Interaction of Isotropic Turbulence with a Dragging Actuator Disk

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Motivation - Wind Energy

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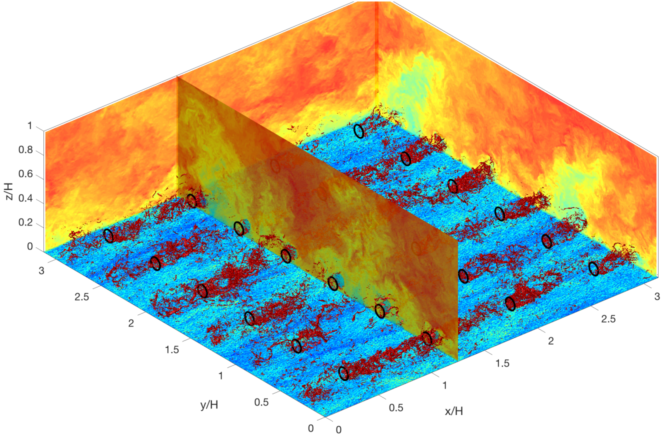
Conventional LES of a *fully developed* WT-array boundary layer

- 6×4 array of of explicitly simulated Actuator disks (Calaf, Meyers & Meneveau, PoF, 2012)
- ≈ 300 **million grid points, needs** $> 250,000$ **CPU hours**
- No-coriolis, no-stratification, $1.5km$ Planetary Boundary Layer (PBL) height, $150m$ Actuator disk (AD) diameter

Consider contours of longitudinal velocity, $\tilde{u}/u_\tau$:



A problem of many scales

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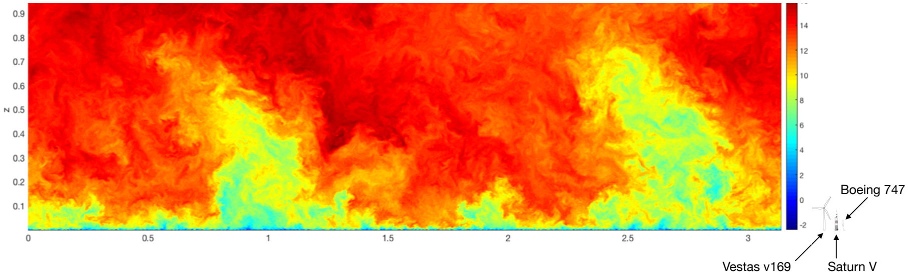
Primary challenge - massive range of relevant scales

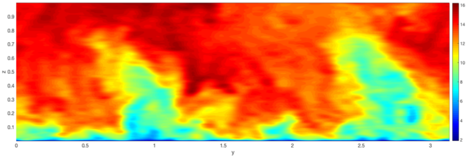
- Turbulence acting at: Integral scales $\mathcal{O}(10^3 m)$, Hub height/diameter $\mathcal{O}(10^2 m)$, chord length $\mathcal{O}(10^0 m)$, Kolmogorov scale $\mathcal{O}(10^{-3} m)$;
- Simulations substantially more challenging in case of stable stratification due to Ozmidov scale phenomena (commonly seen in night-time, off-shore PBLs)
- Scales pertinent for loading/fatigue: $\approx 5m - 20m$ (engineering models for loads)
- Other effects: Mesoscales $> \mathcal{O}(10^4 m)$, Terrain/Waves $\mathcal{O}(10^0 m - 10^2 m) \rightarrow$ need for meso-micro scale coupling in simulations
- Present simulation: grid resolution is approximately 3m (LES)

Contours of $\tilde{u}/u_\tau$ on $y - z$ plane taken at an arbitrary x.



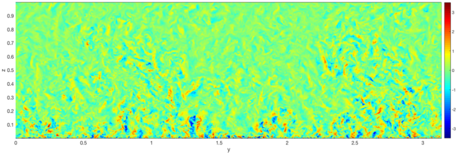
Need for a compressed representation

Now, consider the 2-scale decomposition of this field, again using spatial filtering.



Large Scales: Cost < 1k CPU hours

- $\mathcal{O}(10^6)$ spatial Dofs
- $\mathcal{O}(10^5)$ timesteps
- Very suitable for modern ROMs



Small Scales: Cost > 250k CPU hours

- $\mathcal{O}(10^9)$ spatial Dofs
- $\mathcal{O}(10^6)$ timesteps
- Too big for `#bigdata` (projection ROMs)?

Possible ways to compress the small scales:

- Compressed sensing / Wavelet thresholding
- Simplified engineering models (eg. Veers model, 1988; Mann model, 1994)
- **Our approach - Physics based scale enrichment using Gabor modes**

Outline of the talk

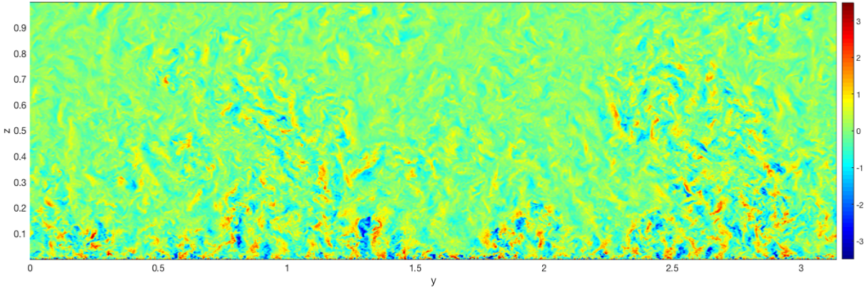
1. Part I: Scale Enrichment
   - Gabor modes
   - WKB-RDT formulation for temporal evolution
   - Some basic validation†
2. Part II: HIT - Actuator disk interaction
   - Flow characteristics: Linear processes, turbulence anisotropy, pressure modulation, wake recovery
   - Is the flow low-rank?
3. Conclusions

† Limited to *a-priori* validation, i.e. an ideal SGS model

Scale Enrichment - The basic idea

What if we try to generate a statistically equivalent (in space and time, 2nd order 1- and 2-pt correlations) field?

Consider the small scale field $u(\boldsymbol{x})$ from the high resolution LES.



The small scales exhibit inhomogeneity over a domain length scale, L.

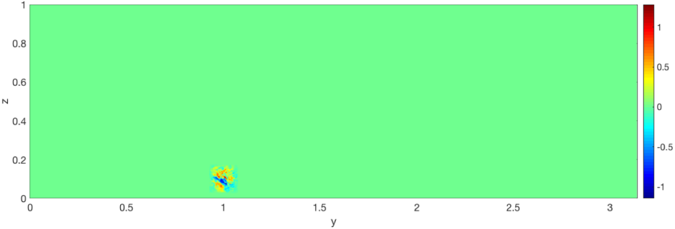
Let's focus on a small neighborhood at an arbitrary location, $\boldsymbol{x}_0$.

Window the small scale field using a spatial window function, $f \in \mathcal{C}_c^\infty \left\{\mathbb{R}^3\right\}$ with support l, such that $l \propto \Delta_{\mathrm{LES}} \ll L$.

$$u_W(\boldsymbol{x}, \boldsymbol{x}_0) = f\left(\varepsilon(\boldsymbol{x} - \boldsymbol{x}_0)\right) u(\boldsymbol{x})$$

The Gabor transform

Contours of the windowed field, $u_W(\boldsymbol{x}, \boldsymbol{x}_0)$.



Definition of the Gabor transform

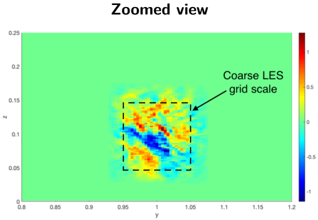
Since the windowed field is homogeneous and periodic, we can take a *shifted* Fourier Transform. (Nazarenko, et. al., J. Fluid Mech., 1999)

Forward transform:

$$\hat{u}(\boldsymbol{x}_0, \boldsymbol{k}) = \int_{\boldsymbol{x} \in \mathbb{R}^3} f\left(\varepsilon(\boldsymbol{x} - \boldsymbol{x}_0)\right) u(\boldsymbol{x}) e^{i\boldsymbol{k}\cdot(\boldsymbol{x} - \boldsymbol{x}_0)} d\boldsymbol{x}$$

Backward transform:

$$u(\boldsymbol{x}) = \frac{1}{f(\boldsymbol{0})} \int_{\boldsymbol{k} \in \mathbb{R}^3} \hat{u}(\boldsymbol{x}, \boldsymbol{k}) d\boldsymbol{k}$$



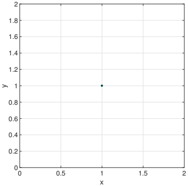
Discrete Gabor modes

Gabor modes are wavepackets that carry:

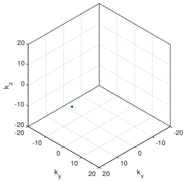
1. Position, $\boldsymbol{x}(t)$
2. Wave vector, $\boldsymbol{k}(t)$
3. Velocity, $\hat{\boldsymbol{u}}(t)$
4. Scalar field(s), $\hat{\theta}(t)$

Single Mode

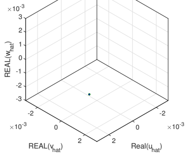
Position $(\hat{\boldsymbol{x}})$



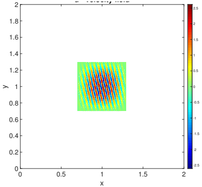
Wave vector $(\hat{\boldsymbol{k}})$



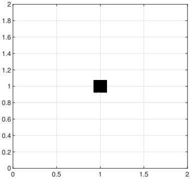
Velocity $(\hat{\boldsymbol{u}})$

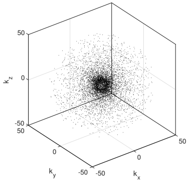


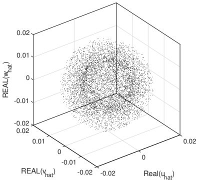
Generated $u(\boldsymbol{x})$ field

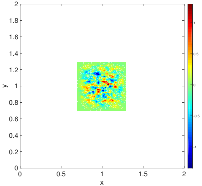


Multiple modes









Initialization of Gabor modes

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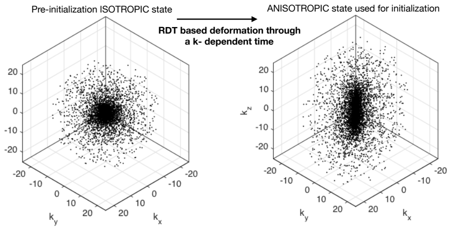
Alternate interpretation of Mann's (J. Fluid Mech., 1994) Eddy Lifetime Hypothesis

A *realizable* local anisotropic state (quantified using the anisotropy tensor $b_{ij} = \dfrac{R_{ij}}{R_{kk}} - \dfrac{1}{3}\delta_{ij}$) can be obtained by straining the isotropic state ($b_{ij} = 0$) using Rapid distortion theory (RDT), through a $\boldsymbol{k}-$ dependent time scale. Mann's model for such a $k-$ dependent time:

$$\tau(k) \propto \frac{1}{k\sqrt{\int_k^\infty E(k)dk}} \implies \tau(k)S = c_\tau\,(kL)^{-2/3}\left[{}_2F_1\left(\frac{1}{3}, \frac{17}{6}; \frac{4}{3}; -(kL)^{-2}\right)\right]^{-1/2}$$



Model constants (L, c_τ) can be determined using information (energy transfer rate, etc.) from the large scales via a least squares minimization. [Ghate & Lele, 2017]

Temporal Evolution of Gabor modes

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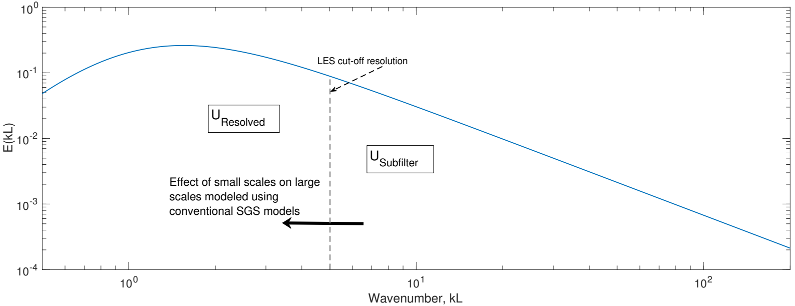
An idealized energy spectrum



Notation: $\widetilde{U}$: Superfilter/Resolved/Large scales, and u: Subfilter/Small scales

Incompressible Navier-Stokes, non-inertial reference frame, buoyancy via Boussinesq approx.

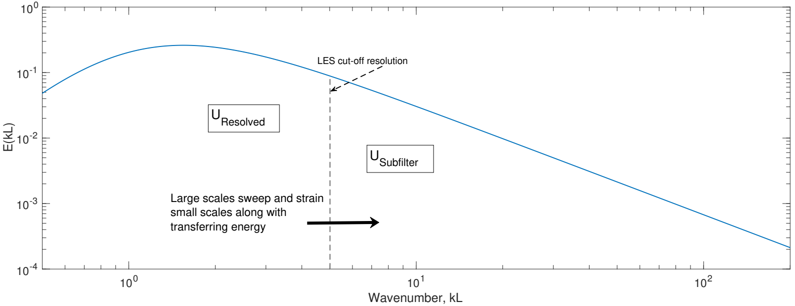
Resolved scale equations

$$\partial_t \tilde{U}_i + \tilde{U}_j \partial_j \tilde{U}_i = -\partial_i \tilde{P} - \partial_j \tau_{ij}^d + \delta_{ij} g_j \left[\frac{\tilde{\Theta} - \Theta_0}{\Theta_0} \right] + 2\varepsilon_{ijk} \Omega_j (G_k - \tilde{U}_k) + \nu \partial_j \partial_j \tilde{U}_i$$

Temporal Evolution of Gabor modes

Stanford University

An idealized energy spectrum



Notation: $\widetilde{U}$: Superfilter/Resolved/Large scales, and u: Subfilter/Small scales

Incompressible Navier-Stokes, non-inertial reference frame, buoyancy via Boussinesq approx.

Subfilter scale equations

$$\partial_t u_i + \tilde{U}_j \partial_j u_i + u_j \partial_j \tilde{U}_i + u_j \partial_j u_i = -\partial_i p + \partial_j \tau^d_{ij} + \delta_{ij} g_j \frac{\theta}{\Theta_0} - 2\varepsilon_{ijk}\Omega_j u_k + \nu \partial_j \partial_j u_i$$

Temporal Evolution of Gabor modes

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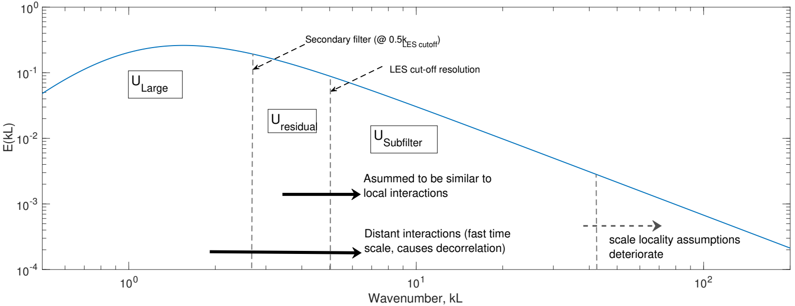
An idealized energy spectrum



Notation: U: Large scales, U^r: Residual scales, and u: Subfilter/Small scales

Incompressible Navier-Stokes, non-inertial reference frame, buoyancy via Boussinesq approx.

Subfilter scale equations

$$\partial_t u_i + U_j \partial_j u_i + u_j \partial_j U_i = -\partial_i p - \partial_j h_{ij} + \partial_j \tau_{ij}^d + \delta_{ij} g_j \frac{\theta}{\Theta_0} - 2\epsilon_{ijk}\Omega_j u_k + \nu \partial_j \partial_j u_i$$

$$h_{ij} = \partial_j (u_i u_j) + \partial_j (u_i U_j^r) + \partial_j (U_i^r u_j)$$

Modeling the small scale evolution

The Quasi-homogeneity assumption

The large scale field, $U(\boldsymbol{x})$ can be expressed in its truncated Taylor series expansion within a neighborhood $||\boldsymbol{x} - \boldsymbol{x_0}|| < l \propto \Delta_{LES}$ where $\boldsymbol{x}, \boldsymbol{x}_0 \in \mathbb{R}^3$.

$$U(\boldsymbol{x}) = U^0 + (\boldsymbol{x} - \boldsymbol{x}_0) \cdot \nabla U|^0$$

Gabor transform of derivatives (WKB asymptotic expansion)

To a leading order in scale separation parameter, ε the Gabor transform of derivatives can be expressed as:

$$\widehat{\partial_m u} = i k_m \hat{u} + \mathcal{O}(\varepsilon)$$

Furthermore, for a solenoidal field ($k_j \hat{u}_j = 0$), pressure non-linearity can be projected out using a Projection tensor

$$\left(\delta_{mj} - \frac{k_m k_j}{k^2}\right) \widehat{\partial_j p} = \left(\delta_{mj} - \frac{k_m k_j}{k^2}\right) (i k_j \hat{p}) = 0$$

Model for the local (in scale space) convective non-linearity

The action of the convective non-linearity due to local triadic interactions will be modeled using a spectral viscosity based on Renormalization Group Theory (RNG) (see Canuto & Dubovikov, PoF, 1996)

$$\widehat{\partial_j h_{ij}}^{\perp} = -\nu_t(k) k^2 \hat{u}_i \quad , \quad \nu_t(k) = \left(\nu^2 + c_\nu \int_k^\infty q^{-2} E(q) dq\right)^{1/2} - \nu$$

Modeling the small scale evolution

Governing equations for subfilter scale

With these assumptions/models the Gabor transform of the subfilter scale equations gives the evolution of a single Gabor mode:

$$\partial_t x_j = U_j^0 \;\; ; \;\; \partial_t k_j = -k_m \partial_j U_m^0$$

$$\partial_t \hat{u}_i = \left(\frac{2k_i k_m}{k^2} - \delta_{im} \right) \hat{u}_j \partial_j U_m^0 + \left(\frac{k_i k_j}{k^2} - \delta_{ij} \right) g_j \beta \hat{\theta} - (\nu + \nu_t) k^2 \hat{a}_i + \hat{f}_i^{\perp} - 2\epsilon_{ijk} \Omega_j \hat{u}_k$$

$$\partial_t \hat{\theta} = -\hat{u}_j \partial_j \Theta^0 - (\kappa + \kappa_t) k^2 \hat{\theta} + \hat{f}_\theta$$

where, $\hat{f}_i$ and $\hat{f}_\theta$ are Gabor projections of the residual stress terms $\partial_j \tau_{ij}^d$ and $\partial_j q_j$ respectively.

Important considerations

1. The ODEs governing evolution of the Gabor modes are only accurate up to leading order in ε, at large times this error is not expected to stay bounded
2. The modeling error using RNG based spectral viscosity is also expected to grow at large times. However, note that the implied non-linear time scale is given as:

$$\tau_D(k) \propto \frac{1}{k^2} \left(\int_k^\infty q^{-2} E(q) dq \right)^{-1/2} \propto k^{-2/3} \left[{}_2F_1 \left(\frac{4}{3}, \frac{17}{6}; \frac{7}{3}; -(kL)^{-2} \right) \right]^{-1/2}$$

which is same as *Coherence destroying diffusion time* defined by Comte-Bello & Corrsin, J. Fluid Mech., 1971.

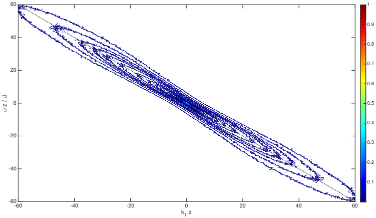
Does the model work?

Half-channel at $Re \to \infty$

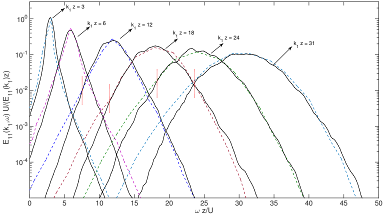
- Validate the space-time behavior by comparison with high-resolution LES data (Wilczek, Stevens & Meneveau, 2015, J. Fluid Mech.) at $z = 0.154H$.
- Gabor mode simulation uses a 3-scale decomposition: a) $kL < 0.6$ (frozen), b) $0.6 < kL < 6$ and c) $6 < kL$ where $L = 0.075H$ is determined from LES data.

$k - \omega$ spectrum

Log-spaced contours of $k - \omega$ spectrum



colored dashes - LES, solid black lines - Gabor modes



Validation - Channel at $\mathrm{Re}_\tau = 1000$

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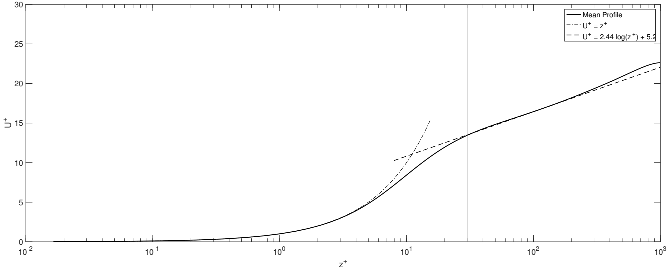
Validation methodology

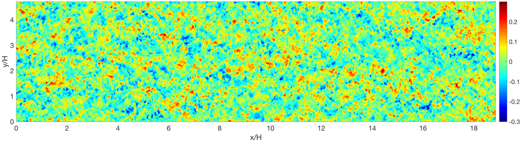
- DNS simulation data available via JHU Turbulence database. DNS performed on a $2048 \times 1536 \times 512$ numerical grid.
- LES emulated by filtering DNS on a $128 \times 96 \times 128$.
- Enriched LES uses 256 Gabor modes in each LES cell; resulting fields resolved on a $2304 \times 1728 \times 1024$ numerical grid.
- Enrichment performed for $z^+ > 25$.

Mean profile

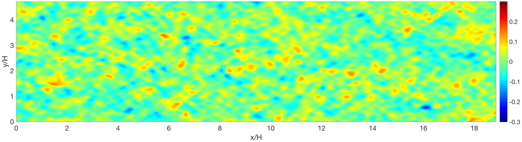


Validation - Channel at $\text{Re}_\tau = 1000$

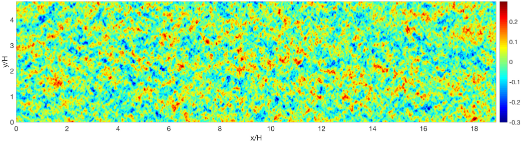
Baseline DNS, contours of u at $z^+ = 80$



LES, contours of u at $z^+ = 80$

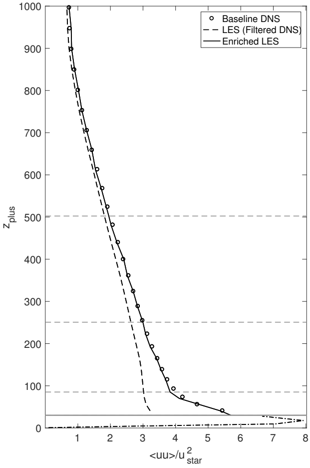


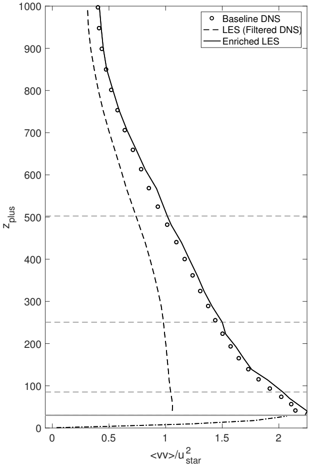
Enriched LES, contours of u at $z^+ = 80$



Validation - Channel at $Re_\tau = 1000$

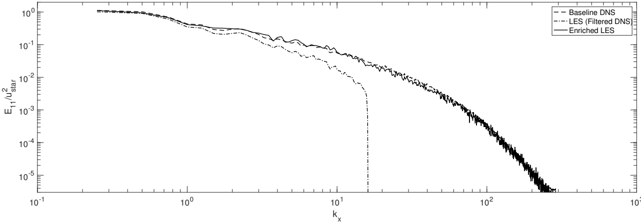
Single point correlations, $< uu >$ and $< vv >$ profiles in z



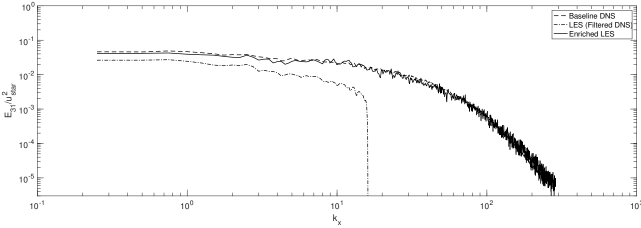


Validation - Channel at $\mathrm{Re}_\tau = 1000$

Spectra of u along k_x at $z^+ = 80$



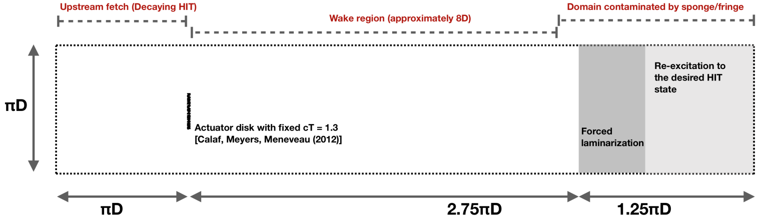
Spectra of w along k_x at $z^+ = 80$



Summary - Part I

1. Also been validated on more realistic PBL problems - high latitude, stably stratified, Ekman layer [Ghate & Lele, 2017]
2. Gabor mode based enrichment extrapolates spectral resolution with accurate second-order space-time correlations
3. Main advantages of describing small scale turbulence using discrete Gabor modes
   - Massive compression in degrees of freedom ($\approx 97\%$)
   - Temporal evolution described via ODEs
   - Fast transform to physical space from Gabor modes using modern **non-uniform FFT algorithms (NUFFTs)** - very HPC friendly
4. Simple boundary conditions like no-penetration (kinematic blocking) and periodicity can be handled rigorously (in simple domains)
5. **Potential use in more complex flow interactions is unclear - For example: wake in ambient turbulence**

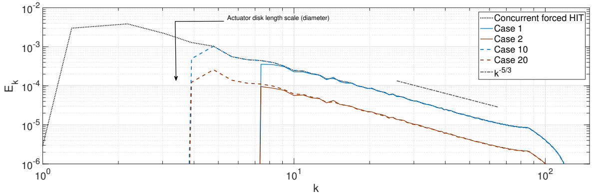
Part II - Actuator disk interaction with HIT



Some pertinent aspects:

- Assumption of periodicity in y and z directions results in a non-negligible blocking effect; the modeled actuator disk in not *isolated*.
- Presence of the sponge/fringe at the exit results in contamination of the domain beyond the support of the sponge/fringe.
- Time is the only homogeneous/stationary direction for computing statistics.

Cases studied



HIT case description	Rel. L_0	Rel. ε	Rel. decay time scale, τ	Turb. Intensity
Case 1 (Small length scale, Large dissipation rate)	1	1	1	3.75%
Case 2 (Small length scale, Small dissipation rate)	1	0.125	2	2.5%
Case 10 (Large length scale, Large dissipation rate)	1.75	1	1.45	5.0%
Case 20 (Large length scale, Small dissipation rate)	1.75	0.125	2.90	3.15%

Cases 1 and 20 have similar turbulence intensity at incidence, but different length scales and dissipation rates

Numerical method / Models

- Spatial differencing: Fourier-collocation (all directions); $2/3$rd dealiasing (*Explictly* filtered LES); rotational formulation
- Time stepping: SSP RK45 (Gottlieb, Shu & Tadmor, 2001)
- SGS model: `sigma` model (Nicoud et al., 2011); no molecular viscosity (Re $\rightarrow \infty$)
- Fringe method for forcing (Nordström, Nordin & Henningson, 1999)

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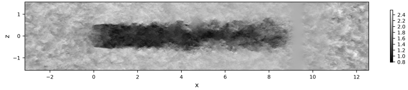
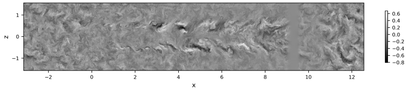
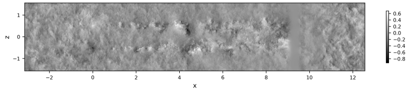
Instantaneous fields on $x - z$ plane through the centerline



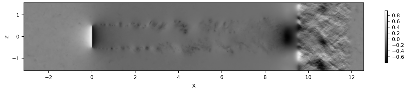
(a) u



(b) v

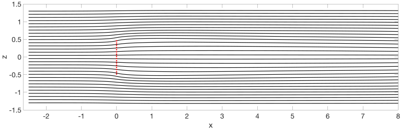


(c) w

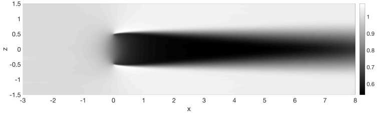


(d) p (pressure)

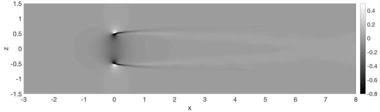
Time averaged (mean) flow



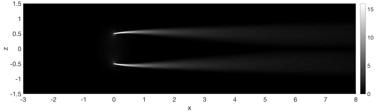
(a) Streamlines, U



(b) Axial velocity, U



(c) dU/dx

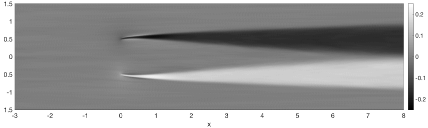


(d) $|dU_i/dx_j|_F$

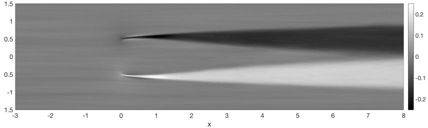
Characterizing the wake recovery/entrainment

- In this problem *wake recovery* is essentially equivalent to *momentum entrainment* by the shear layer.
- Easy to characterize entrainment using anisotropy, $b_{ij} = R_{ij}/R_{kk} - \delta_{ij}/3$.

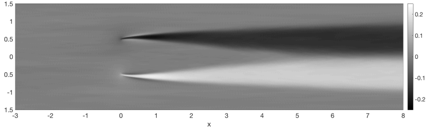
Contours of b_{13}



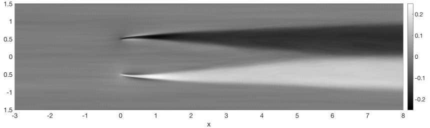
(a) Case 1



(b) Case 2



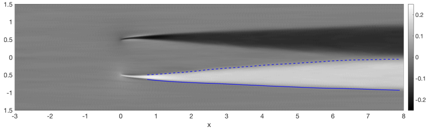
(c) Case 10



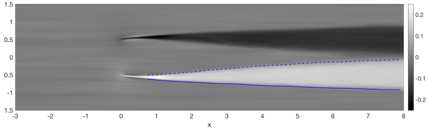
(d) Case 20

Characterizing the wake recovery/entrainment

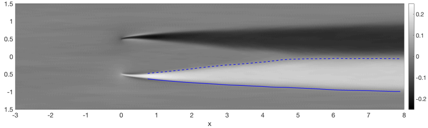
Contours of b_{13}, shear layer marked by $|b_{13}| < 0.03$



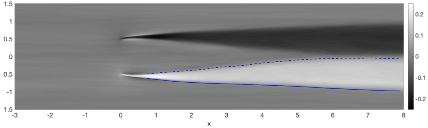
(a) Case 1



(b) Case 2



(c) Case 10



(d) Case 20

The entrainment of the inner core is highly sensitive to the length scale of the incident turbulence.

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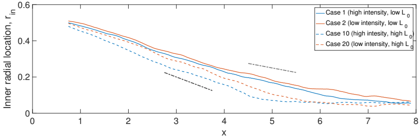
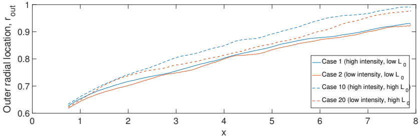
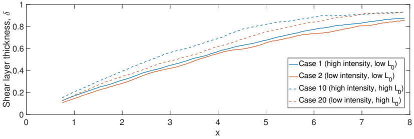
Shear layer growth



(a) Inner radius

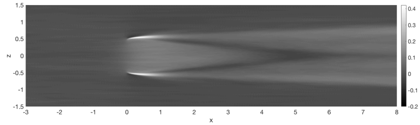


(b) Outer radius

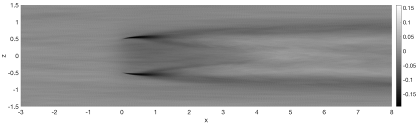


(c) Shear layer thickness

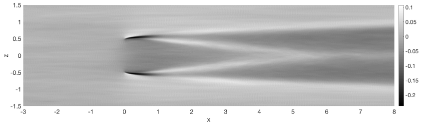
Case 10 - overall turbulence characteristics



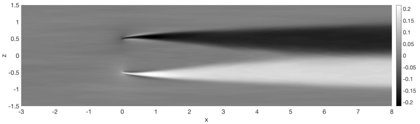
(a) b_{11} (axial)



(b) b_{22} (azimuthal)



(c) b_{33} (radial)

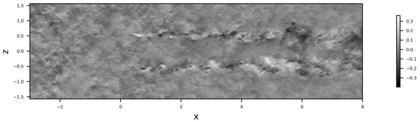


(d) b_{13} (radial-axial)

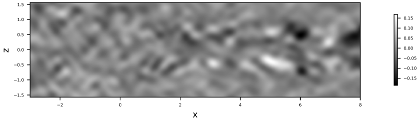
- Four distinct flow regimes
  1. Decaying incident HIT (isotropic)
  2. Axisymmetric expansion in core (amplified axial component, damped radial and azimuthal components)
  3. Shear layer/wake turbulence (highly damped radial component, very non-gaussian)
  4. Inner entrainment interface (entrainment of anisotropic turbulence by the shear layer)

Multiscale decomposition

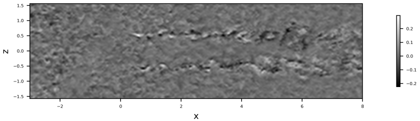
Consider a 3 scale decomposition (5-pt implicit *Pade* filter, Sypropoulos and Blaisdell, 1996)
X-Z contours through the centerline



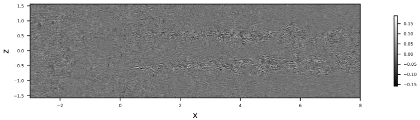
(a) **Axial velocity fluctuations**



(b) Scale A



(c) Scale B



(d) Scale C

Anisotropy at the 3 scales

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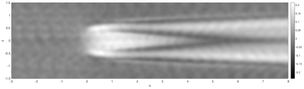
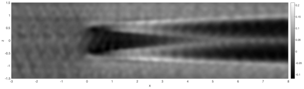
Consider a 3 scale decomposition (5-pt implicit *Pade filter*, Sypropoulos and Blaisdell, 1996)

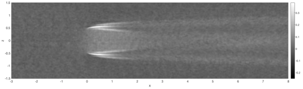
X-Z contours through the centerline



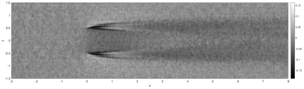
(a) Scale A, b_{11}



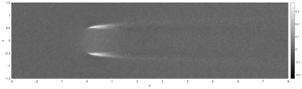
(b) Scale A, b_{33}



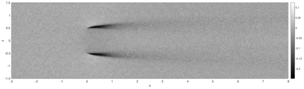
(c) Scale B, b_{11}



(d) Scale B, b_{33}



(e) Scale C, b_{11}



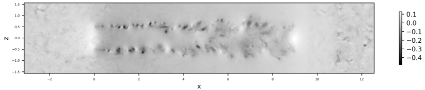
(f) Scale C, b_{33}

Pressure fluctuations

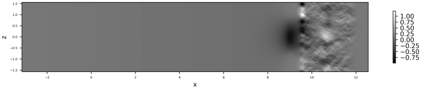
The computed pressure field (incompressible flow) in the LES can be expressed as a superposition of 4 contributions:

1. *True* pressure: $\partial_j \partial_j \overline{p}_{\text{true}} = -\partial_j \partial_i \overline{\overline{u}_i \overline{u}_j}$
2. Contribution from the *Fringe*: $\partial_j \partial_j \overline{p}_{\text{fringe}} = \left(\overline{u}_j - \overline{u}_j^{\text{targ}} \right) \partial_j g^{\text{fringe}}$
3. Contribution from SGS closure: $\partial_j \partial_j \left(\overline{p}_{\text{SGS}} + \frac{1}{3} \tau_{kk}^{\text{sgs}} \right) = -\partial_i \partial_j \tau_{ij}^{\text{d}}$
4. Contribution from the actuator disk: $\partial_j \partial_j \overline{p}_{\text{AD}} = \partial_j f_j^{\text{AD}}$

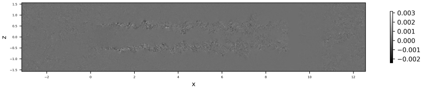
X-Z contours through the centerline



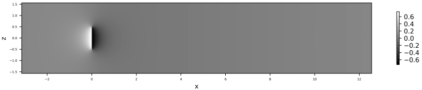
(a) True pressure



(b) Fringe contribution



(c) SGS contribution

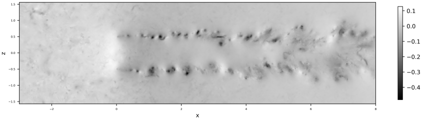


(d) Actuator disk contribution

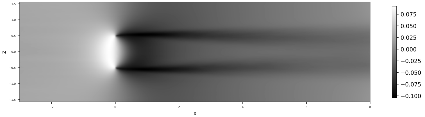
Further decompose *true* pressure

True pressure can be further decomposed into: a) Mean, b) Rapid component and c) Slow component

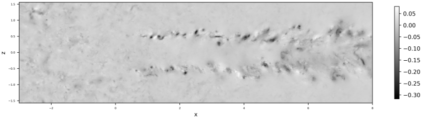
$$\partial_j \partial_j \overline{p}_{\text{true}} = -\left\langle \left(\partial_j \overline{u}_i\right)\left(\partial_i \overline{u}_j\right)\right\rangle - 2\left(\partial_j \left\langle \overline{u}_i \right\rangle\right)\left(\partial_i \overline{u}'_j\right) - \partial_i \partial_j \left(\left\langle \overline{u}'_i \overline{u}'_j \right\rangle - \overline{u}'_i \overline{u}'_j\right)$$



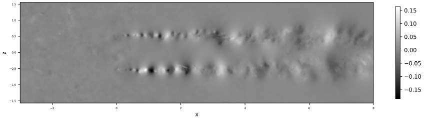
(a) True pressure



(b) Mean (Time averaged)



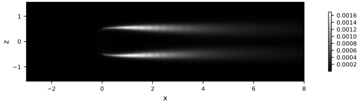
(c) Slow component



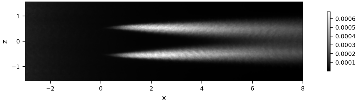
(d) Rapid component

The rapid component does not seem to overwhelm the slow component in any region of the flow.

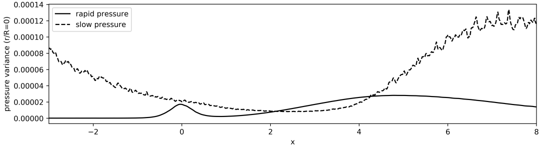
Pressure variance contributions



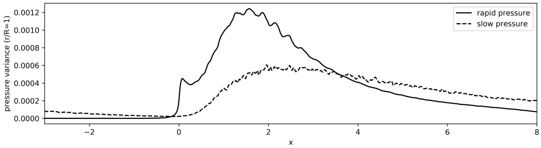
(a) Rapid component



(b) Slow component



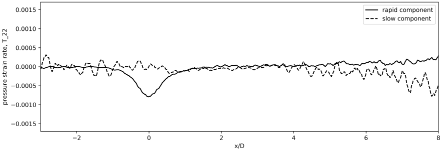
(c) Centerline



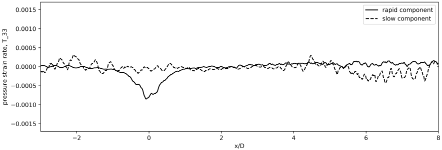
(d) Shear layer

Pressure-strain rate correlations

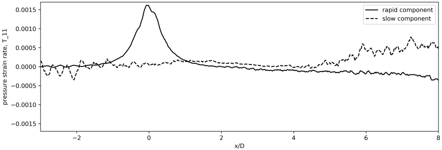
Now consider $T_{ij} = \left\langle \bar{p}' \left(\partial_j \bar{u}_i' + \partial_i \bar{u}_j' \right) \right\rangle$ along the **centerline**



(a) T_{22}



(b) T_{33}



(c) T_{11}

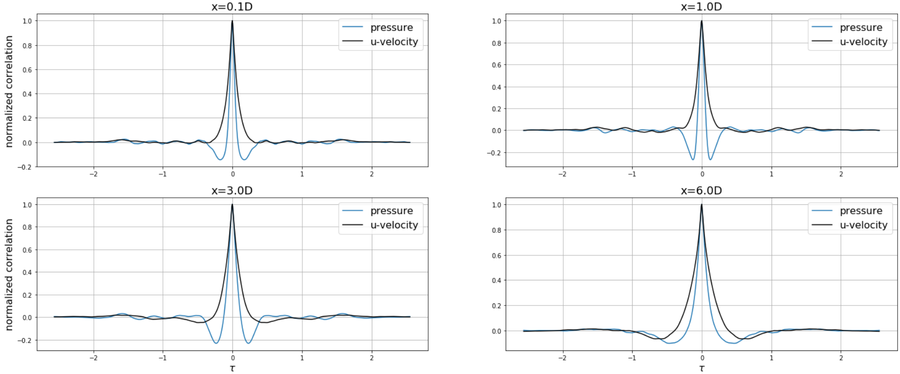
The distortion of the isotropic turbulence as it enters the core, does appear to be due to linear processes

Shear Layer: 2pt correlations

Define 2pt auto-correlations of a field variable, $\boldsymbol{q}(\boldsymbol{x}, t)$ as:

$$\boldsymbol{C}(\boldsymbol{x}, \boldsymbol{x}', \tau) = \left\langle \boldsymbol{q}(\boldsymbol{x}, t)\boldsymbol{q}(\boldsymbol{x}', t+\tau) \right\rangle$$

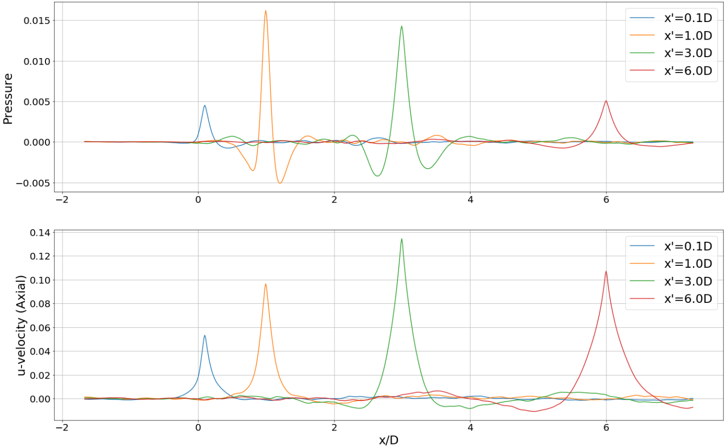
Now let's consider **temporal correlations** (i.e. $\boldsymbol{x} - \boldsymbol{x}' = 0$) along the shearline $(r/D = 0.5)$ at various x locations



Non-monotonic auto-correlations in time (especially pronounced in pressure)

Shear Layer: 2pt correlations

Now let's consider **axial (spatial) correlations** (i.e. $\tau = 0$) along the shearline ($r/D = 0.5$) at various x locations



Non-monotonic spatio-temporal correlations (especially in u component) suggest wave-packet like features in the shear layer.

Space-time modal decomposition (SPOD)

Lumley's (1970) original space-time POD (NOT to be confused with the more commonly used *snapshot POD*)

- Wavepacket like features in 2pt correlations suggest the need to consider the **principle components** of the 2pt. space-time correlation tensor.
- More convenient to consider principle components of its temporal Fourier transform

$$\boldsymbol{S}(\boldsymbol{x},\boldsymbol{x}',f) = \int_{-\infty}^{\infty} \boldsymbol{C}(\boldsymbol{x},\boldsymbol{x}',\tau)e^{-i2\pi f\tau}d\tau = \sum_{j=1}^{\infty}\lambda_j(f)\boldsymbol{\psi}_j(\boldsymbol{x},f)\boldsymbol{\psi}_j^{\star}(\boldsymbol{x}',f)$$

- The complex valued function, $\psi_j(\boldsymbol{x},f)$ is the j^{th} **mode shape** at the frequency, f and the real value λ_j is its **modal energy**.
- Modes at frequency, f are **orthonormal** in the spatial norm, $\langle\boldsymbol{\psi}_j(\boldsymbol{x},f)\boldsymbol{\psi}_k(\boldsymbol{x},f)\rangle_x = \delta_{jk}$
- Equivalently, we seek a modal expansion for the temporal Fourier transform of the field variable(s)

$$\hat{\phi}(\boldsymbol{x},f) = \sum_{j=1}^{\infty} a_j(f)\psi_j(\boldsymbol{x},f)$$

where the coefficients $a_j(f) = \left\langle\hat{\phi}(\boldsymbol{x},f),\psi_j(\boldsymbol{x},f)\right\rangle_x$ are uncorrelated, i.e. $\overline{a_j a_m^{\star}} = \lambda_j\delta_{jm}$

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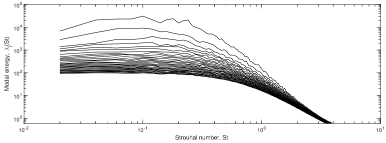
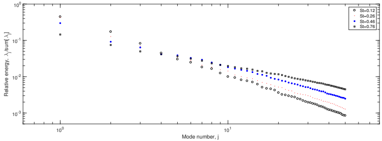
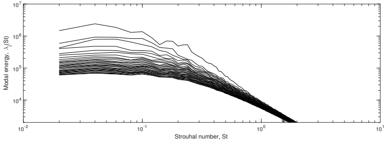
Modal energies for Case 10 inflow



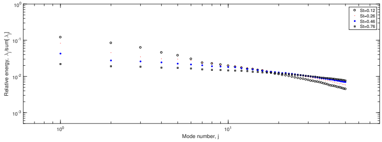
Modal energies using pressure norm



Rel. energies at 4 frequencies (pressure norm)



Modal energies using the TKE norm



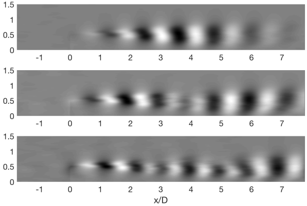
Rel. energies at 4 frequencies (TKE norm)

Low rank?

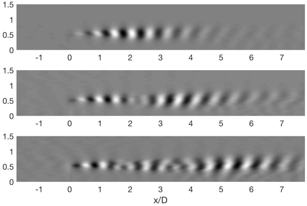
- At **St=0.12, 15 modes** contribute **95% pressure variance**, and **55% of TKE**
- At **St=0.76, 15 modes** contribute **65% pressure variance**, and **30% of TKE**

SPOD - Pressure wavepackets

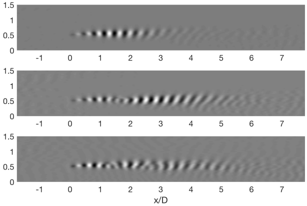
First 3 pressure mode shapes $(\texttt{REAL}(\psi(x, f)))$ for Case 10 inflow



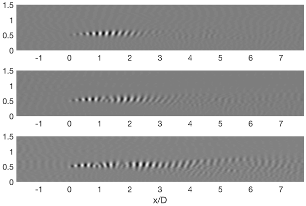
St=0.12



St=0.26

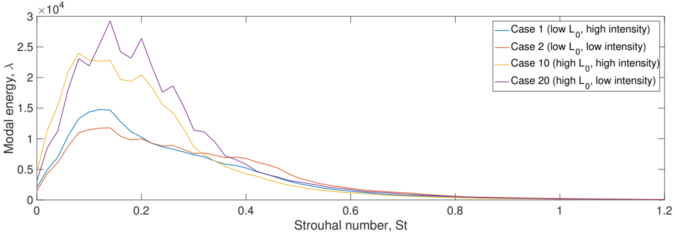


St=0.46

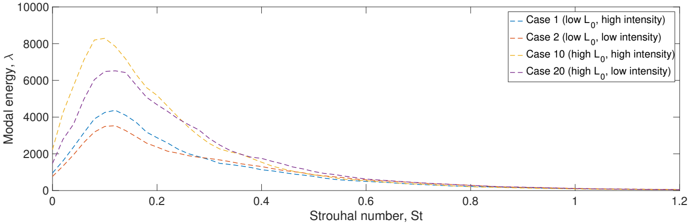


St=0.76

SPOD - Sensitivity to HIT inflow



Modal energies for the 1st mode



Modal energies for the 2nd mode

Part II - Conclusions

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Sensitivity to upstream HIT

1. The wake recovery is strongly influenced by the *length scale* of the inflow HIT, as opposed to its *intensity*.
2. Larger length scale HIT inflow results in a larger projection of the initial perturbation onto low Strouhal number, large wavelength KH wavepackets, thereby entraining momentum more rapidly and causing faster wake recovery.

Flow physics

1. The AD induces axisymmetric expansion of HIT which is mostly felt by the largest incident scales. Intercomponent energy transfer occurs via linear processes.
2. The space-time modal decomposition using the TKE norm does not suggest that the overall flow physics is low rank, and a simple projection based ROM for the problem may not be conceivable.
3. The shear layer instability triggered by the incident turbulence has a very broadband character; this is unlike the more tonal behavior seen in jets (relevant for *jet noise*)
4. Many more peculiar phenomena (especially along the centerline) yet to be explained

Concluding remarks - Moving forward

1. Full scale problem has a rich interplay between local secondary instabilities and turbulent scales in free stream; different phenomena at different scales, even when mesoscales are neglected.
2. Gabor modes: Excellent basis for broadband inertial range turbulence. But, enrichment relies on robust prediction of large scale features; can LES accurately predict 2pt correlations?
3. Gabor modes: More work needed for large time dynamics (better closure model for non-linear terms)
4. Data driven projections / Modal bases with optimal forcing: Excellent for isolating secondary flow instabilities
5. Potential to exploit this synergy between physics-based modeling strategies for secondary instabilities using ideas based on Global modes (Nichols & Lele, 2011) and Resolvent operators (Gomez et. al., 2016; Schmidt, et. al., 2017) and Gabor mode enrichment for turbulence

Questions?

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Further information on Gabor mode enrichment

Ghate, A., & Lele, S., *Subfilter-scale enrichment of planetary boundary layer large eddy simulation using discrete Fourier-Gabor modes*, Vol. 819, J. Fluid Mech., 2017

Further information on HIT - Actuator disk interactions

Ghate, A., Ghaisas, N., Towne, A. & Lele, S., *Interaction of small scale Homogenenous Isotropic Turbulence with an Actuator Disk*, AIAA Scitech, 2018 (Paper No. 2018-0753)

Further information on Spectral POD

Towne, A., Schmidt, O. T., and Colonius, T., *Spectral proper orthogonal decomposition and its relationship to dynamic mode decomposition and resolvent analysis*, arXiv preprint arXiv:1708.04393, 2017.