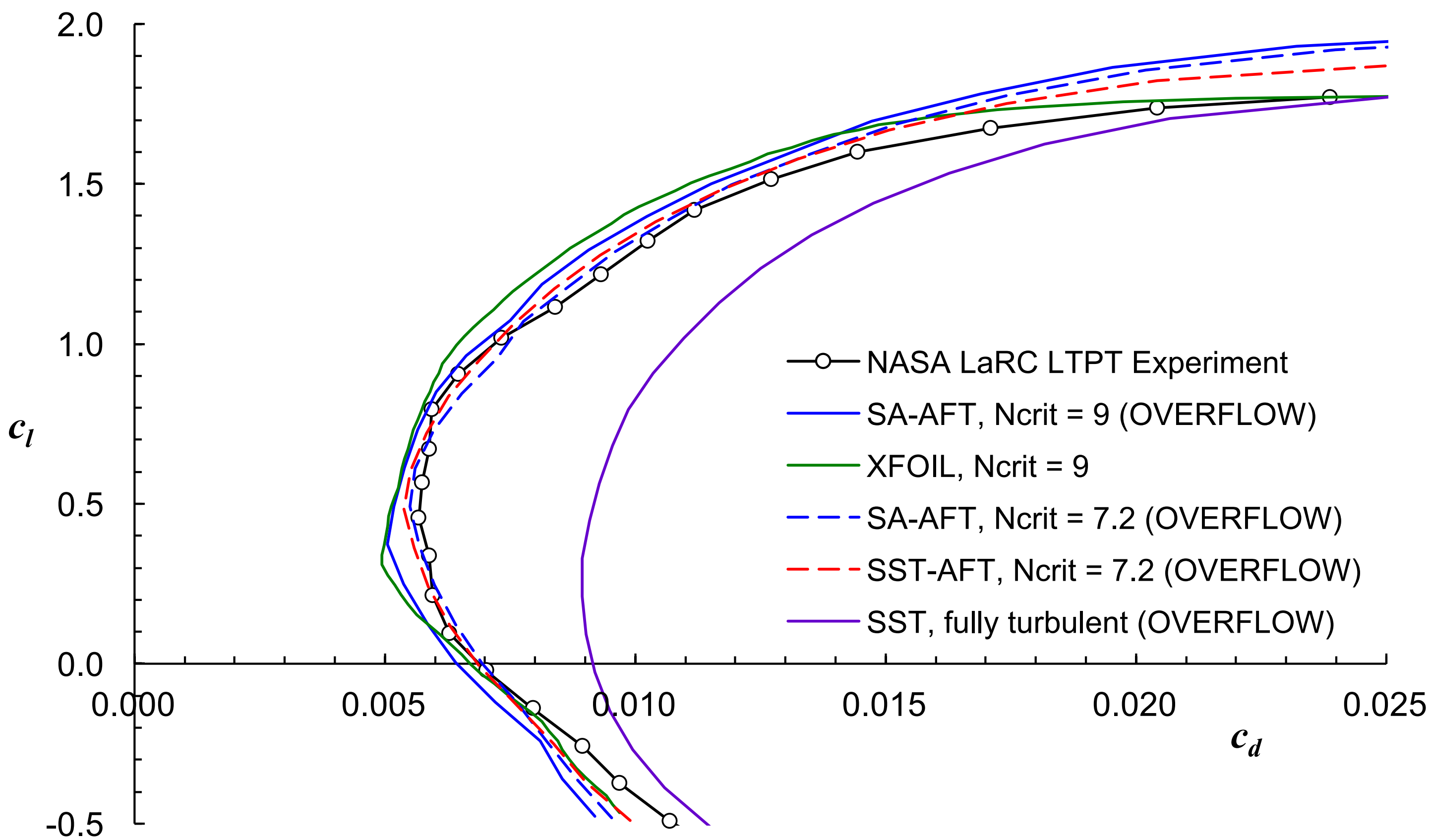
Zero-Pressure-Gradient Flat Plate

Skin-friction distribution, N_{crit} = 10.30



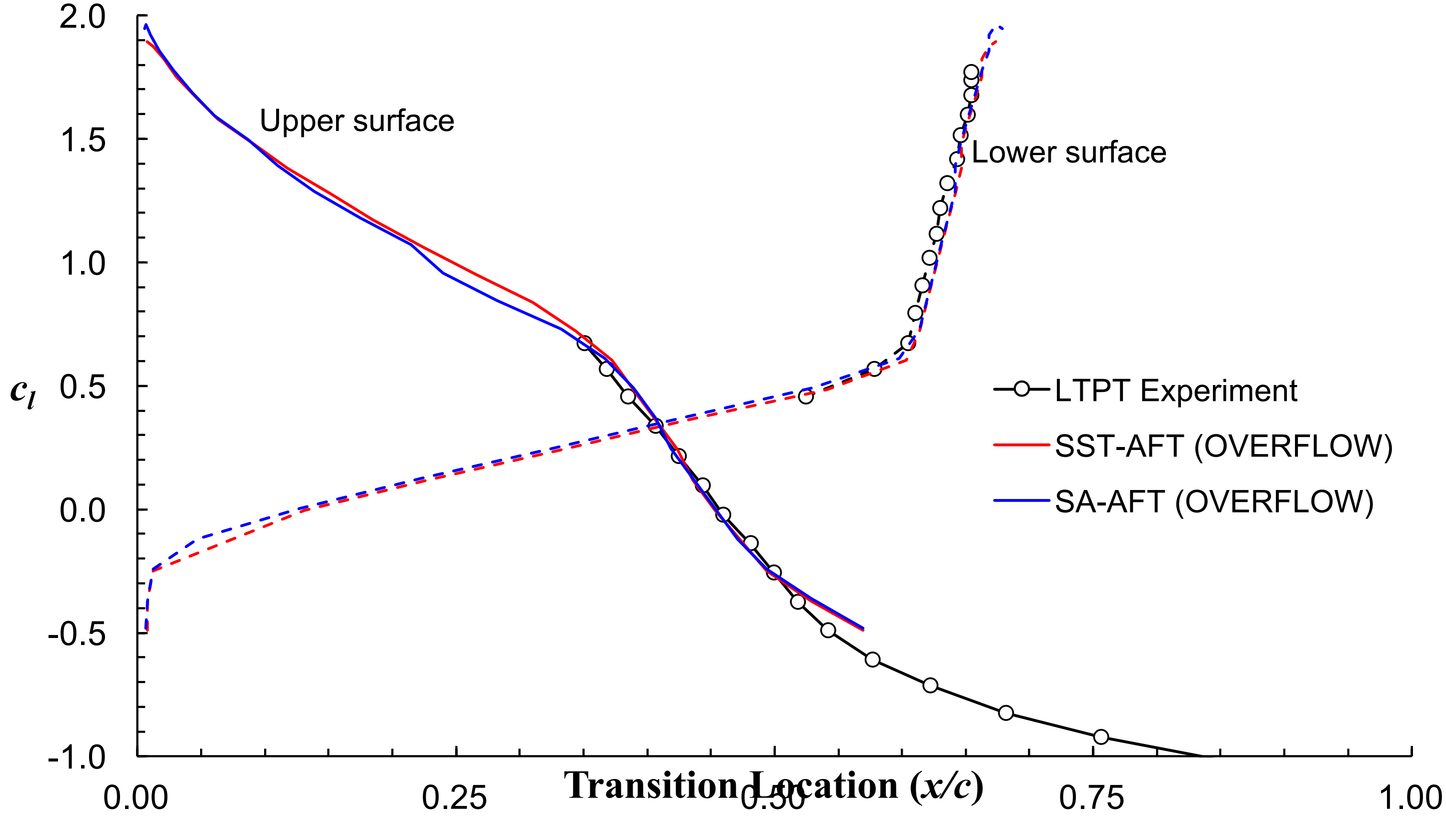
NASA NLF(1)-0416 Airfoil

Drag polar, unforced transition, Re = 4.0×10^6



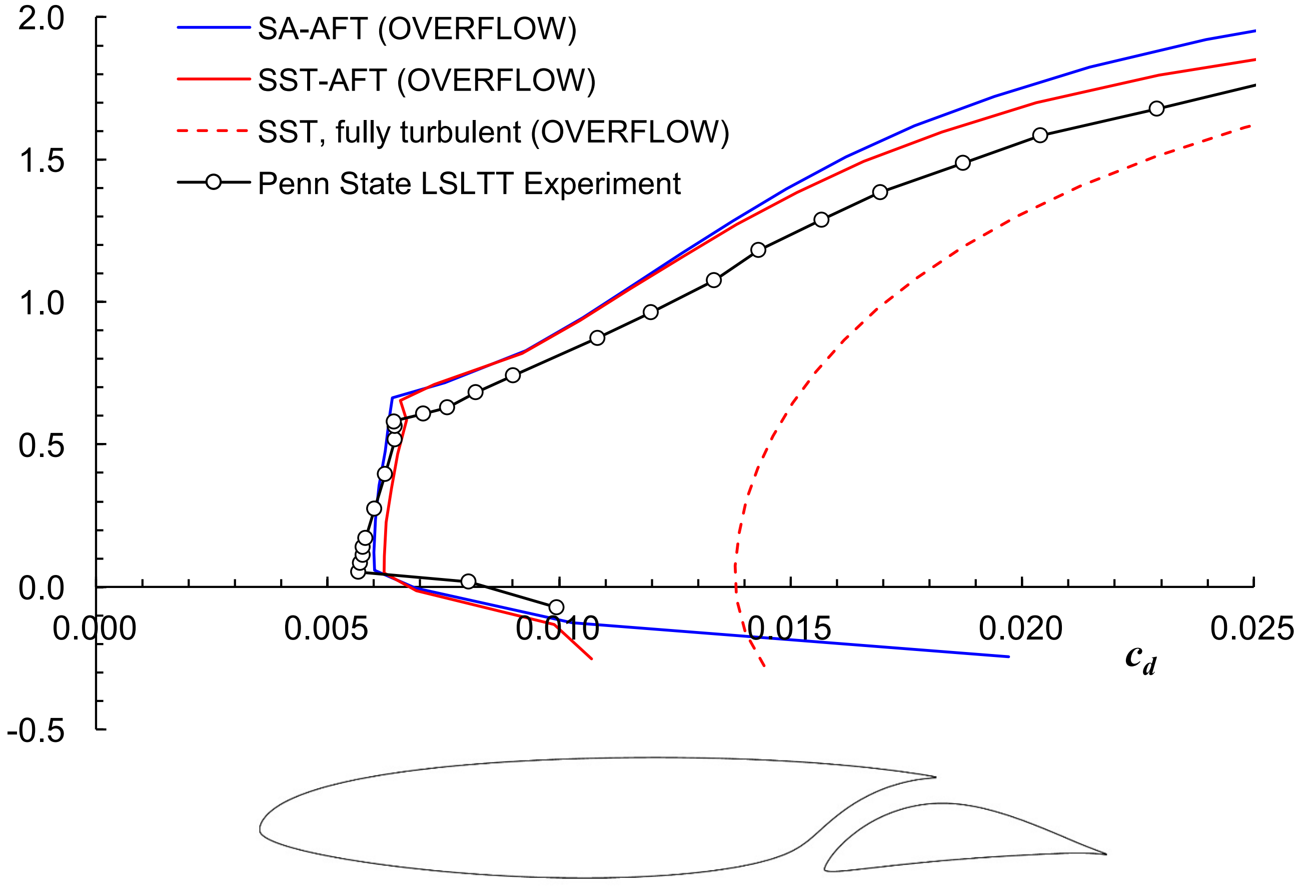
NASA NLF(1)-0416 Airfoil

Transition locations, Re = 4.0 x 10^6



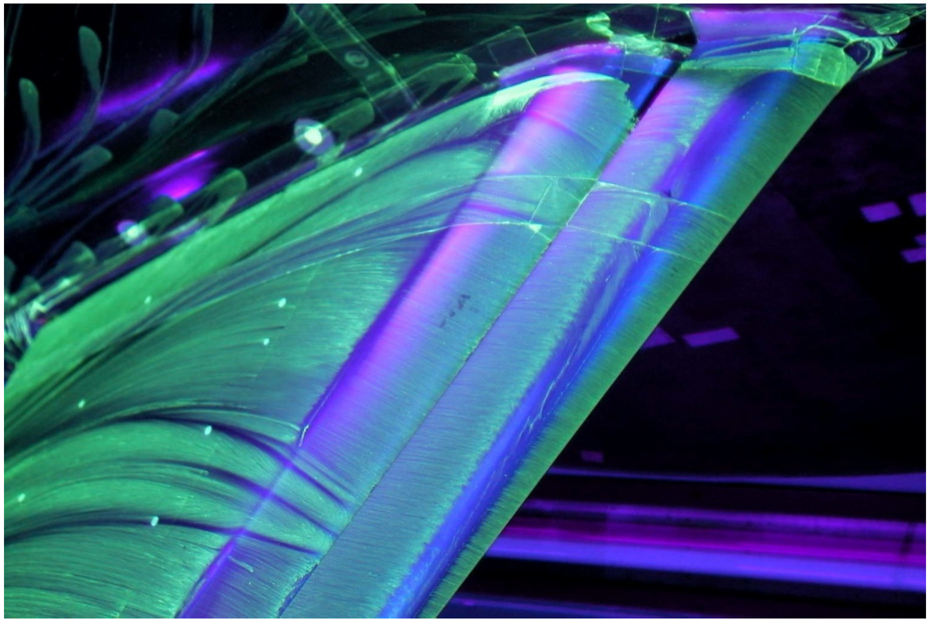
S414, SNLF Airfoil

Drag polar, unforced transition, Re = 1.5×10^6

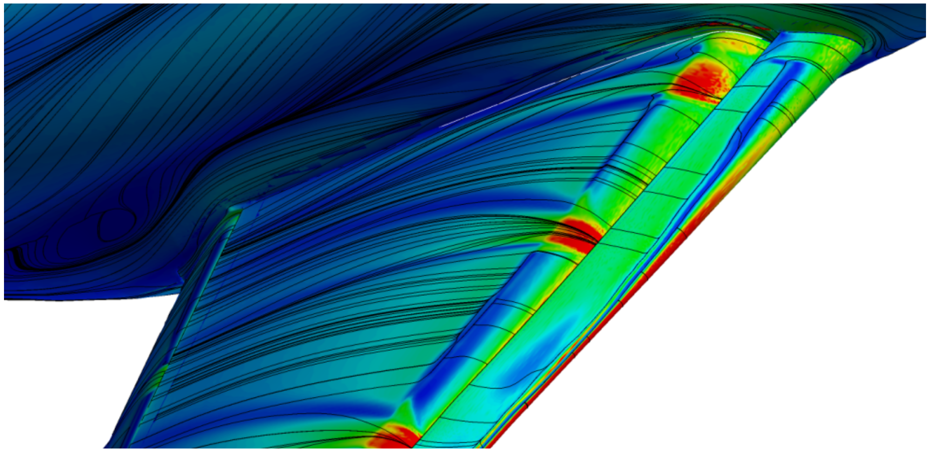


DLR-F11 from HiLiftPW-2

Experiment vs. CFD, Re = 1.35×10^6, $\alpha = 21°$ - SA-RC-AFT2014



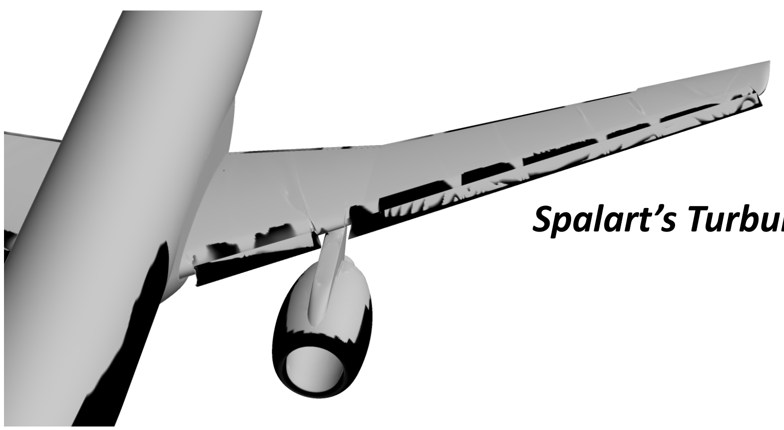
Experimental Oil Flow
(Rudnik, R., "Experimental Analysis of Separation and Transition Phenomena for the DLR-F11 High Lift Configuration," AIAA 2013-3035)



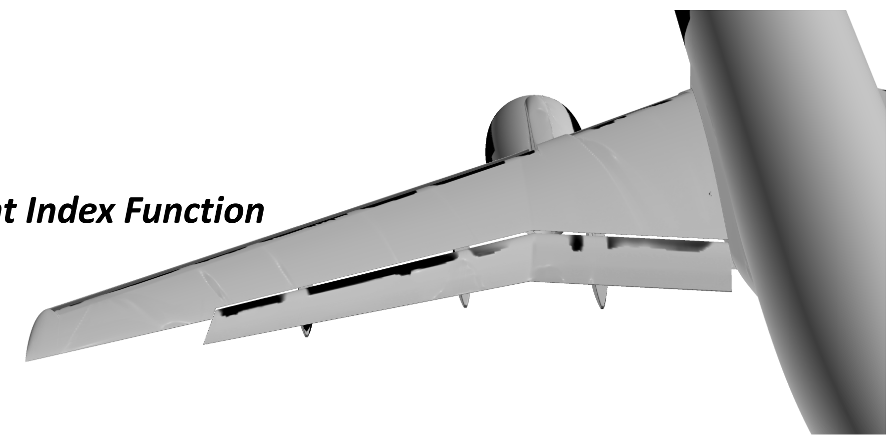
Computational Vorticity Contours and Surface Flow Patterns (OVERFLOW)

JSM from HiLiftPW-3

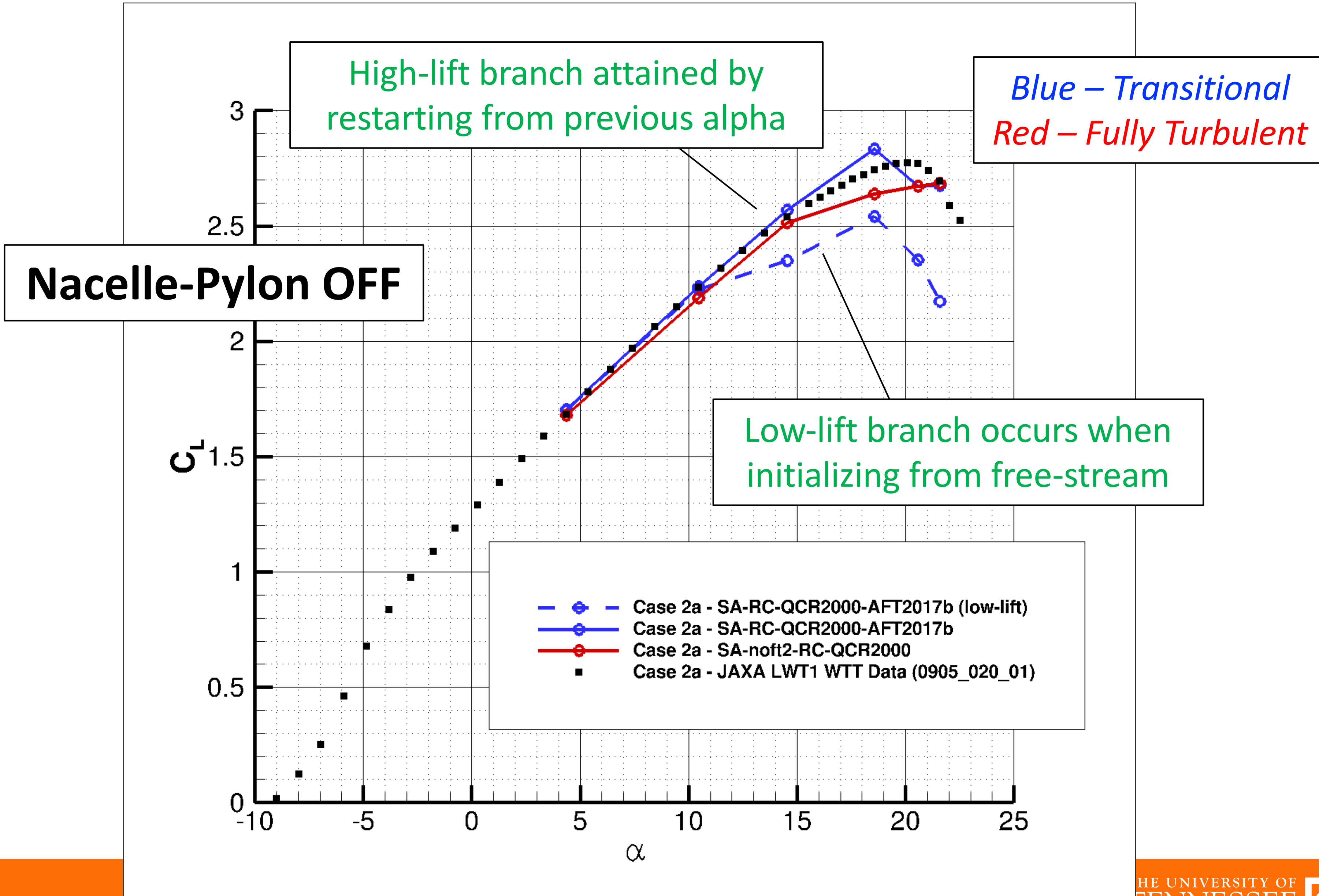
Experimental china clay vs. CFD, α = 18.58° - SA-RC-AFT2017b



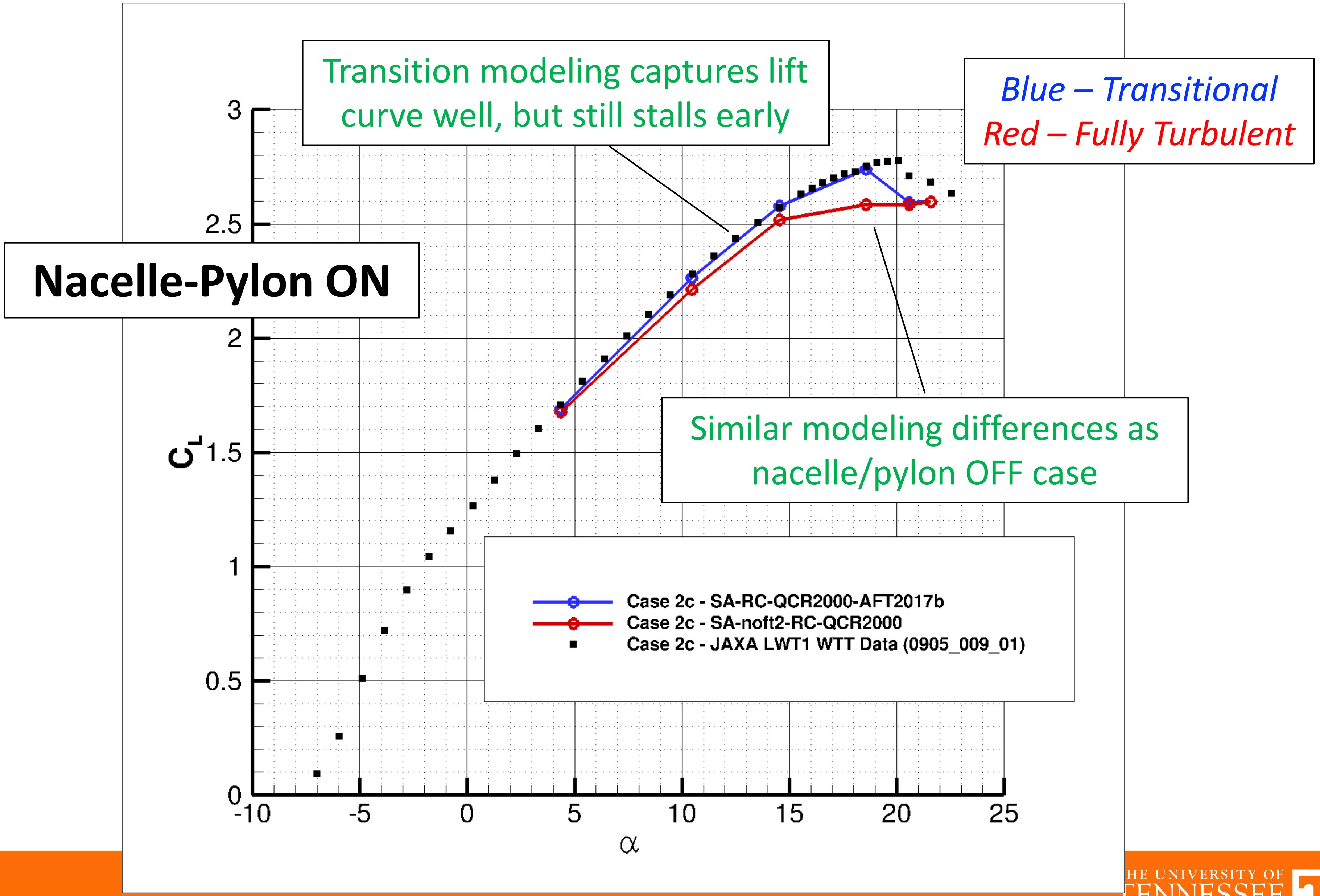
Spalart's Turbulent Index Function



JSM from HiLiftPW-3



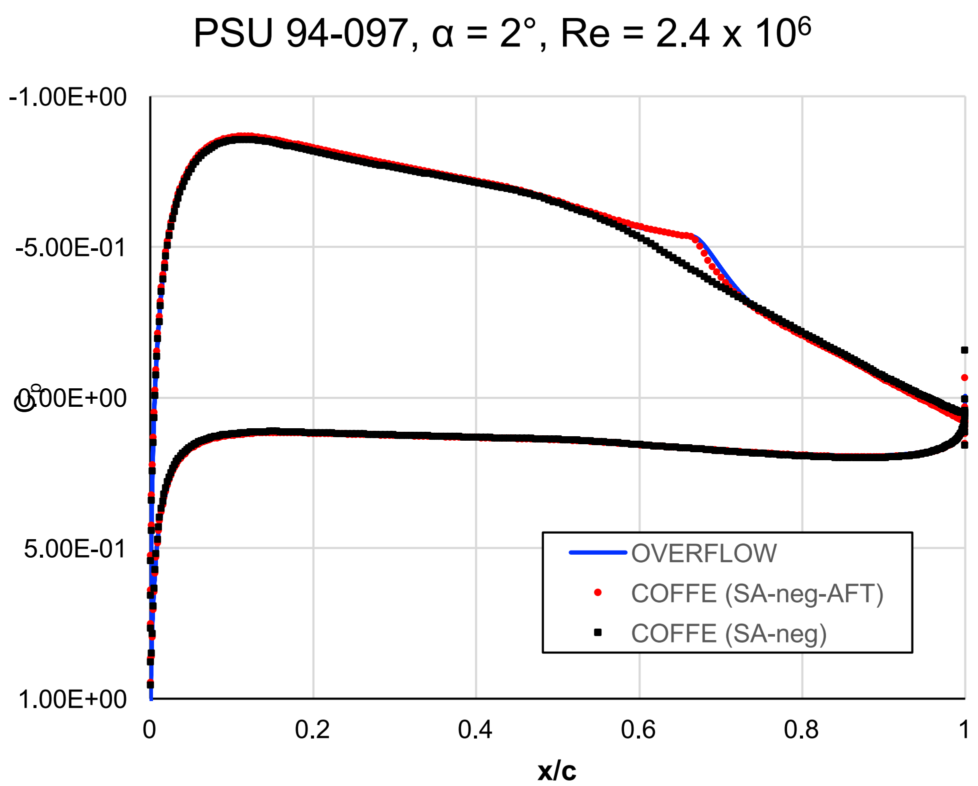
JSM from HiLiftPW-3

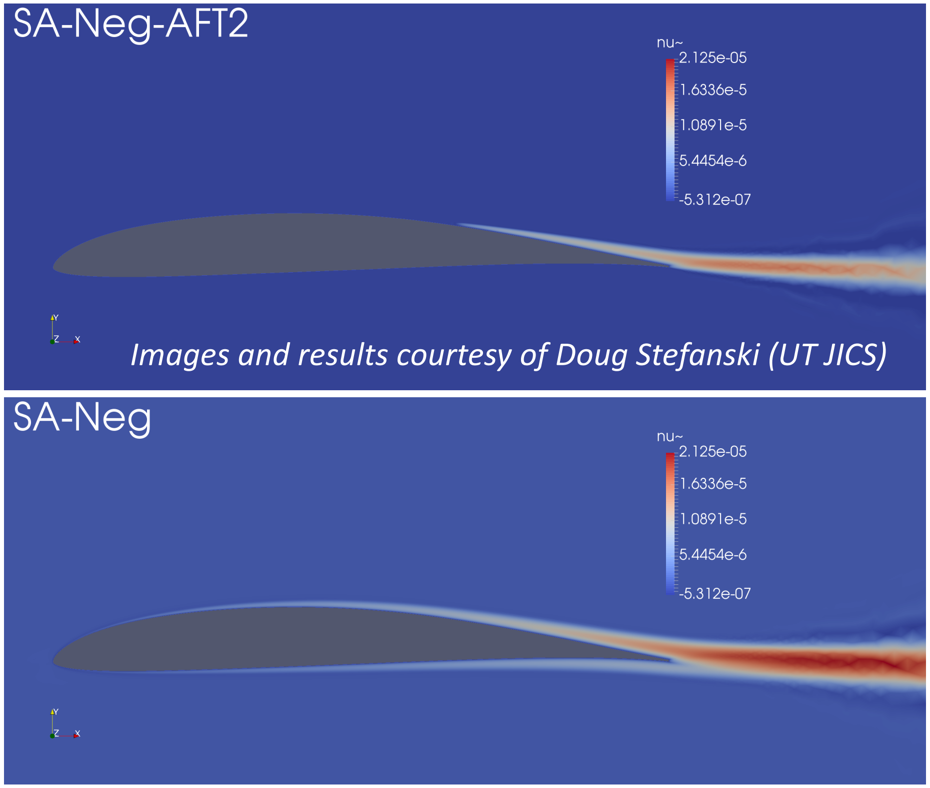


Implementation in Other Codes

HPCMP CREATE™-AV Kestrel component COFFE

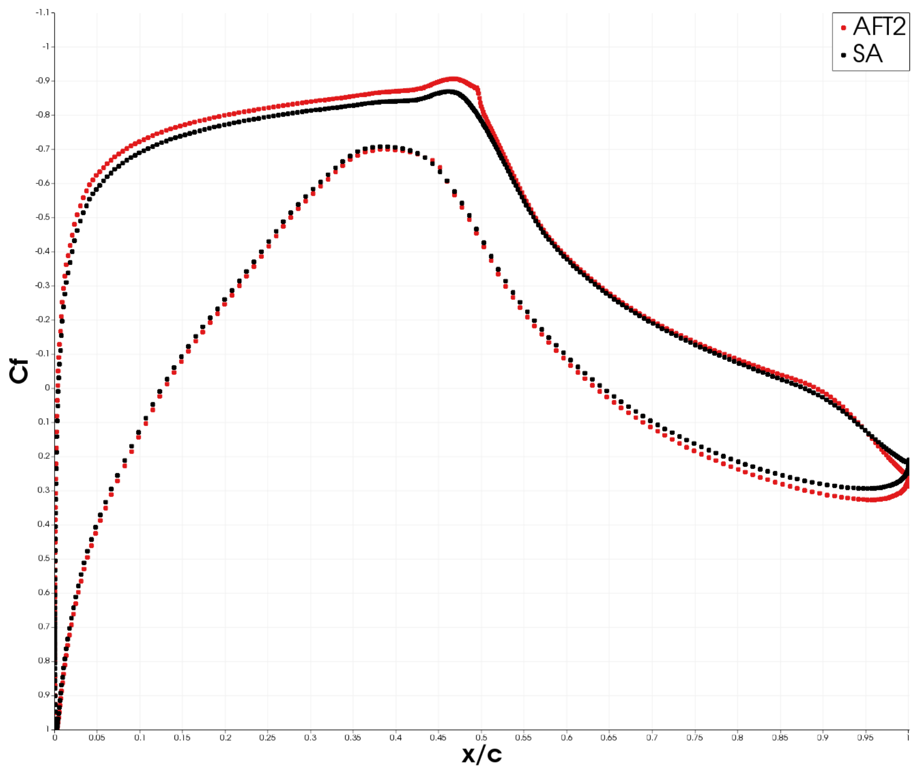
- Streamwise-upwind/Petrov-Galerkin finite element

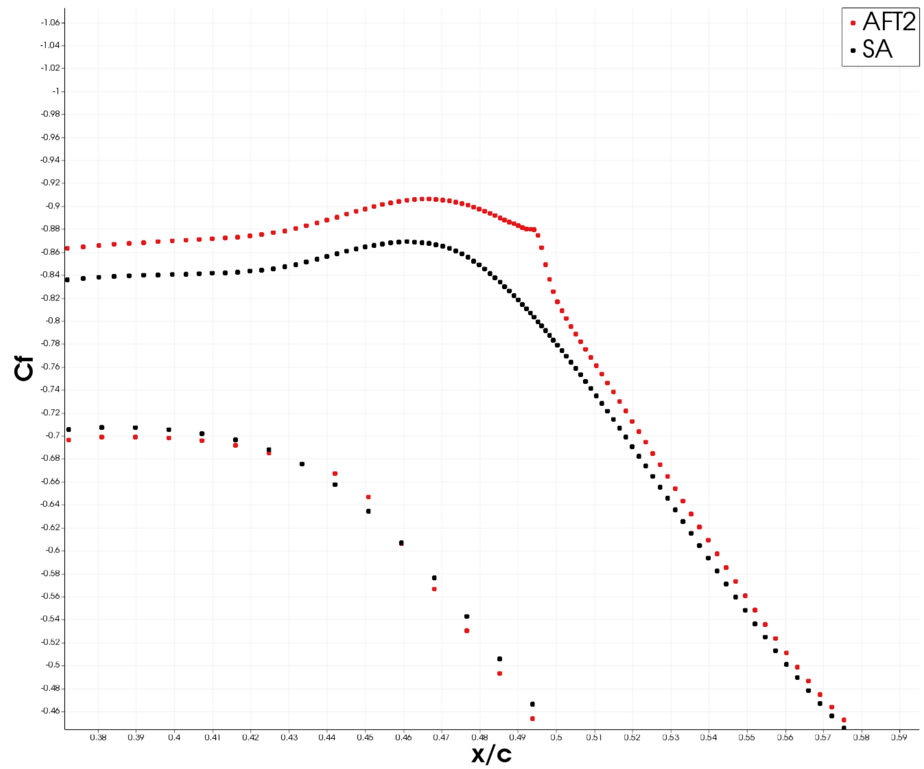




Implementation in Other Codes

COFFE predictions for S809 wind-turbine airfoil





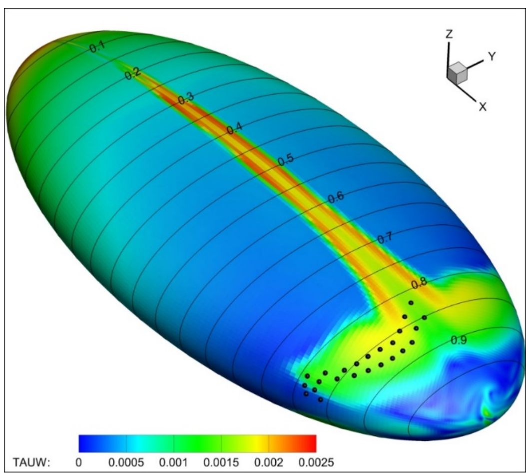
Images courtesy of Doug Stefanski (UT JICS)

Implementation in Other Codes

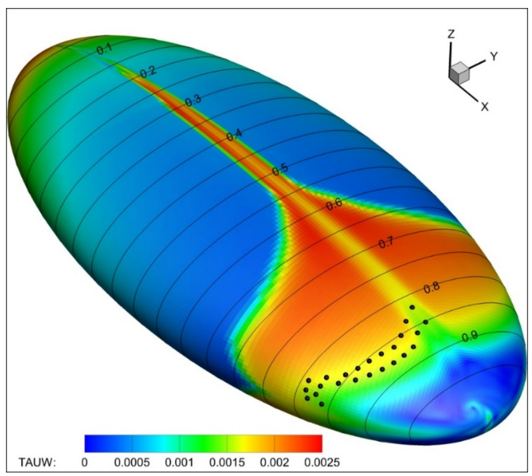
REX - Structured, incompressible/PISO solver

- Development supported by ONR over past 25 years
- Based on CFDShip-Iowa
  - E. Paterson (1994-2004), F. Stern (1994-Present), R. Wilson (1996-2005), P. Carrica (1996-2005)

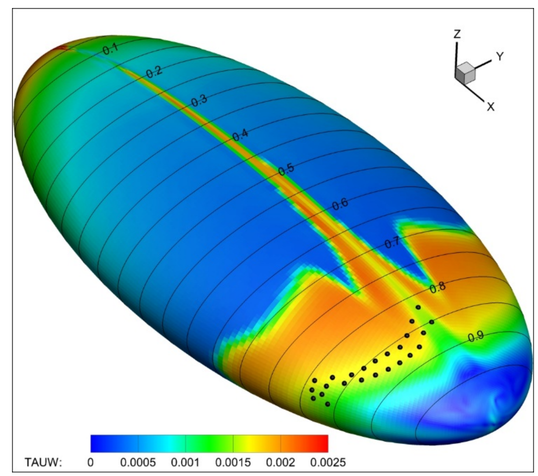
Transition Model Comparison for 3D Ellipsoid, Re = 2.5 x 10^6



Langtry-Menter (2009)



Menter (2015)



Coder (2017a)

Images courtesy of Bob Wilson (ORNL)

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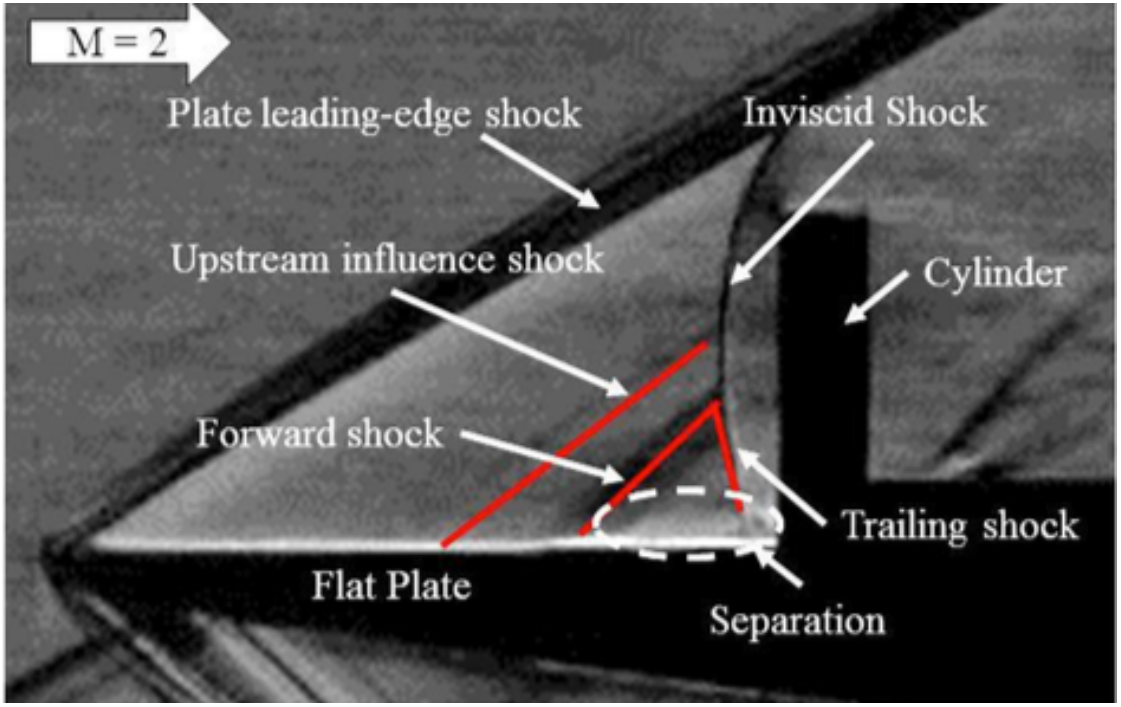
Other Application Areas

Hypersonics

- Strong shockwaves and SBLI phenomena on complex geometries
- Locally separated flows



Source: Lash et al. (2017)

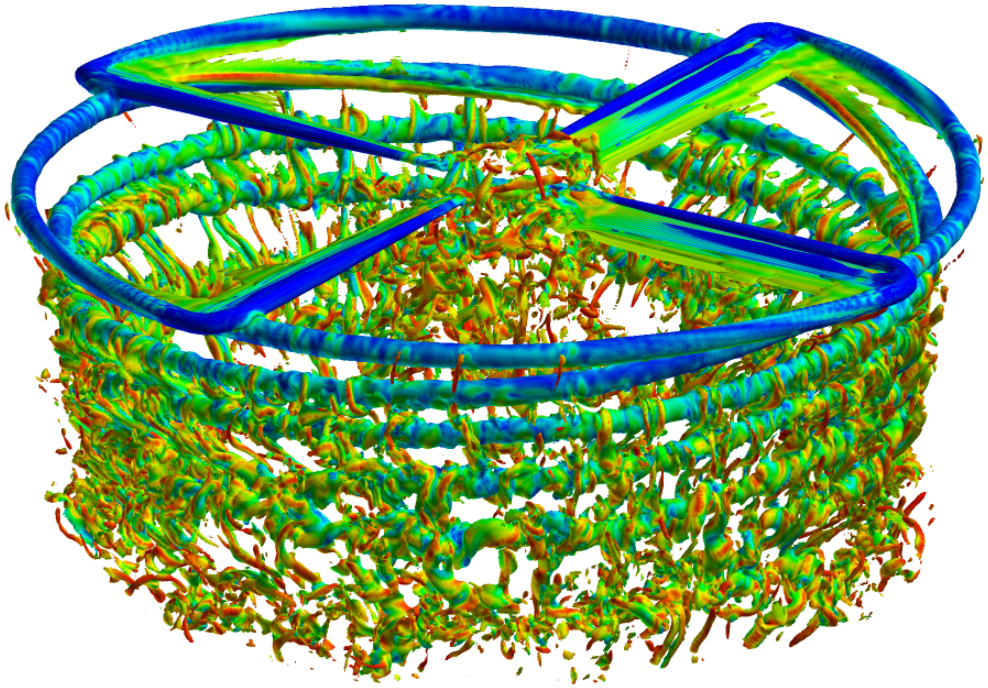
Other Application Areas

Transitional SBLI

Other Application Areas

Rotorcraft

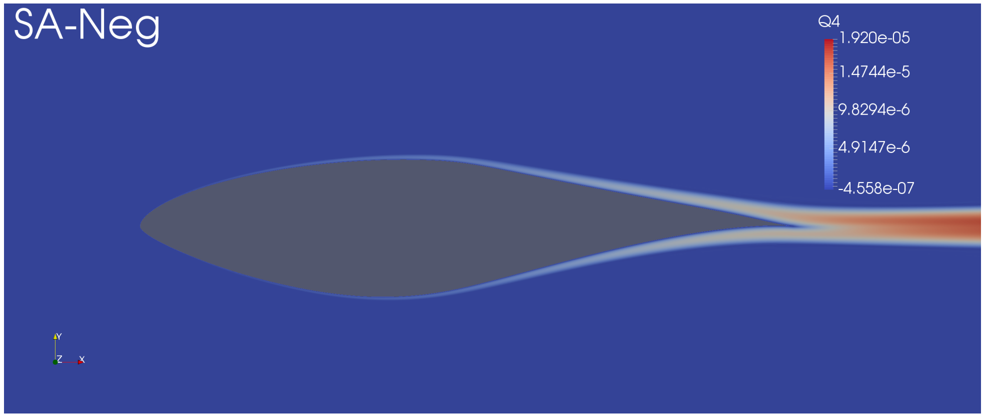
- Unsteady, rotating system
- Multiple transition mechanisms
- Demand for hybrid RANS/LES capabilities

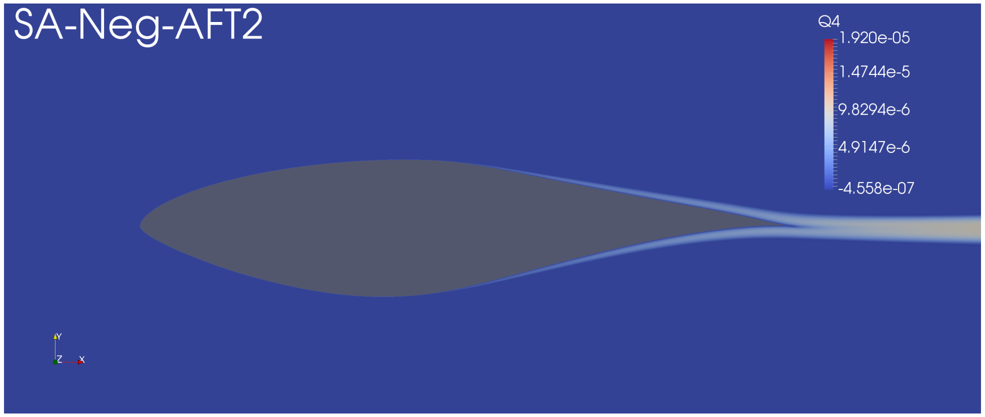


New Solver Technologies

Finite-Element Methods

- Robust convergence properties
- Clear path to higher-order accuracy
- Open frontier in implementation of transition (and turbulence) models





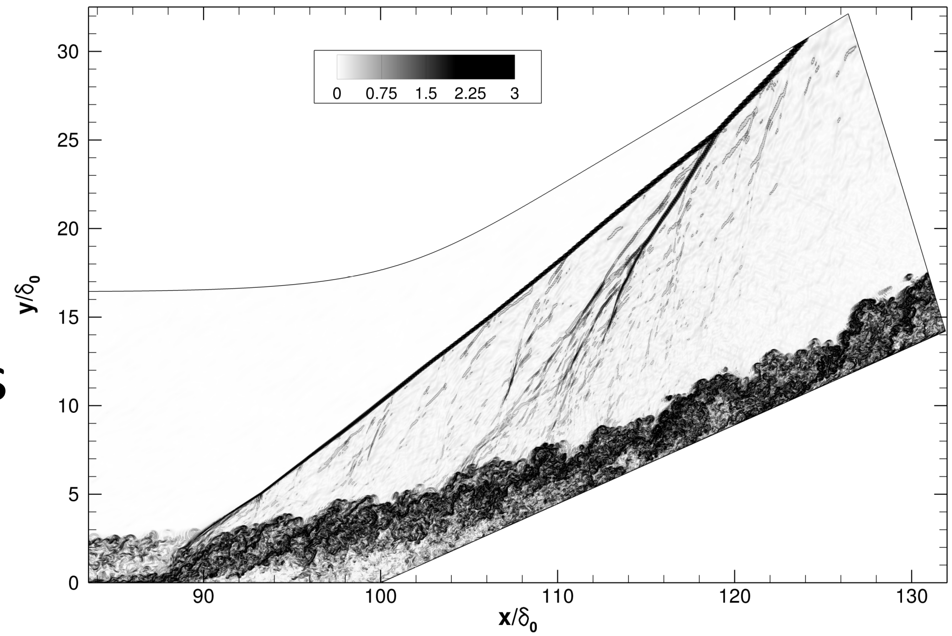
COFFE solution images courtesy of Doug Stefanski (UT JICS)

Life After RANS (into the Exascale Era)

Steady-state CFD calcs with $O(10^8)$ points are common

Turbulence-resolving methods becoming practical for select cases

- Hybrid RANS/LES
- Wall-modeled LES (WMLES)
- Implicit LES (ILES)



Source: Porter and Poggie, AIAA 2017-0533 (2017)

Transition ***modeling*** still necessary!

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Conclusion

- Laminar-turbulent transition modeling has enjoyed a rich theoretical development that is just now propagating into CFD-based methods

- Phenomenological transition models show the greatest potential for CFD applications
  - PDE implementations of classical models
  - Reynolds averaging excludes the physics of linear stability theory!

CFD-based transition models are still a work in progress

Observations for Moving Forward

1. Fresh ideas are always welcome for CFD-based transition modeling

2. Be mindful of both how and why certain modeling terms/constants are calibrated the way they are

3. Use boundary-layer theory whenever possible

4. Pay close attention to the numerical formulation and the convergence properties of the models

Acknowledgments

- Development of the AFT transition model supported in part by funding from
  - NDSEG Fellowship sponsored by HPCMP (PhD research)
  - Office of Naval Research
  - U.S. Army (VLRCOE)

Questions?