# Towards a Viscous Wall Model for Immersed Boundary Methods

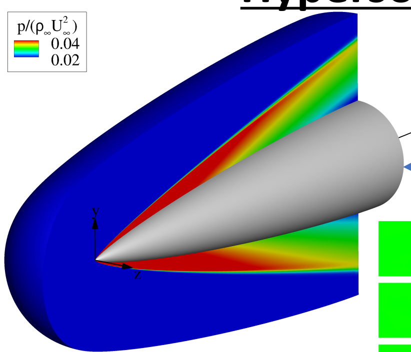
***C. Brehm*** ***and O.M.F. Browne***
Mechanical Engineering, University of Kentucky, Lexington, USA

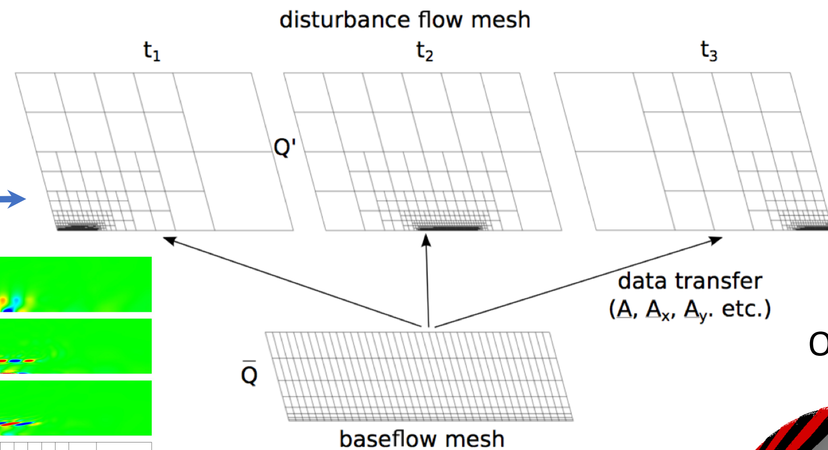
***N. Ashton***
Oxford Thermofluids Institute, University of Oxford, Oxford, UK

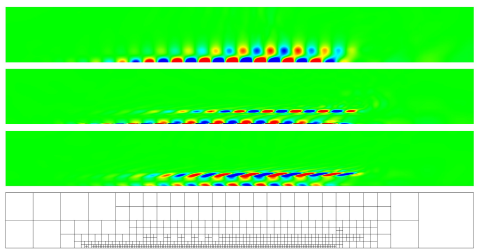
NASA AMS Seminar Series – 05/03/2018

*Initially presented at 2018 AIAA SciTech Forum, Orlando, Florida*
*Session FD-48: Cartesian, Overset, and Meshfree CFD Methods*

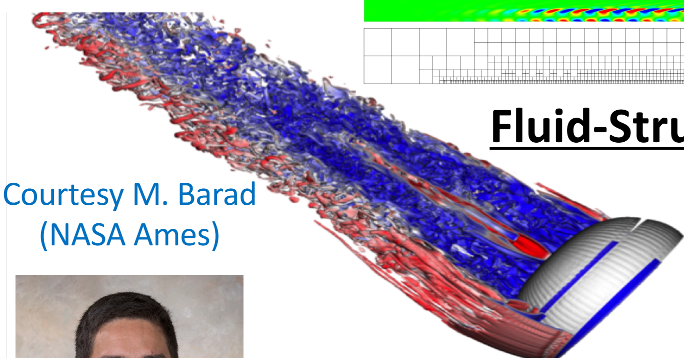
## Hypersonic Transition



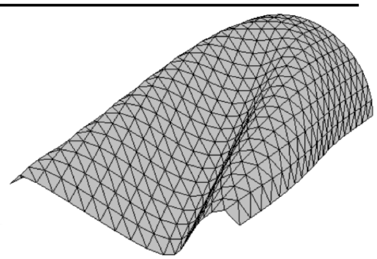



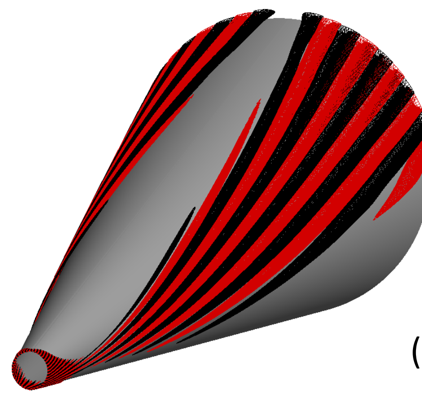
Oliver M. F. Browne (Postdoc)



Courtesy M. Barad (NASA Ames)

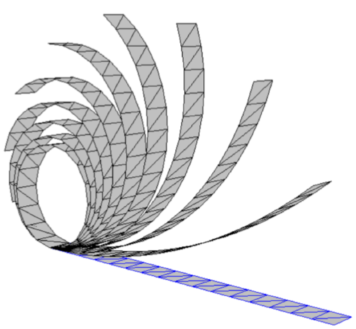
## Fluid-Structure Interaction





Anthony Haas (PhD student at UofA, Prof. H.F. Fasel )

Jonathan Boustani (PhD student)



O.M.F. Browne, A.P. Haas, H.F. Fasel, and C. Brehm, "An Efficient Strategy for Computing Wave-Packets in High-Speed Boundary Layers Over Complex Geometries", AIAA Aviation, 2018

O.M.F. Browne, A.P. Haas, H.F. Fasel, and C. Brehm, "An Efficient Strategy for Simulating Nonlinear Wave-Packets in High-Speed Boundary Layers", AIAA Aviation, 2018

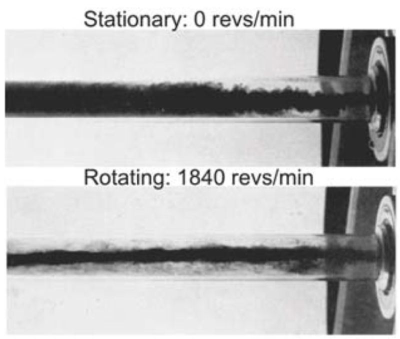
O.M.F. Browne, A.P. Haas, H.F. Fasel, and C. Brehm, "A Global Stability Solver on Block-Structured Cartesian Domain-Decomposed Irregular Domains", ParCFD 2018

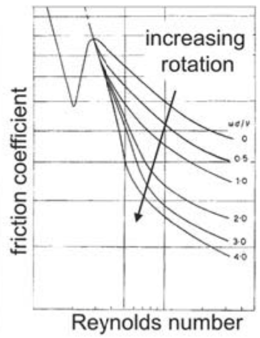
J. Boustani, O. Browne, J. Wenk, M. Barad, C. Kiris, and C. Brehm, "A Numerical Method for Fluid-Structure Interactions with Large Deformations", AIAA Aviation, 2018

# Relaminarization and Turbulence Suppression

J. Davis, S. Ganju, S. Bailey, C. Brehm, "Direct Numerical Simulations of Turbulence Suppression in Rotating Pipe Flows", ParCFD 2018

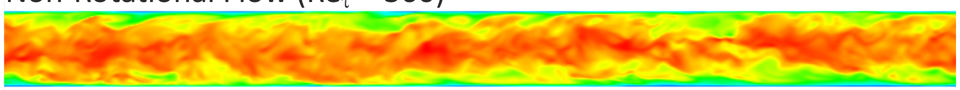
Jeff Davis (PhD student, since 08/17)



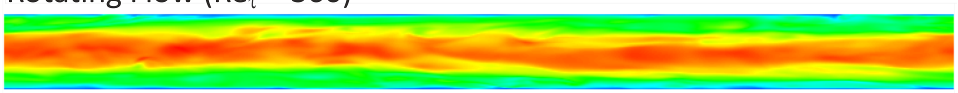




adjusted from Imao et al.

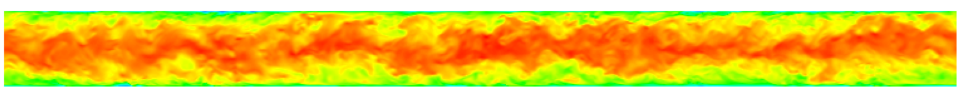
Non-Rotational Flow ($Re_\tau$ = 360)



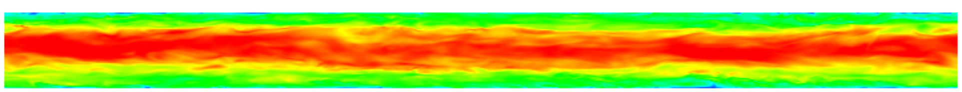
Rotating Flow ($Re_\tau$ = 360)



Non-Rotational Flow ($Re_\tau$ = 590)



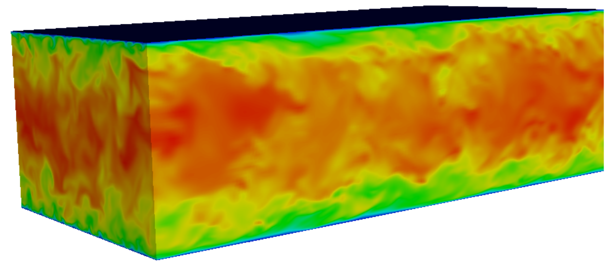
Rotating Flow ($Re_\tau$ = 590)

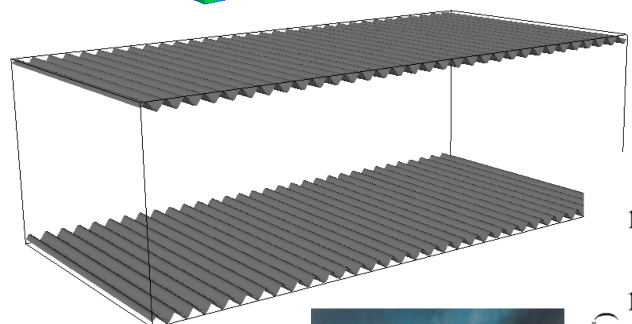


Collaboration with Prof. Bailey (experiments at UK)

# DNS of Wall-Bounded Turbulent Flows

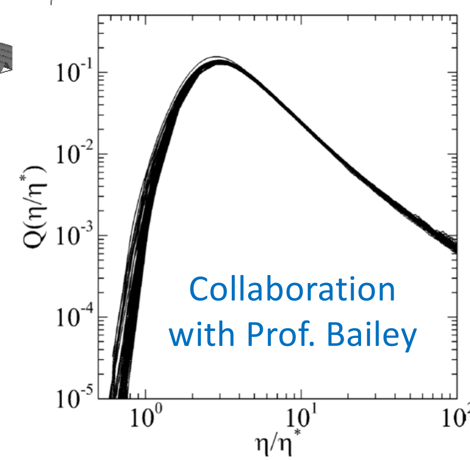
S. Ganju, C. Brehm, S. Bailey, "Examination of the Distribution of Dissipative Scales within Turbulent Wall-Bounded Flow"





Sparsh Ganju
(PhD student)

John Higgins
(Incoming PhD student, research topic undecided)




Collaboration with Prof. Bailey

# Outline

## Motivation/Introduction
Current state and challenges for IBM.

## Immersed Boundary Method (IBM)
*Introducing conservative FD IB scheme.*

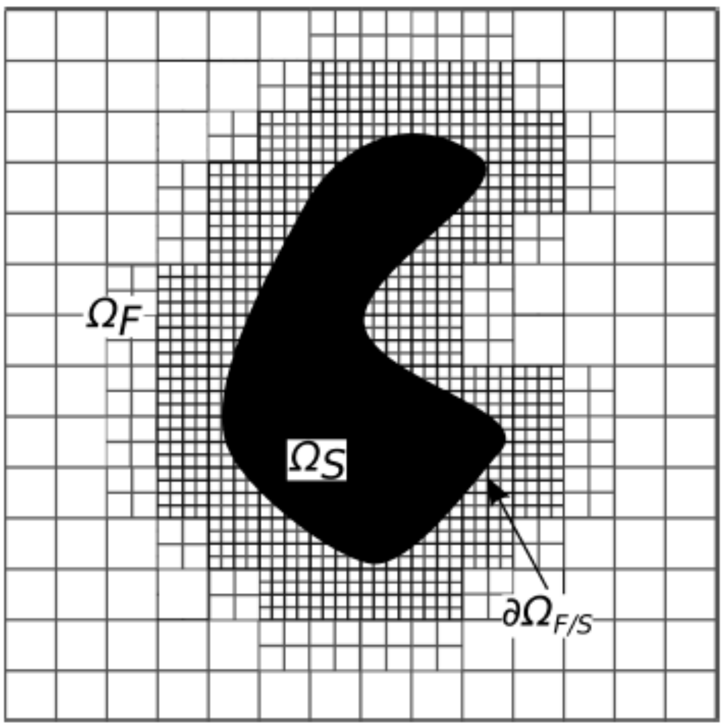
## Viscous Wall Model (VWM)
*Discussion of different viscous wall modeling approaches.*

## IBM & VWM Coupling
*Introduces basic idea of immersed boundary method.*

## Validation Study
*Validation of newly developed method.*

## Final Discussion and Conclusion
What is the current state and what is next? Additional Challenges.

# Motivation for Immersed Boundary Methods




*Arbitrary geometry immersed into a Cartesian grid, where fluid and solid domains are marked with $\Omega_F$ and $\Omega_S$, and immersed boundary as $\Omega_{S/F}$.*

Why Cartesian mesh methods?

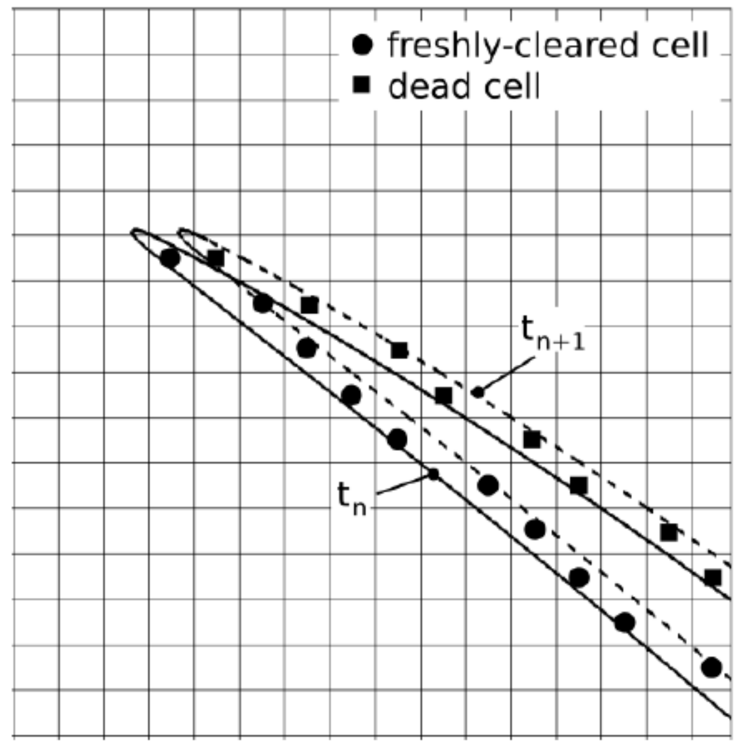
- ❑ Grid generation process can be fully automated
- ❑ Cartesian mesh provides excellent numerical solution properties (although boundary operators can be problematic)
- ❑ Higher-Order accuracy can be obtained in a straight-forward fashion for interior operators
- ❑ Well-suited for exa-scale computing (data locality, tree-structure, *etc*.)
- ❑ Fully-Eulerian solver approach for fluid-structure interaction problems (eliminating procedures for mesh deformation, transfer of solution from $\Omega^n$ to $\Omega^{n+1}$, *etc*.)

# Immersed Boundary Methods

Immersed Boundary Methods (IBMs) have been developed and extended for a number of years (Peskin et al. , Goldstein et al. , LeVeque and Li, Wiegmann and Bube, Linnick and Fasel, Johansen and Colella, Mittal and Iaccarino, Zhong, Duan et al., and many more.)

**Algorithmic challenges for IB Cartesian Mesh Methods:**

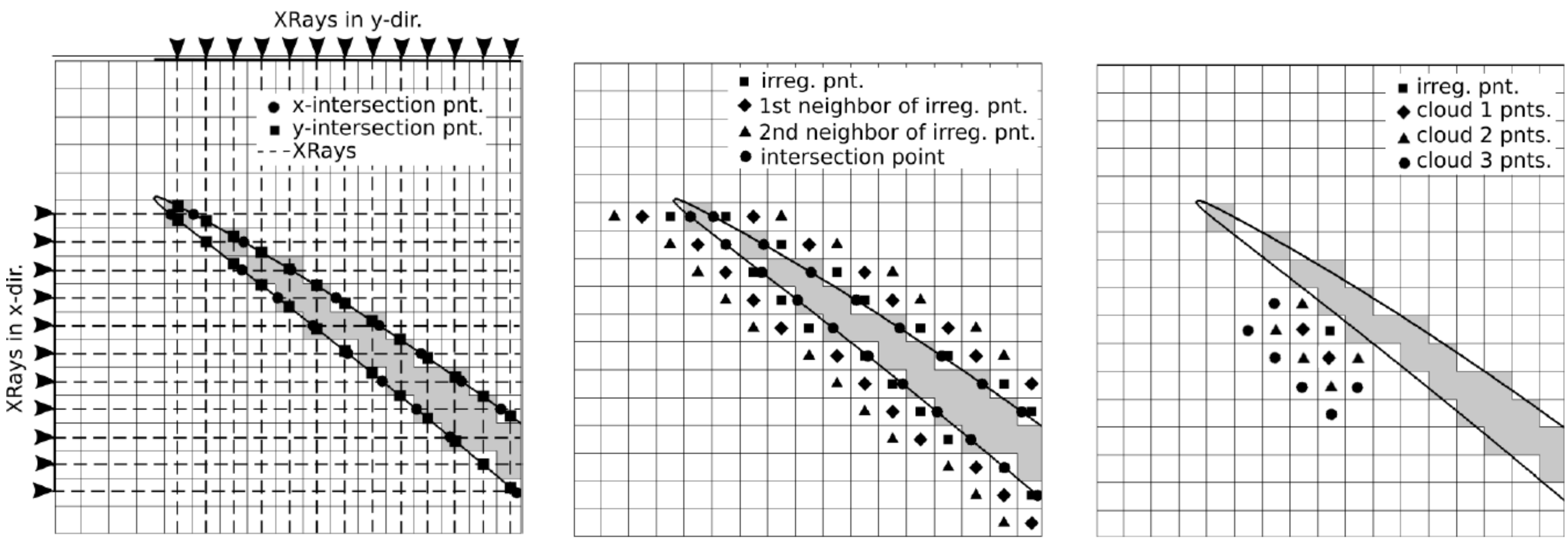
- ❑ Grid stretching approaches are not efficient and defeat the purpose of IBM methods
  - ➢ Some type of block-structured Cartesian mesh topology is advantageous (AMR)
- ❑ Higher-order boundary operators (preferably provable stable) not straightforward to obtain
  see for example, Linnick and Fasel, Zhong, Duan et al. , Brehm and Fasel, Brehm et al. , and others
- ❑ Dynamic load balancing (especially for moving or deforming boundary problems)



# Immersed Boundary Methods

**Algorithmic challenges for IB Cartesian Mesh Methods (cont'd):**

- Automation of volume mesh generation is traded against algorithmic complexity to handle complex geometries (in-out testing, cloud stencil search algorithms, grid-line intersection, distance function, *etc.*)
  - Geometry is required to be watertight and resolution of surface grid affects accuracy

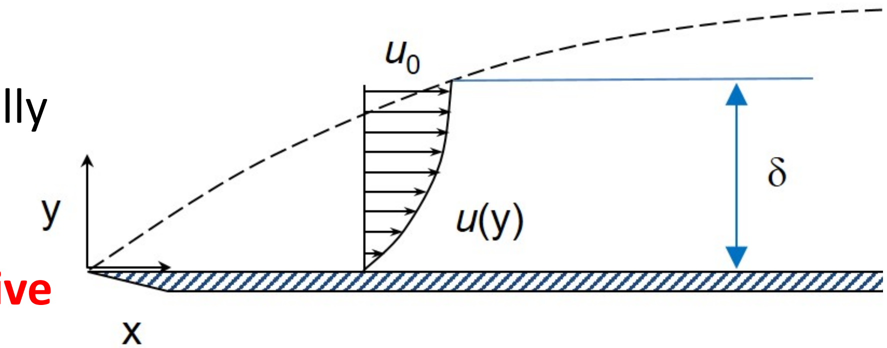



Brehm, Barad, and Kiris (JCP 2018, submitted)

# Immersed Boundary Methods

**Physical limitations of current Cartesian grid methods:**

- ❑ Mesh-alignment with shocks (especially problematic for hypersonic flows)
- ❑ Method is currently limited to solution of Euler equation and low Reynolds number applications
  - ➢ IB approaches are inefficient in resolving boundary layers since Cartesian mesh generally does not allow the use of (wall-normal) high aspect ratio cells at the wall
    **→ wall-resolved simulations are too expensive**

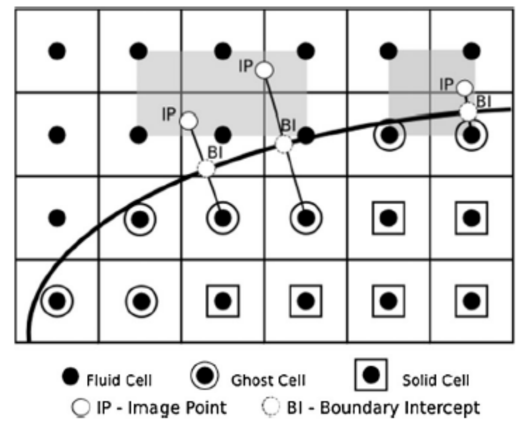



  - ➢ Consider that for DNS $N_{Grid} \sim Re_L^{37/14}$ and for LES $N_{Grid} \sim Re_L^{13/7}$ Choi & Moin, 2011
  - ➢ With Wall-modeled LES the grid resolution can be significantly reduced with $N_{Grid} \sim Re_L$
  - ➢ Practical importance of wall-modeling in LES for high Reynolds number flows (even for body-fitted meshes)

# Viscous Wall Extensions

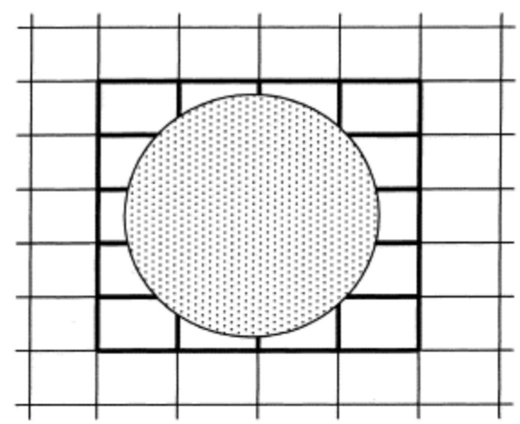
- Ruffin and Lee (2009) combined IB ghost cell approach with standard k-ε turbulence model by Launder and Spalding with Spalding's wall model formulation
  - Fair agreement for subsonic 2D and axisymmetric test cases (flat plate & airfoil)
- Berger and Aftosmis (2012) combined Cartesian cut-cell finite volume method with analytic wall model that is based on the Spalart-Allmaras turbulence model
  - Good agreement for flow over a flat plate and sub- and transonic airfoils
  - Convergence of surface pressure $\Delta x = 0.1\% \times$ chord and converged surface skin friction for 2-4 smaller grid spacings

Ghost Cell Method




Taken from Kiris *et al.* (2016)

Cut Cell Method

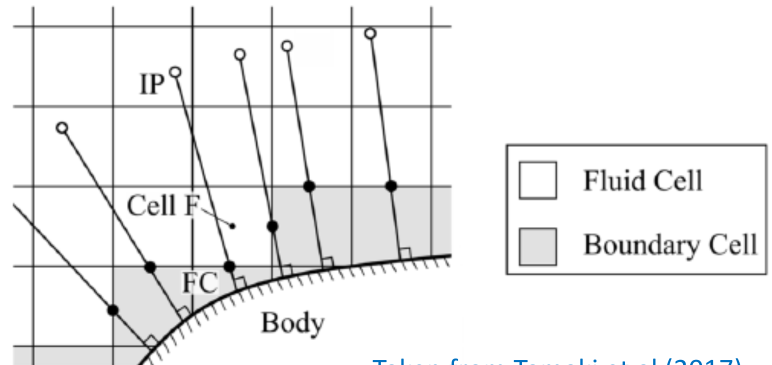


Taken from Tucker and Pan *et al.* (2000)

# Viscous Wall Extensions

- Tamaki et al. (2017) developed an IBM for turbulent flow simulations based on analytical S-A turbulence model
  - Two key modifications of an earlier 2016 version: (1) linearly extrapolate velocity of the forcing point to the wall and (2) modification of eddy-viscosity profile to maintain balance of the shear stress
  - good validation results against body-fitted results for flat plate, NACA0012 airfoil, and turbulent flow over bump in the channel

IBM for turbulent flows




Taken from Tamaki et al (2017)

- Berger and Aftosmis (2017) followed up on previous work from 2012
  - System of ODEs coupling the streamwise velocity and the turbulent viscosity replaced analytic wall function
  - Streamwise momentum equation included the pressure gradient and convection terms
  - excellent comparison against body-fitted CFD results for 2D test cases even for y+ > 100.
- Another strategy is to couple the IB Navier-Stokes solver to an integral boundary layer (IBL) method (see Drela (1987), Aftosmis (2006), Rodriguez (2012))

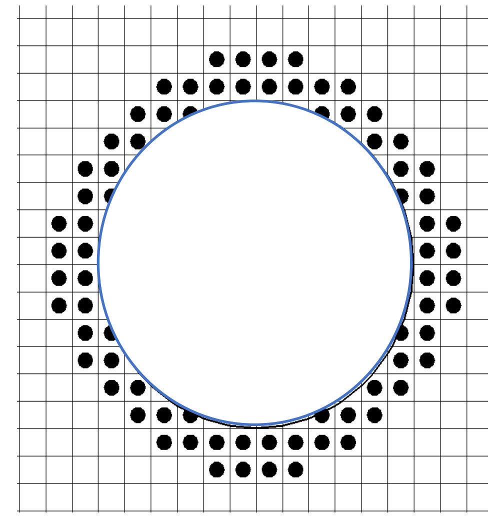
# Outline

# Basics of Stability Enhancement Approach

❑ Initial observation:

*“Stability of numerical scheme can be formulated as N-dimensional optimization problem”*

(N=number of irregular grid points)



● Irregular grid point

❑ Derivation of stencil coefficients:

- Enforce order-of-accuracy

$$\tau_i = \left(-c_x\frac{\partial\phi}{\partial x} + \frac{\partial}{\partial x}\left(\beta\frac{\partial\phi}{\partial x}\right) + f\right)_{x=x_i} - \left(\hat{\phi}_B\tilde{\alpha}_1 + \hat{\phi}_i\tilde{\alpha}_2 + \hat{\phi}_{i+1}\tilde{\alpha}_3 + \hat{\phi}_{i+2}\tilde{\alpha}_4 + \hat{\phi}_{i+3}\tilde{\alpha}_5 + C\right)$$

- Additional grid point is needed to introduce free parameter
- Objective function depends on the nature of the PDE, e.g., $\lambda_{r,max}$ or $\rho(A)$

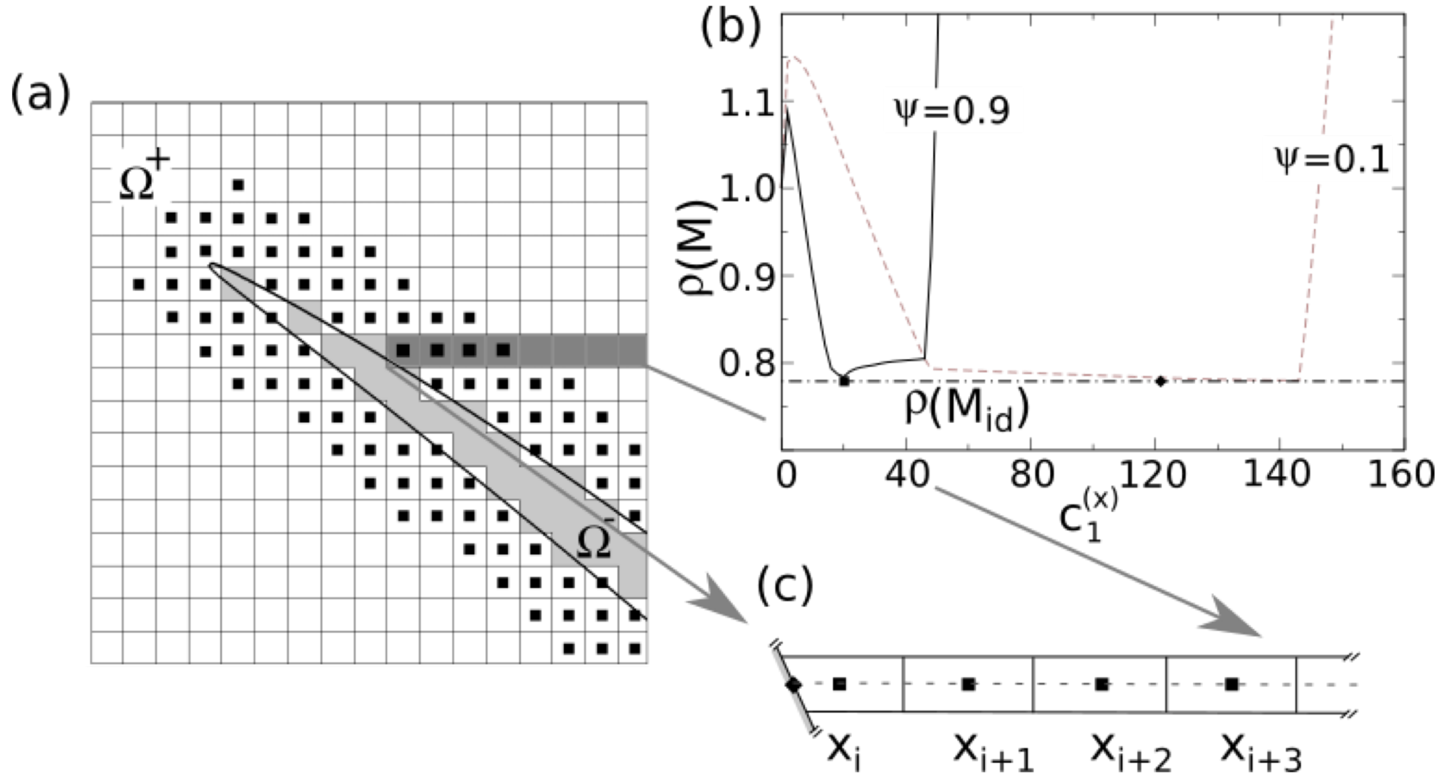
$$\frac{\partial\phi}{\partial t} = -c_x\frac{\partial\phi}{\partial x} + \frac{\partial}{\partial x}\left(\beta\frac{\partial\phi}{\partial x}\right) + f \xrightarrow{\mathit{discr.}} \underline{\mathbf{B}}\hat{\boldsymbol{\phi}}^{n+1} = \underline{\mathbf{A}}\hat{\boldsymbol{\phi}}^{n} + \hat{\mathbf{f}}$$

- Extract perturbation of irregular finite difference stencil (assume $\underline{\mathbf{B}}=\underline{\mathbf{I}}$)

$$\underline{\mathbf{A}} = \underline{\mathbf{E}}\underline{\mathbf{A}}_{id} \xrightarrow{\mathit{cond.}} \rho(\underline{\mathbf{E}}) \leqslant 1$$

*Brehm & Fasel (JCP, 2013, 2015), applications: Brehm, Barad, & Kiris (JCP 2018, IJFH 2017), Brehm, Barad, Housman & Kiris (JFM 2016)*

# Basics of Stability Enhancement Approach




- ❑ Solving N-dimensional optimization problem is too expensive
- ❑ Apply localization of FD stencil which turns N-dimensional problem
  - into N x 1-D problems (localization was demonstrated) Brehm and Fasel (JCP, 2013)

# Conservative Finite-Difference Method – Convective Terms

- Conservative Finite Difference Operator:

$$\frac{\partial f}{\partial x} = \frac{1}{\Delta x}(h_{i+1/2} - h_{i-1/2}) = \frac{1}{\Delta x}(\hat{f}_{i+1/2} - \hat{f}_{i-1/2}) + \mathcal{O}\left(\Delta x^{2n-1}\right)$$

- Scheme relies on error cancellation
  - Numerical flux derivative at $x_i$:

$$\left.\frac{\partial f}{\partial x}\right|_{x=x_i} = \frac{h_{i+1/2} - h_{i-1/2}}{\Delta x} \approx \frac{\hat{f}_{i+1/2} - \hat{f}_{i-1/2}}{\Delta x} \quad (1)$$

  - Truncation error obtaining flux at $x_{i+1/2}$

$$\hat{f}_{i+1/2} = h_{i+1/2} - \frac{1}{60}\left.\frac{\partial^5 f}{\partial x^5}\right|_{x=x_i} \Delta x^5 \text{ (scheme A)} \quad (2)$$

  - Truncation error obtaining flux at $x_{i-1/2}$

$$\hat{f}_{i-1/2} = h_{i-1/2} - \frac{1}{600}\left.\frac{\partial^5 f}{\partial x^5}\right|_{x=x_i} \Delta x^5 \text{ (scheme B)} \quad (3)$$

  - Substituting (2) and (3) in (1) leads to:

$$\left.\frac{\partial f}{\partial x}\right|_{x=x_i} = \frac{h_{1+1/2} - h_{1-1/2}}{\Delta x} - \frac{3}{200}\left.\frac{\partial^5 f}{\partial x^5}\right|_{x=x_i} \Delta x^4$$

  - *To recover formal order-of-accuracy match not only order but also leading term of truncation error* **(Brehm, JCP 2017)**

# Conservative Finite-Difference Method – Convective Terms

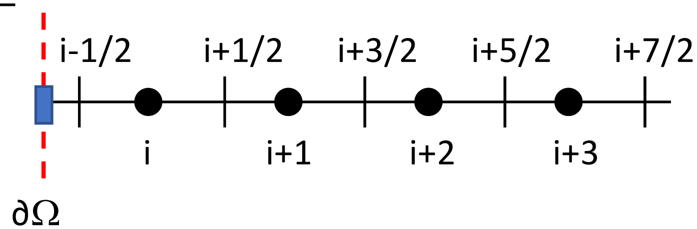
- ❑ For third-order accurate scheme we implicitly define primitive function h(ξ) of flux

$$h(x) = h_0 + h_1 x + h_2 \frac{x^2}{2!} \longrightarrow \hat{f} = \frac{1}{\Delta x}\int_{x-\Delta x/2}^{x+\Delta x/2} h(\xi)d\xi = h_0 + \ldots.$$

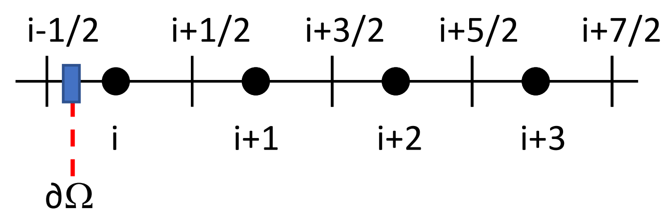
  - Use over-determined stencil to obtain free parameter

- ❑ Need to define appropriate interpolation operators at domain boundaries

ψ>0.5:




ψ<0.5:



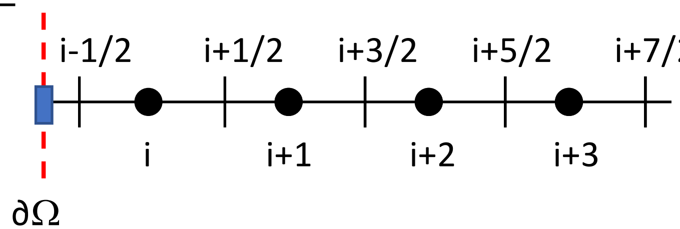
# Conservative Finite-Difference Method – Convective Terms

- For third-order accurate scheme we use

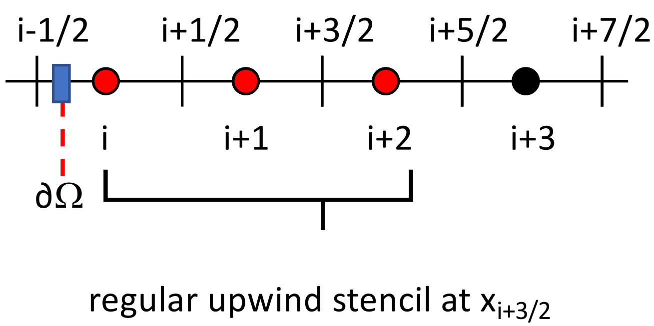
$$h(x) = h_0 + h_1 x + h_2 \frac{x^2}{2!} \longrightarrow \hat{f} = \frac{1}{\Delta x}\int_{x-\Delta x/2}^{x+\Delta x/2} h(\xi)d\xi = h_0 + \ldots.$$

  - Use over-determined stencil to obtain free parameter
- Need to define appropriate interpolation operators at domain boundaries

<u>ψ>0.5:</u>




<u>ψ<0.5:</u>



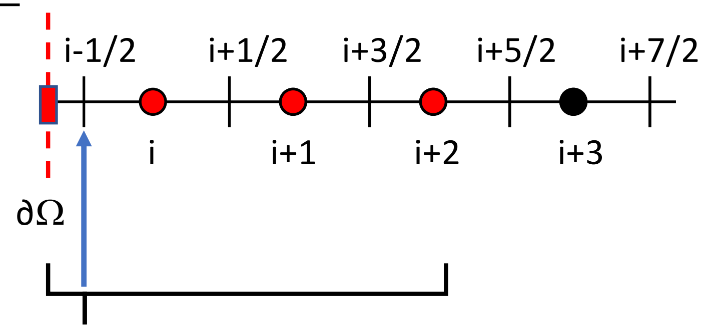
# Conservative Finite-Difference Method – Convective Terms

- For third-order accurate scheme we use

$$h(x) = h_0 + h_1 x + h_2 \frac{x^2}{2!} \longrightarrow \hat{f} = \frac{1}{\Delta x} \int_{x-\Delta x/2}^{x+\Delta x/2} h(\xi) d\xi = h_0 + \ldots .$$

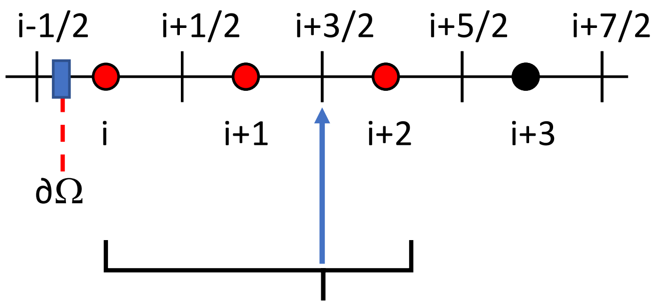
  - Use over-determined stencil to obtain free parameter
- Need to define appropriate interpolation operators at domain boundaries

$\psi$>0.5:




irregular upwind stencil at $x_{i-1/2}$

$\psi$<0.5:




regular upwind stencil at $x_{i+3/2}$

# Conservative Finite-Difference Method – Convective Terms

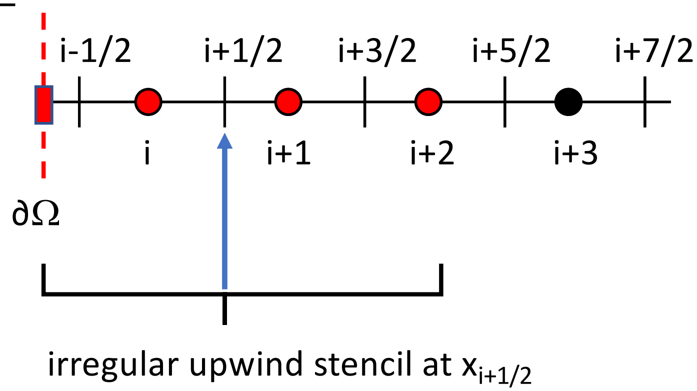
- For third-order accurate scheme we use

$$h(x) = h_0 + h_1 x + h_2 \frac{x^2}{2!} \longrightarrow \hat{f} = \frac{1}{\Delta x}\int_{x-\Delta x/2}^{x+\Delta x/2} h(\xi)d\xi = h_0 + \ldots.$$

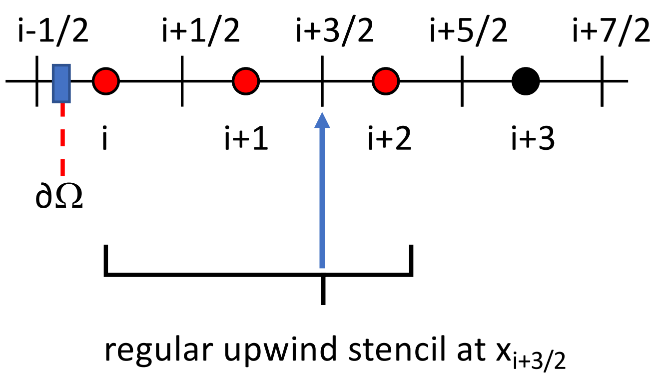
  - Use over-determined stencil to obtain free parameter

- Need to define appropriate interpolation operators at domain boundaries

$\psi$>0.5:




$\psi$<0.5:



# Conservative Finite-Difference Method – Convective Terms

- Conservative FD operator is derived such that
  1) Accuracy constraints (for derivative $\underline{D}$ and appropriate interpolation $\underline{\mathbb{I}}$ operators)

$$\frac{\partial f}{\partial x} = \underline{D}\, f + \mathcal{O}(\Delta x^p) \quad \text{or} \quad \widehat{f} = \underline{\mathbb{I}}\, f + \mathcal{O}(\Delta x^p)$$

  2) Telescoping derivative operator, $\underline{D}$ ("*telescopes flux from boundary to boundary*")

Fisher *et al.* (2012)

$$\frac{\partial f}{\partial x} = \underline{D}\, f = \underline{\Delta}\, \widehat{f} = \underline{\Delta}\, \underline{\mathbb{I}}\, f + \mathcal{O}(\Delta x^p)$$

$$\underline{\Delta} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad \text{and} \quad \underline{D} = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 & 0 & 0 \\ a_2 & b_2 & c_2 & d_2 & 0 & 0 \\ 0 & a & b & c & 0 & 0 \\ 0 & 0 & a & b & c & 0 \\ 0 & 0 & 0 & a & b & c \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix}$$

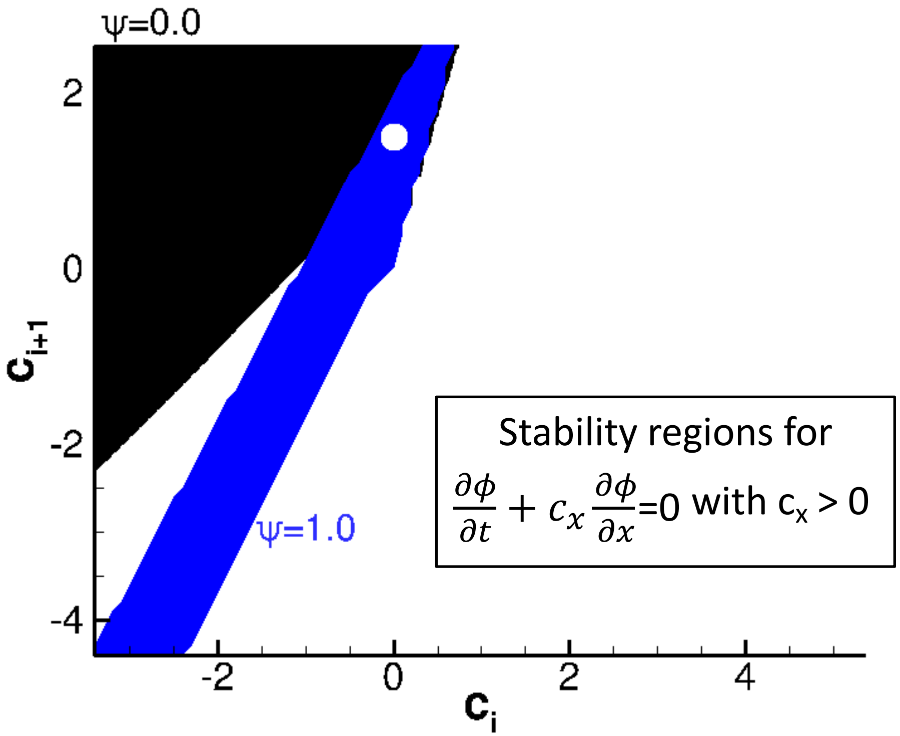
with end point flux consistency $\widehat{f}_{1/N} = f_{1/N}$

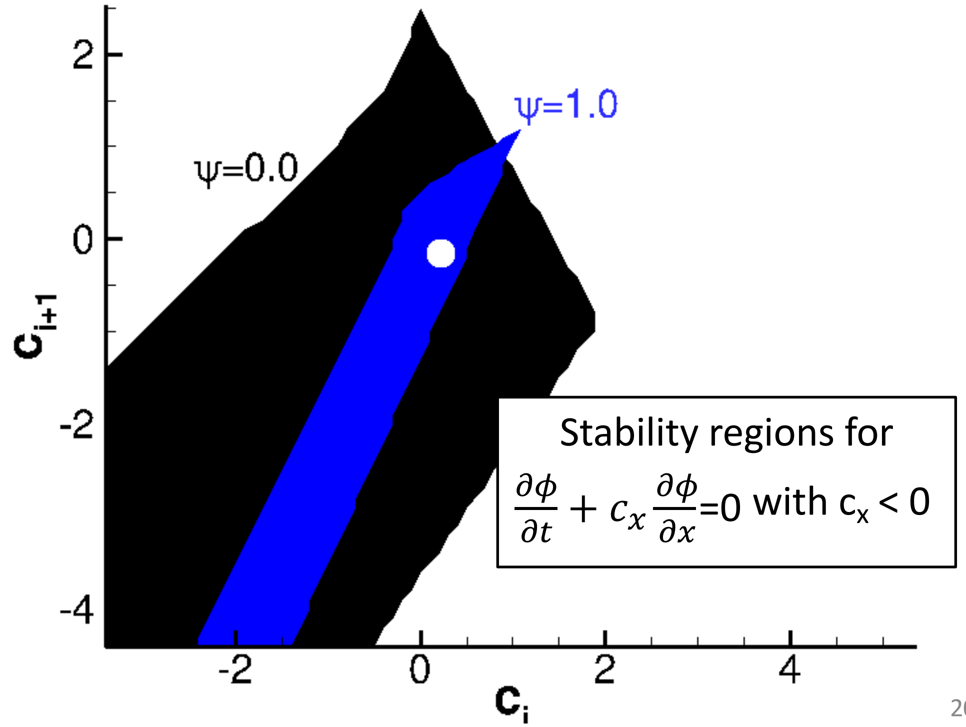
# Conservative Finite-Difference Method – Convective Terms

- Matrix Stability Analysis - Spectral radius of update matrix

$$\underline{\tilde{B}}\frac{d\varepsilon}{dt} = -\frac{1}{\Delta x}\underline{\tilde{A}}\varepsilon, \quad \text{with BC:} \quad \varepsilon_0(t) = 0, \quad \text{and IC:} \quad \varepsilon(t) = f(x_i)$$

- Stability regions for advection equation with advection speed $c_x$ (using third-order accurate extrapolation operators)



# Conservative Finite-Difference Method – Viscous Terms

- Original Non-conservative approach:

$$\frac{\partial\left(\tau_{xy}\right)}{\partial y}=\frac{d\mu}{dT}\frac{\partial T}{\partial y}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\mu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial y\partial x}\right) \text{ with } \left.\frac{\partial\phi}{\partial x}\right|_k=c_{\partial\Omega}\phi_{\partial\Omega}+\sum_{m=1}^{p_k+1}c_{i+m-1}\phi_{i+m-1}+\mathcal{O}\left(\Delta x^{2n}\right)$$

$$\left.\frac{\partial^2\phi}{\partial x^2}\right|_k=c_{\partial\Omega}\phi_{\partial\Omega}+\sum_{m=1}^{p_k+2}c_{i+m-1}\phi_{i+m-1}+\mathcal{O}\left(\Delta x^{2n}\right) \text{ and } \left.\frac{\partial^2\phi}{\partial x\partial y}\right|_{i,j,k}=c_{\partial\Omega}\phi_{\partial\Omega}+\sum_{m=1}^{n_R-1+n_A}c_{i_m,j_m,k_m}\phi_{i_m,j_m,k_m}+\mathcal{O}\left(\Delta x^{2n}\right)$$

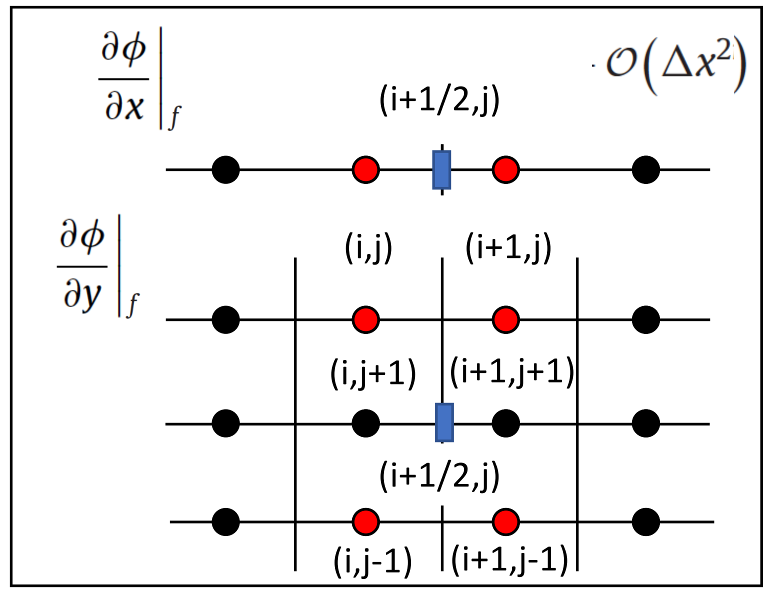
  - Velocity and temperature are provided $\partial\Omega$
  - ➢ **Problem**: How to provide viscous flux at the wall?

- Conservative approach

$$\frac{\partial f}{\partial x}=\frac{1}{\Delta x}\left(\hat{f}_{i+1/2}-\hat{f}_{i-1/2}\right)+\mathcal{O}\left(\Delta x^{2n}\right)$$

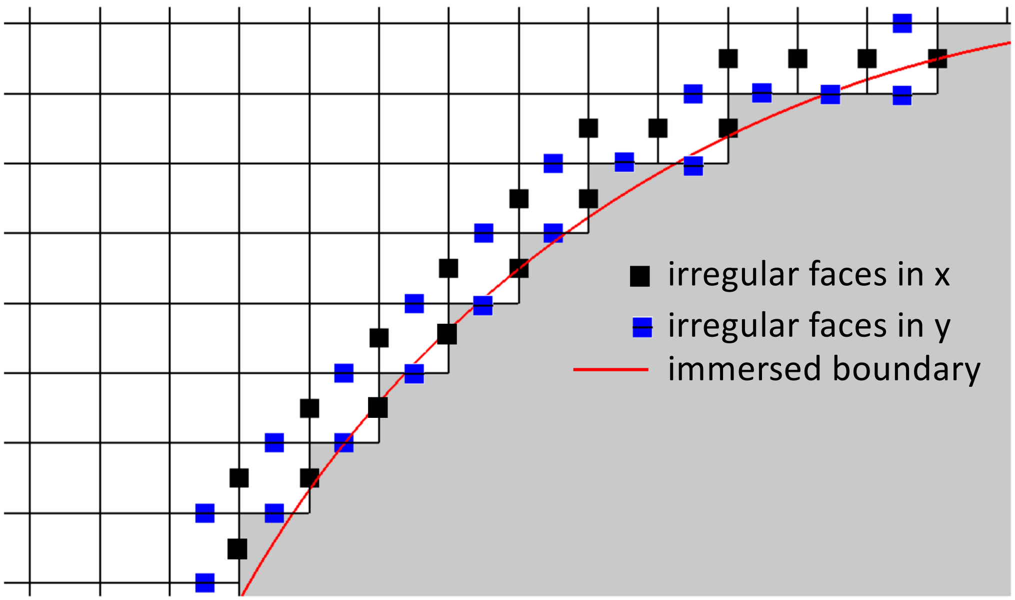
  - Need to compute viscous fluxes at faces
  - Treat viscous fluxes consistently at irregular faces
  - Truncation error for all derivatives need to match

$$\left.\frac{\partial\phi}{\partial x}\right|_f=c_{\partial\Omega}\phi_{\partial\Omega}+\sum_{m=1}^{p_k+1}c_{i+m-1}\phi_{i+m-1}+C\Delta x^{2n}$$

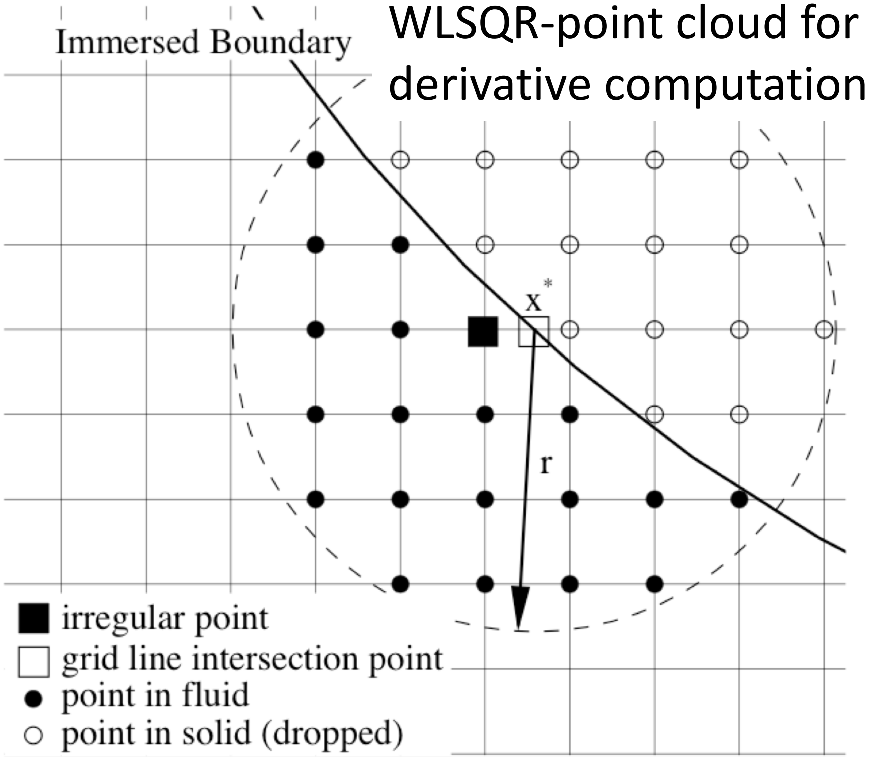


# Conservative Finite-Difference Method – Viscous Terms

Irregular faces in vicinity of IB




WLSQR-point cloud for derivative computation




- ❑Viscous flux are computed at irregular faces and grid line intersection points (for wall-resolved)
- ❑Using weighted-least squares stencils to compute derivatives at irregular faces

# Finite-Difference IB Method – Turbulence Model

- Solve standard SA transport model

$$\nu_t = \tilde{\nu} f_{v1}, \quad f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3}$$

$$\frac{\partial}{\partial t}(\tilde{\nu}) + \frac{\partial}{\partial x_j}(u_j \tilde{\nu}) = \boxed{\mathcal{D}_i} + \boxed{\mathcal{P}_r} - \boxed{\mathcal{D}_e}$$

$$f(\chi) = \begin{cases} 1 & \text{for } \chi > 0, \text{ and} \\ 1 + \chi + \frac{\chi^2}{2} & \text{otherwise.} \end{cases}$$

Diffusion term

$$\mathcal{D}_i = \frac{1}{\sigma}\left[(1 + c_{b2})\left(\frac{\partial \tilde{\nu}}{\partial x_j}\right)^2 + (\nu + \tilde{\nu} f(\chi))\frac{\partial^2 \tilde{\nu}}{\partial x_j^2}\right]$$

Dissipation term

$$\mathcal{D}_e = \begin{cases} c_{w1} f_w \left(\frac{\tilde{\nu}}{d}\right)^2 & \text{for } \chi \geq 0, \text{ and} \\ -c_{w1}\left(\frac{\tilde{\nu}}{d}\right)^2 & \text{otherwise.} \end{cases}$$

Production term

$$\mathcal{P}_r = \begin{cases} c_{b1} \hat{S} \tilde{\nu} & \text{for } \chi > 0, \text{ and} \\ c_{b1} \Omega g_n \tilde{\nu} & \text{otherwise,} \end{cases}$$

$$\hat{S} = \begin{cases} \Omega + \bar{S} & \text{for } \bar{S} > -c_{v2}\omega, \text{ and} \\ \Omega + \frac{\Omega(c_{v2}^2 + c_{v3}\bar{S})}{(\Omega(c_{v3} - 2c_{v2}) - \bar{S})} & \text{otherwise,} \end{cases}$$

- Immersed boundary operators are the same as for Navier-Stokes equations
  - using new conservative formulation

# Outline

Motivation/Introduction

Current state and challenges for IBM.

Immersed Boundary Method (IBM)

*Introducing conservative FD IB scheme.*

Viscous Wall Model (VWM)

*Discussion of different viscous wall modeling approaches.*

IBM & VWM Coupling

*Introduces basic idea of immersed boundary method.*

Validation Study

*Validation of newly developed method.*

Final Discussion and Conclusion

What is the current state and what is next? Additional Challenges.

# Wall Modeling Approaches – Near-wall analytical solution

- Boundary layer simplification for x-momentum equation, *i.e.*, simple diffusion equation and mixing length assumption for turbulent viscosity

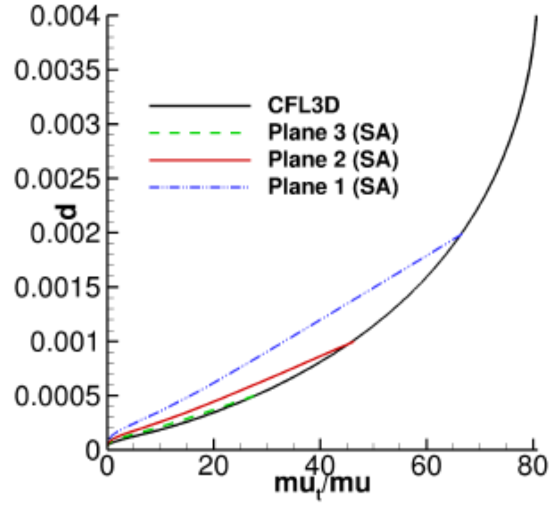
$$\frac{d}{dy}\left[(\nu+\nu_t)\frac{du}{dy}\right]=0,\ \text{or}\ (\nu+\nu_t)\frac{du}{dy}=u_\tau^2 \qquad \tilde{\nu}=\kappa u_\tau y=\kappa\nu y^+$$

- Integrating and manipulating terms gives the following analytical expression

$$u^+=\bar{B}+c_1\log((y^++a_1)^2+b_1^2)-c_2\log((y^++a_2)^2+b_2^2)-c_3\arctan(y^++a_1,b_1)-c_4\arctan(y^++a_2,b_2)$$

- Analytical SA wall function constants

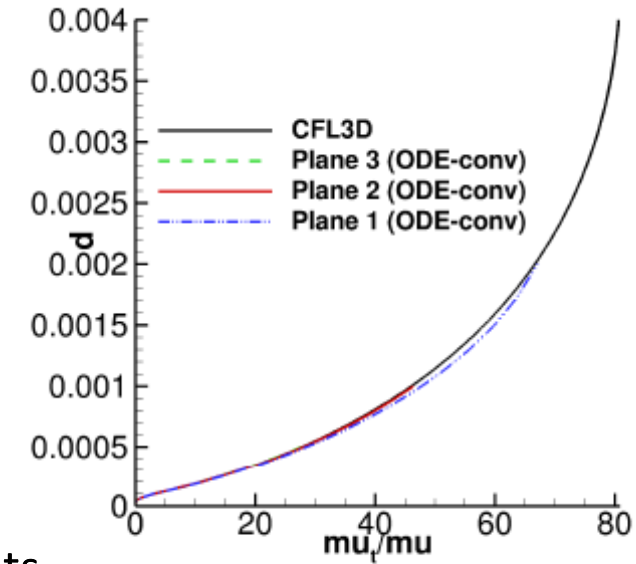
$$\bar{B}=5.03339088,\quad a_1=8.14822158,\quad a_2=-6.92870938,$$
$$b_1=7.46008761,\quad b_2=7.46814579,\quad c_1=2.54967735,$$
$$c_2=1.33016516,\quad c_3=3.59945911,\quad c_4=3.63975319,$$



# Wall Modeling Approaches – ODE-based wall model

- We include streamwise pressure gradient and convective terms to capture greater physics

$$\underbrace{\frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right)}_{\textit{Diffusion only models}} = \frac{\partial p}{\partial x} + \psi(y)\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]_F$$




- Empirical cutoff function for convective terms along the ray, using the velocity derived from analytical wall function, previously described $\psi(y) = u_{SA}/u_F$

- Solve a simplified SA model - neglecting streamwise gradients

$$\frac{\partial}{\partial y}\left[(\nu+\tilde{\nu})\frac{\partial \tilde{\nu}}{\partial y}\right] = \frac{1}{\sigma}\left[(1+c_{b2})\left(\frac{\partial \tilde{\nu}}{\partial y}\right)^2 + (\nu + \tilde{\nu} f(\chi))\frac{\partial^2 \tilde{\nu}}{\partial y^2}\right]$$

- Second-order central FD scheme, inverted via Newton iteration (with exact linearization)

Berger and Aftosmis (2017)

# Outline

## Motivation/Introduction
Current state and challenges for IBM.

## Immersed Boundary Method (IBM)
*Introducing conservative FD IB scheme.*

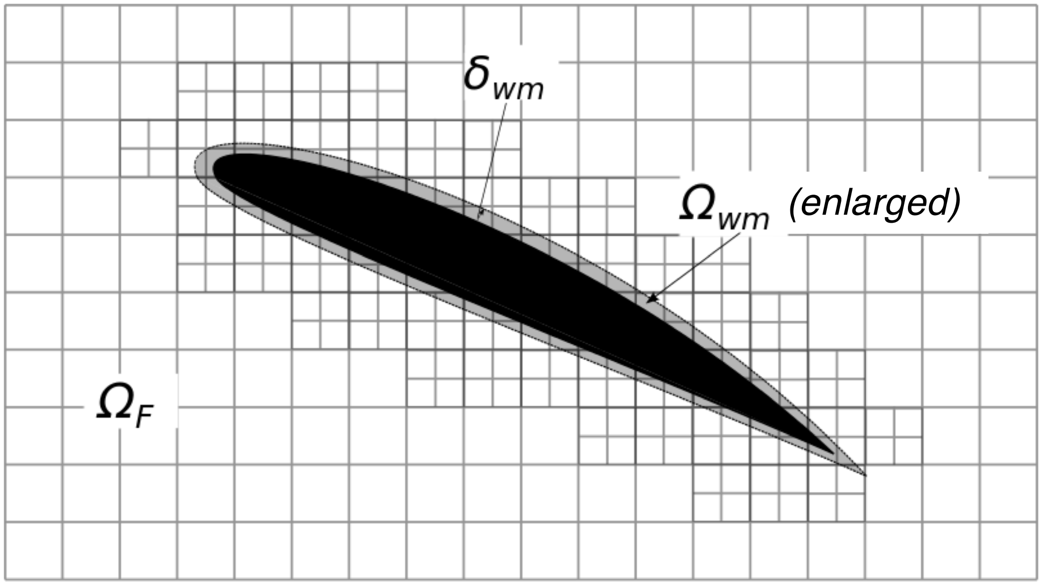
## Viscous Wall Model (VWM)
*Discussion of different viscous wall modeling approaches.*

## IBM & VWM Coupling
*Introduces basic idea of immersed boundary method.*

## Validation Study
*Validation of newly developed method.*

## Final Discussion and Conclusion
What is the current state and what is next? Additional Challenges.

# IBM-VWM Coupling




*$\Omega_{wm}$: approximate boundary layer solution domain*
*$\Omega_F$: Cartesian grid solution domain*

- ❑ Within $\Omega_{wm}$ domain, wall model is assumed to provide valid flow information
- ❑ Important to provide smooth transition between $\Omega_{wm}$ and $\Omega_F$
- ❑ Strong interplay between numerical implementation details of IBM and wall model is expected
- ❑ Grid resolution at boundary layer edge is crucial (see also Spalart 2015 on turbulence modeling for body-fitted mesh)

❑ Simplistic way of viewing the viscous wall effects -- *flow partially slips past wall*:
   (1) Supply the “right” force to the under-resolved flow to control BL growth
   (2) Obtain friction force on immersed geometry → requires some type of wall model

# IBM-VWM Coupling – Approximate BL Solution Domain

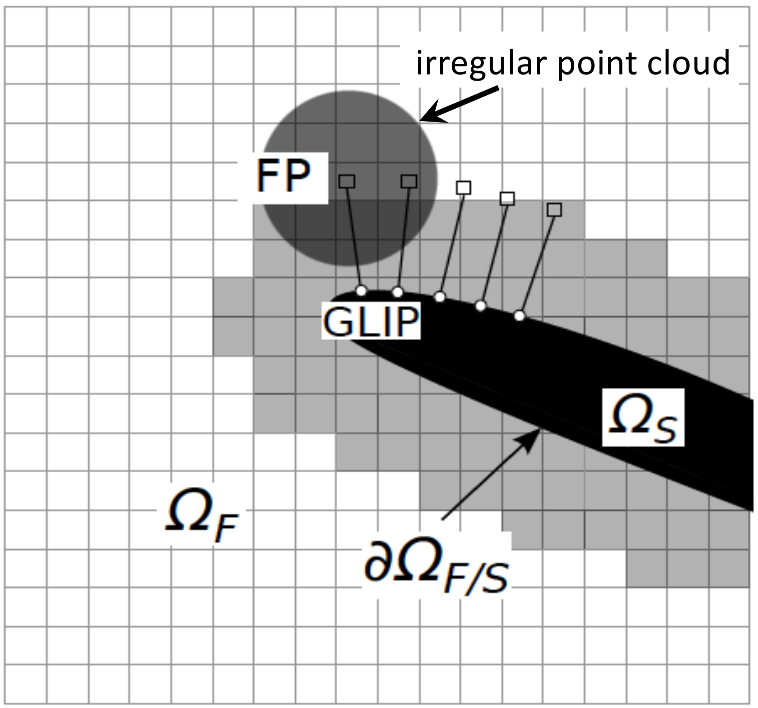
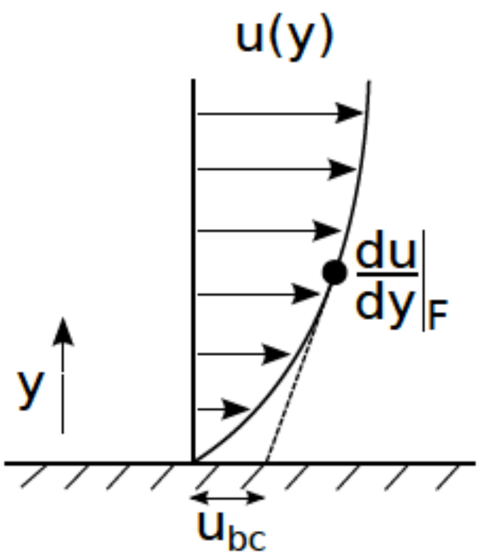



*Illustration of coupling between wall modeling approaches and IB solver*

*FP: forcing point*
*GLIP: gridline intersection point*

- ❑ WM solved along rays with constant $\delta_{wm}$
  - Directional/wall-normal distances vary non-smoothly causing oscillations
  - WMs display dependencies on wall distances where external flow data is provided
- ❑ WM is solved as BVP (BCs at $y_n$=0 & $y_n$=$\delta_{wm}$)
- ❑ Transfer of data between $\Omega_{wm}$ and $\Omega_f$:
  - (1) flow data is interpolated to FP
  - (2) WM solution is transfer to IB solver at GLIP
- ❑ WLSQR stencil cloud provides flow data at $y_n$=$\delta_{wm}$
  - data needed depends on wall model
  - with this data WM solution can be obtained

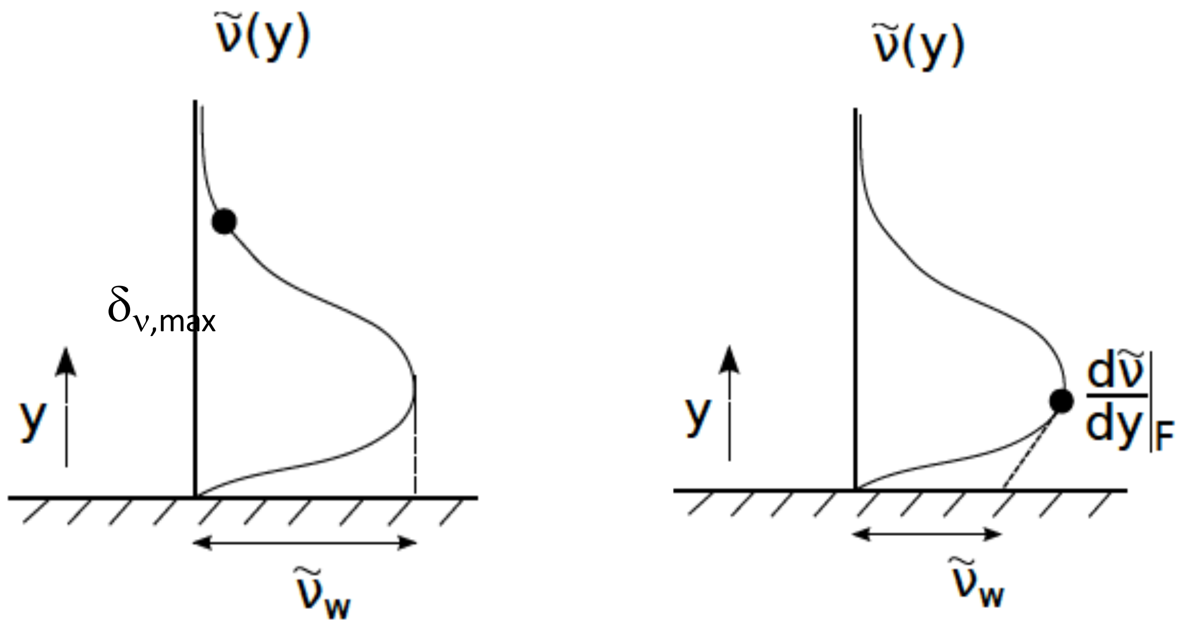
# IBM-VWM Coupling – Cartesian Grid Solution Domain




- ❑ WM solution data needs to be fed back to Cartesian grid solver
- ❑ Boundary conditions for Cartesian solver are provided at GLIPs
- ❑ Linearly extrapolate velocity to the wall and use as slip-wall velocity ($u_{bc}$)
  - Dissipation from convective flux will be introduced depending on numerical flux and discretization scheme
- ❑ Viscous flux at GLIP is obtained from $\tau_w$ which is obtained from WM
  - Rotate viscous flux from wall-aligned coordinate system into Cartesian coordinate system
  - assuming that all diagonal components of stress tensor are small
  - Other irregular fluxes use slip-velocity to compute velocity derivatives
- ❑ Pressure is assumed to stay constant through boundary layer

# IBM-VWM Coupling – Cartesian Grid Solution Domain

How to obtain eddy viscosity at GLIP?




- ❑ Eddy viscosity is obtained in similar way as slip-velocity
  - Linearly extrapolate eddy viscosity if $y_n < \delta_{\nu,max}$
  - Use $\nu_{max}$ if $y_n > \delta_{\nu,max}$

# Outline

Motivation/Introduction

Current state and challenges for IBM.

Immersed Boundary Method (IBM)

*Introducing conservative FD IB scheme.*

Viscous Wall Model (VWM)

*Discussion of different viscous wall modeling approaches.*

IBM & VWM Coupling

*Introduces basic idea of immersed boundary method.*

**Validation Study**

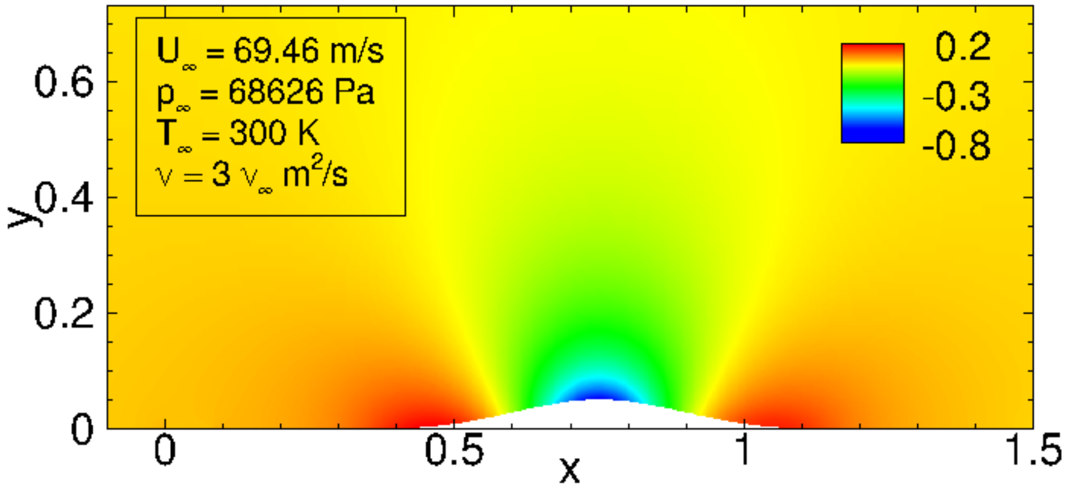
*Validation of newly developed method.*

Final Discussion and Conclusion

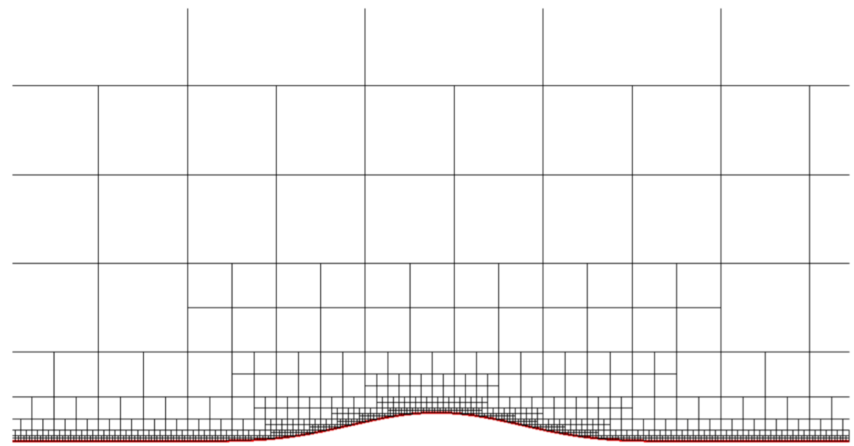
What is the current state and what is next? Additional Challenges.

## Validation Results – Bump in Channel

Bump in Channel ($Re=3\times10^6$)




Block-Structured Cartesian Mesh

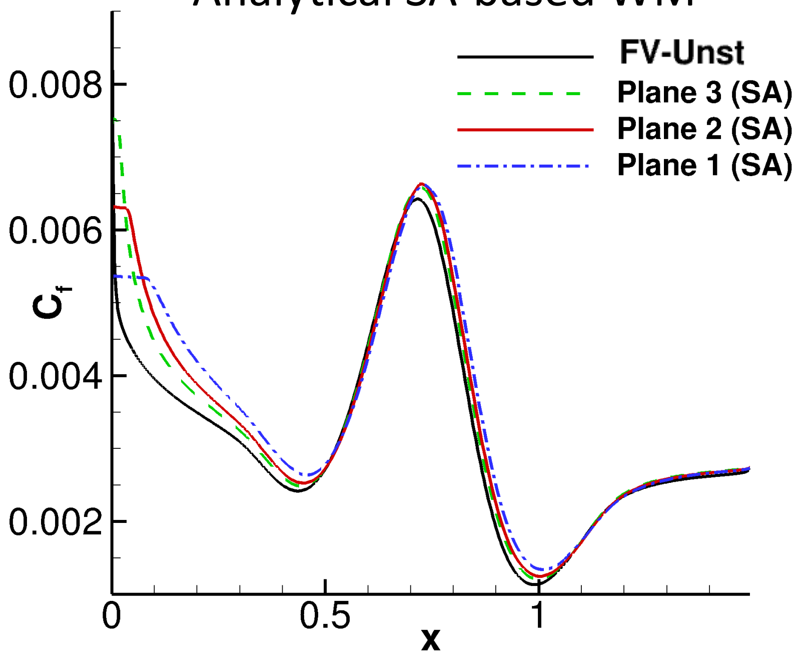


- Assess a flow with non-zero pressure gradients from NASA TMR website*
- Inlet/Outlet , no-slip from x=0 to x=1.5 (symmetry prior)
- Symmetry top-wall

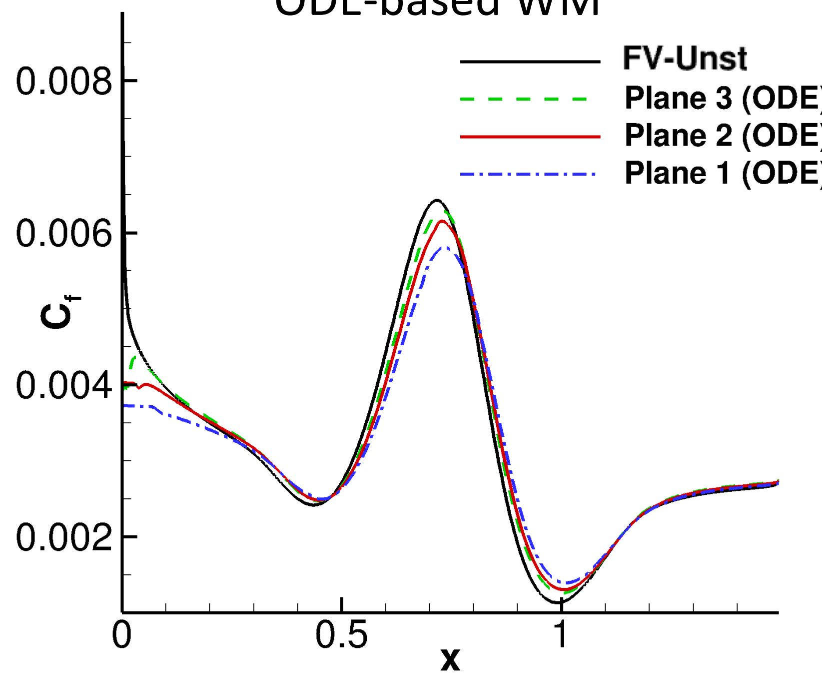
*https://turbmodels.larc.nasa.gov/

# Validation Results - Bump in Channel (Standalone)

Analytical SA-based WM




ODE-based WM

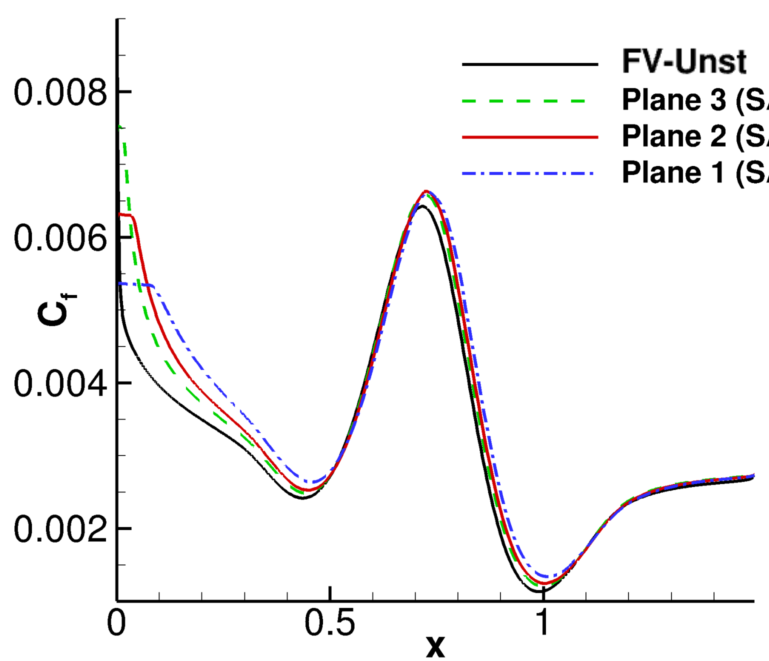



| Plane | $d$ | $y^+_{x=0.75}$ |
|---|---|---|
| 1 | $2 \times 10^{-3}$ | 329.2 |
| 2 | $1 \times 10^{-3}$ | 164.6 |
| 3 | $5 \times 10^{-4}$ | 82.3 |

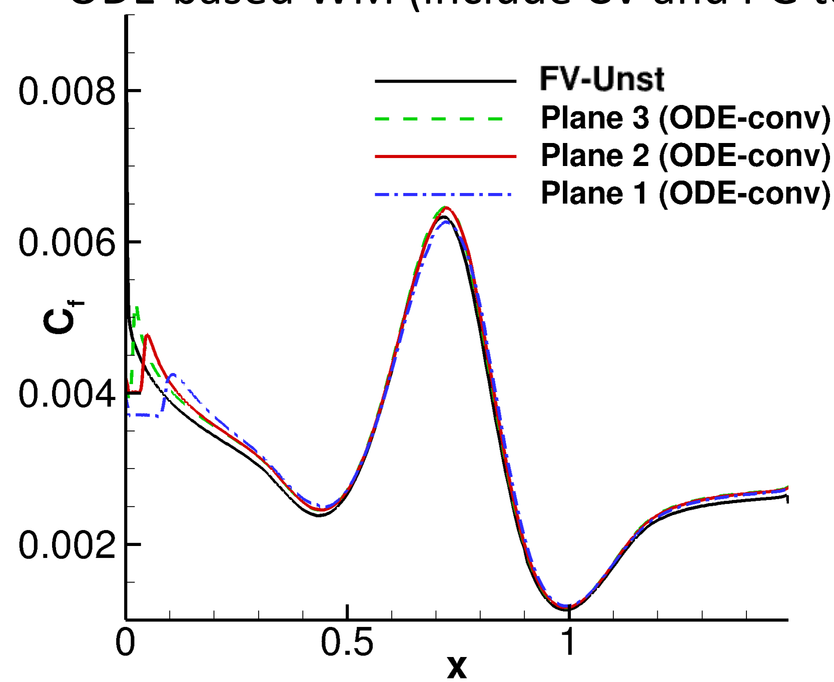
- Reference solution at several constant $y^+$ planes taken as input into standalone wall-model
- Differences in ODE based wall model and analytical SA model
- Less effective $y^+$ sensitivity for ODE model

# Validation Results - Bump in Channel (Standalone)

Analytical SA-based WM




ODE-based WM (include CV and PG terms)

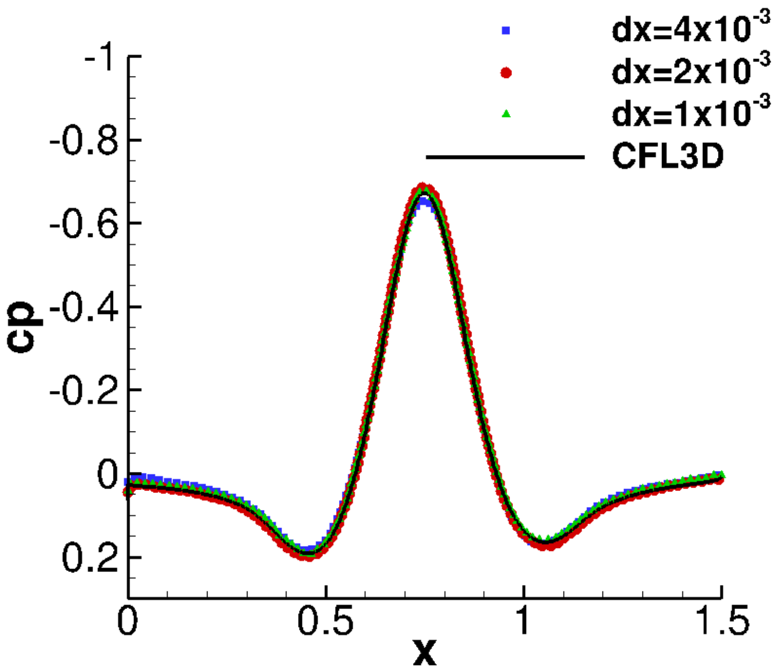



| Plane | $d$ | $y^+_{x=0.75}$ |
|---|---|---|
| 1 | $2 \times 10^{-3}$ | 329.2 |
| 2 | $1 \times 10^{-3}$ | 164.6 |
| 3 | $5 \times 10^{-4}$ | 82.3 |

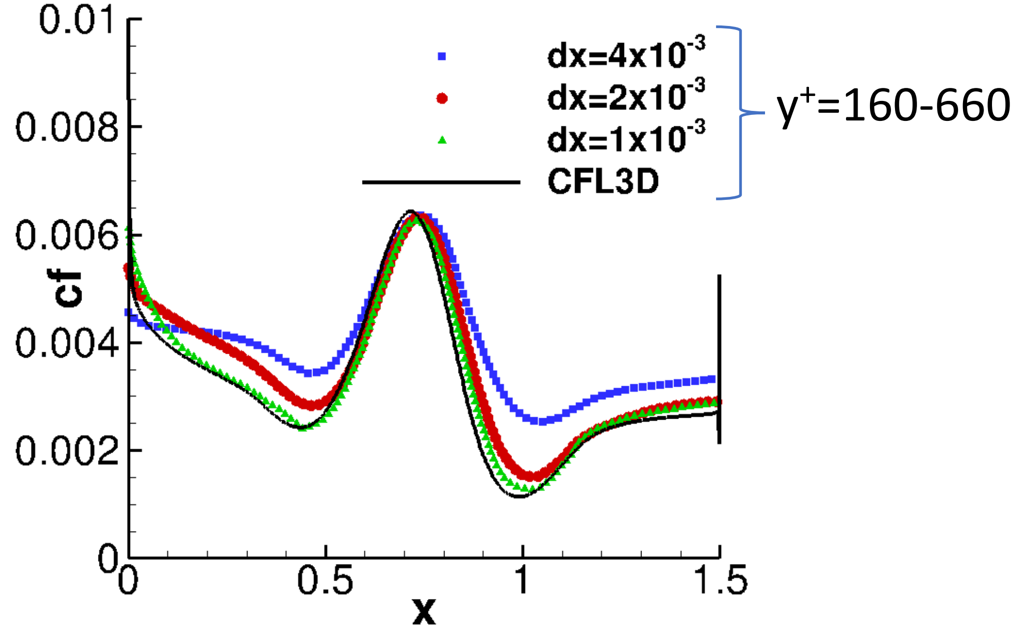
- Including pressure gradient and convective terms improves accuracy and reduces $y^+$ sensitivity

# Validation Results – Bump in Channel (Fully Coupled)

Pressure distribution



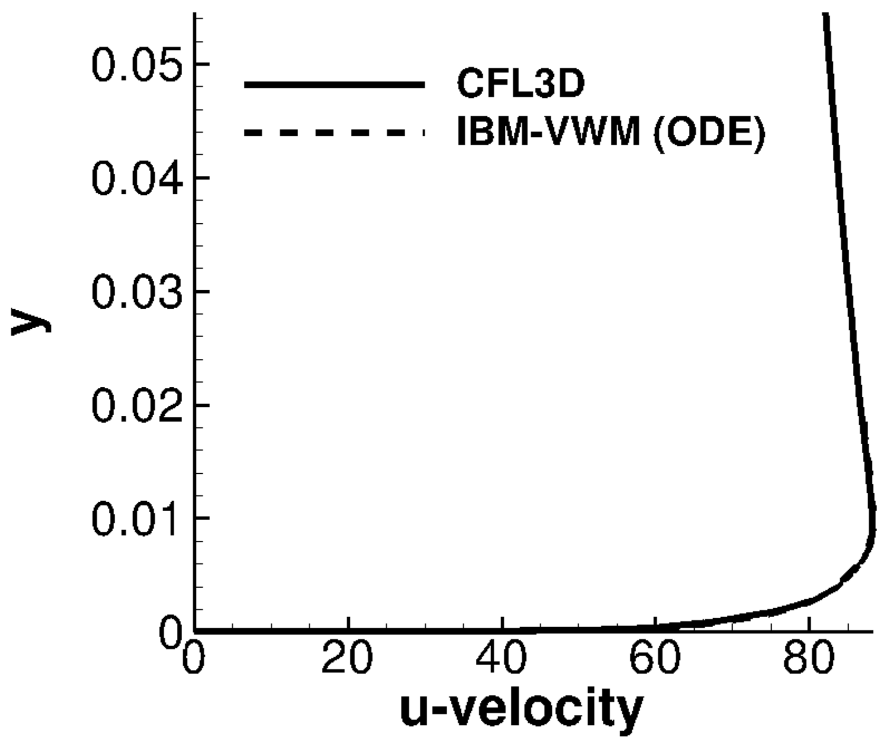

Skin friction curves



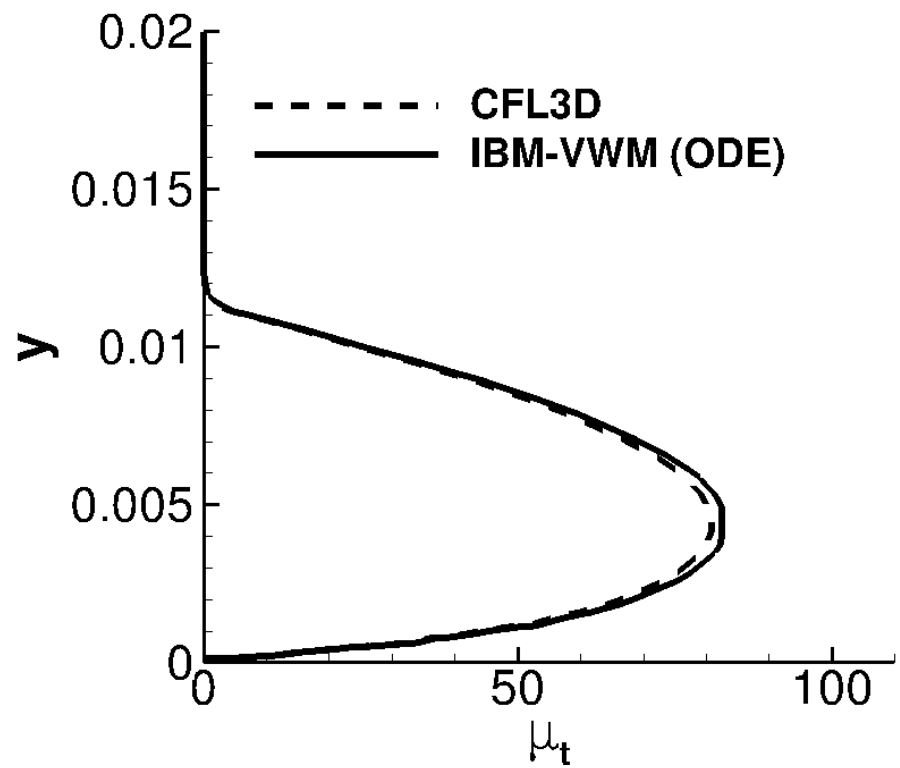

- ❑ Fully-coupled results with ODE wall-model demonstrate very good agreement for $y^+$=160, although higher $y^+$ values remain a challenge

# Validation Results – Bump in Channel

Streamwise Velocity Profile



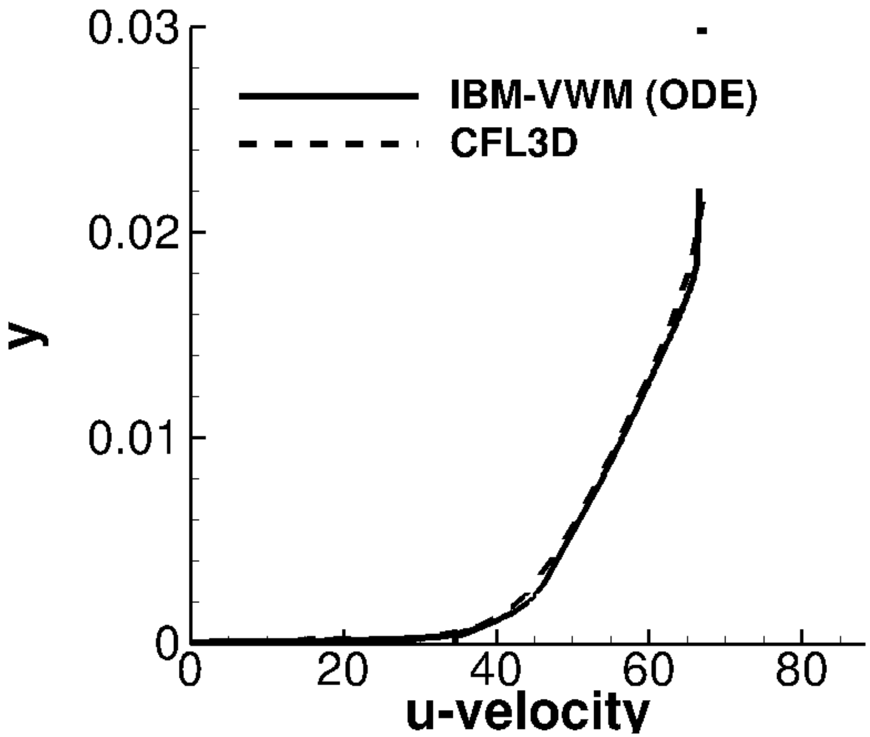

Eddy Viscosity



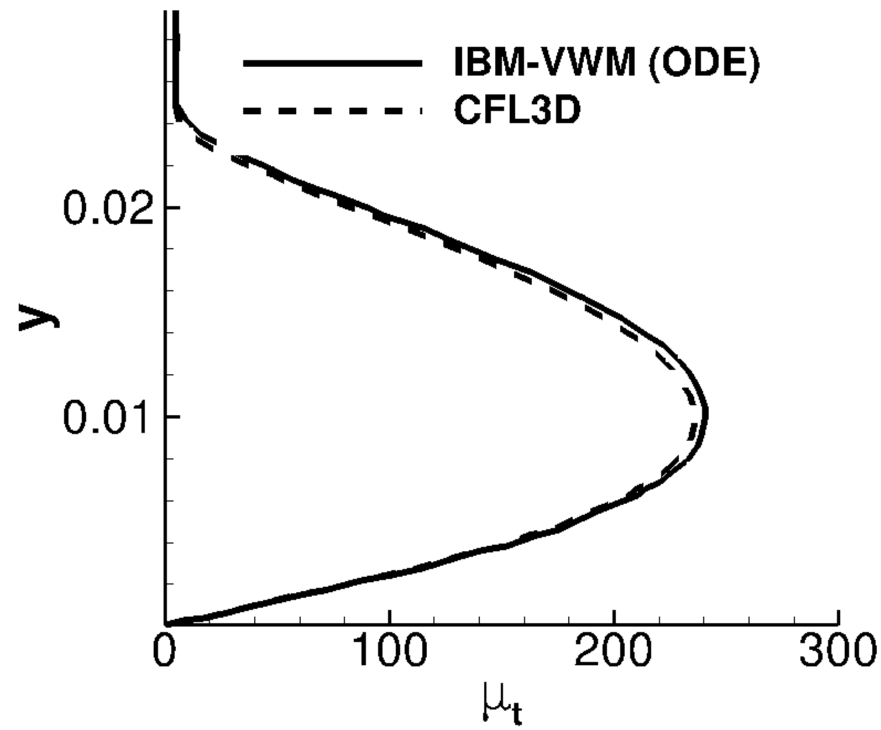

- Good comparison between IBM-VWM and CFL3D at x=0.75

# Validation Results – Bump in Channel

Streamwise Velocity Profile



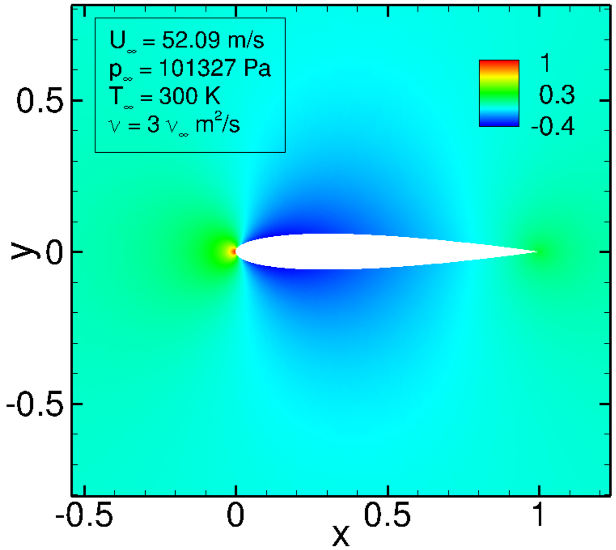

Eddy Viscosity



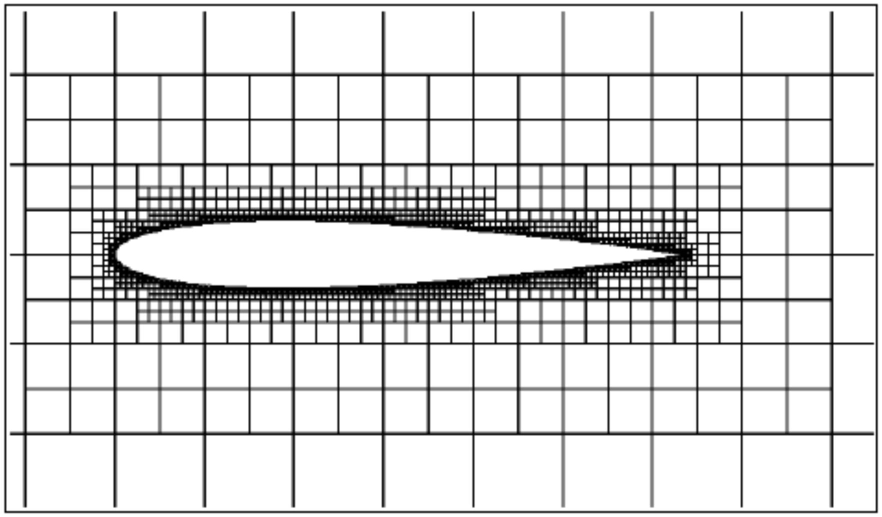

- ❑ Good comparison between IBM-VWM and CFL3D at x=1.2

# Validation Results – NACA0012

NACA0012 (Re=6×10^6)



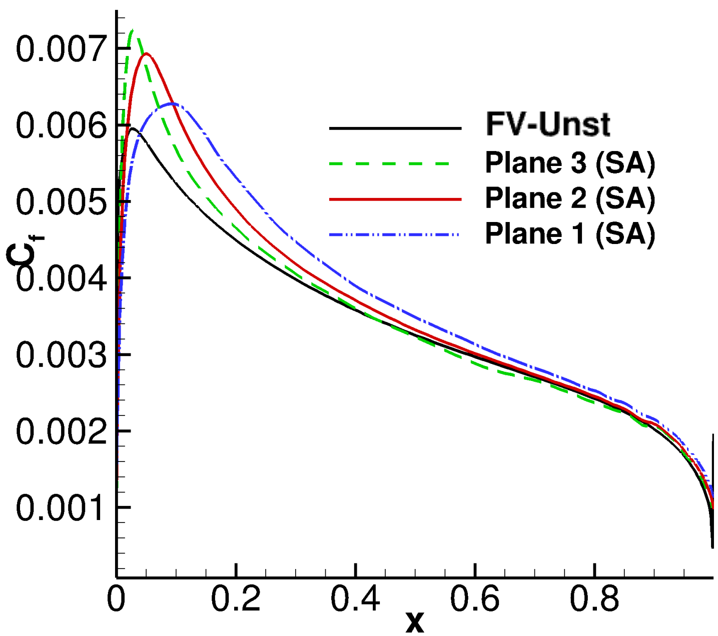

Block-Structured Cartesian Mesh

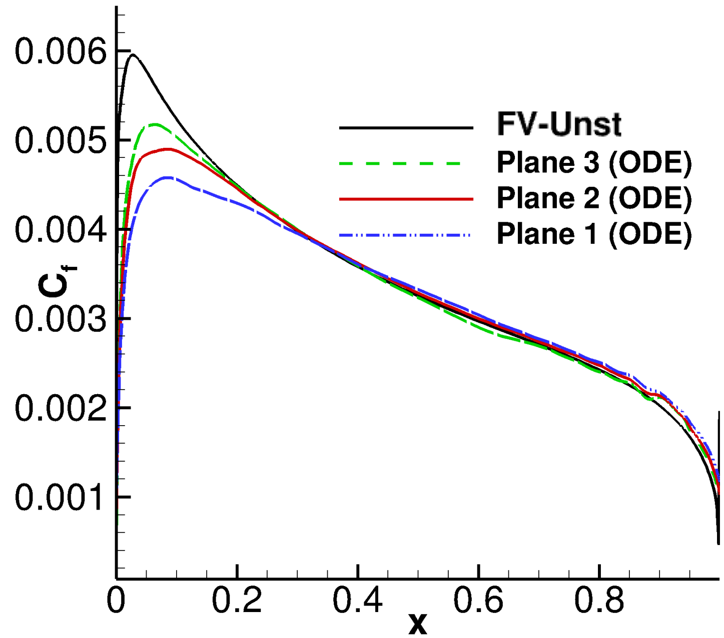


- Assess a higher Reynolds number case with variable AoA from NASA TMR website*
- No-slip on wall, farfield conditions 500×C away
- Stepping-stone for more complex aerospace cases

*https://turbmodels.larc.nasa.gov/

# Validation Results – Wall Model Only



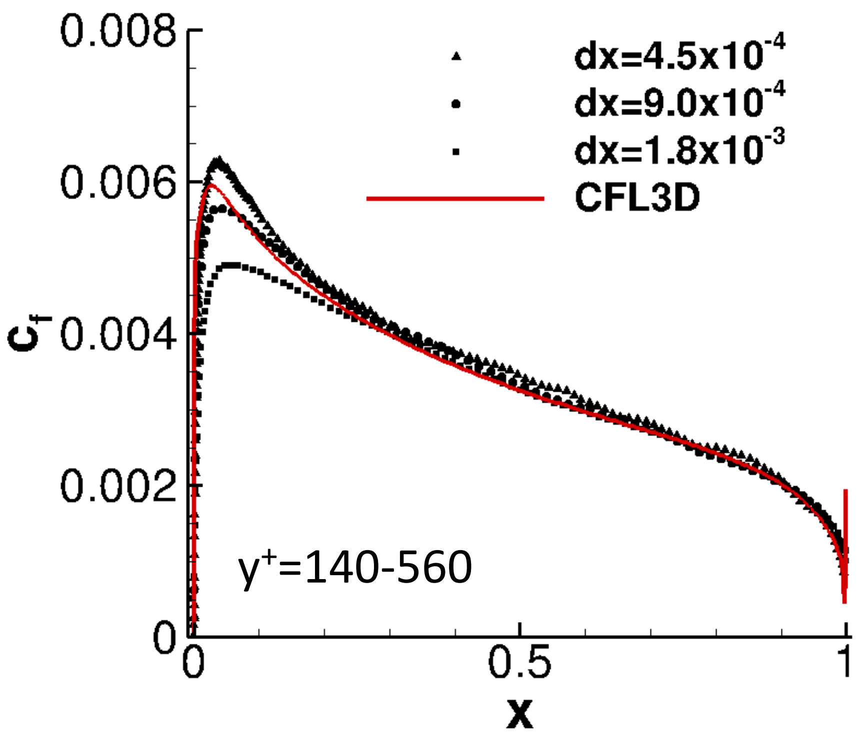




| Plane | $d$ | $y^+_{x=0.3}$ |
|---|---|---|
| 1 | $2 \times 10^{-3}$ | 572 |
| 2 | $1 \times 10^{-3}$ | 286 |
| 3 | $5 \times 10^{-4}$ | 143 |

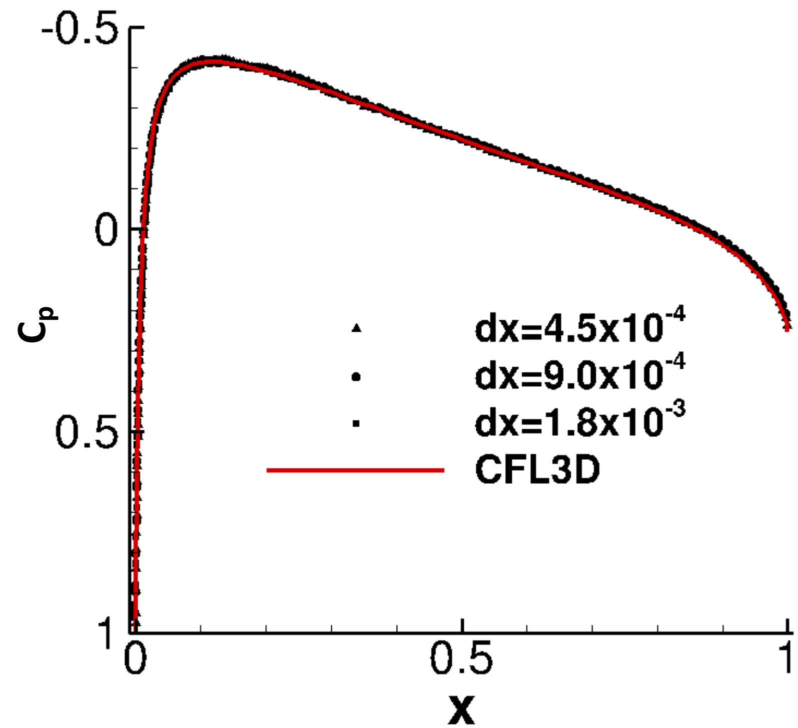
- Reference solution at several constant $y^+$ planes taken as input into standalone wall-model
- SA analytical model over-predicts skin-friction but ODE more consistent and trending towards correct profile

# Validation Results – NACA0012 – Fully Coupled

Skin friction curves

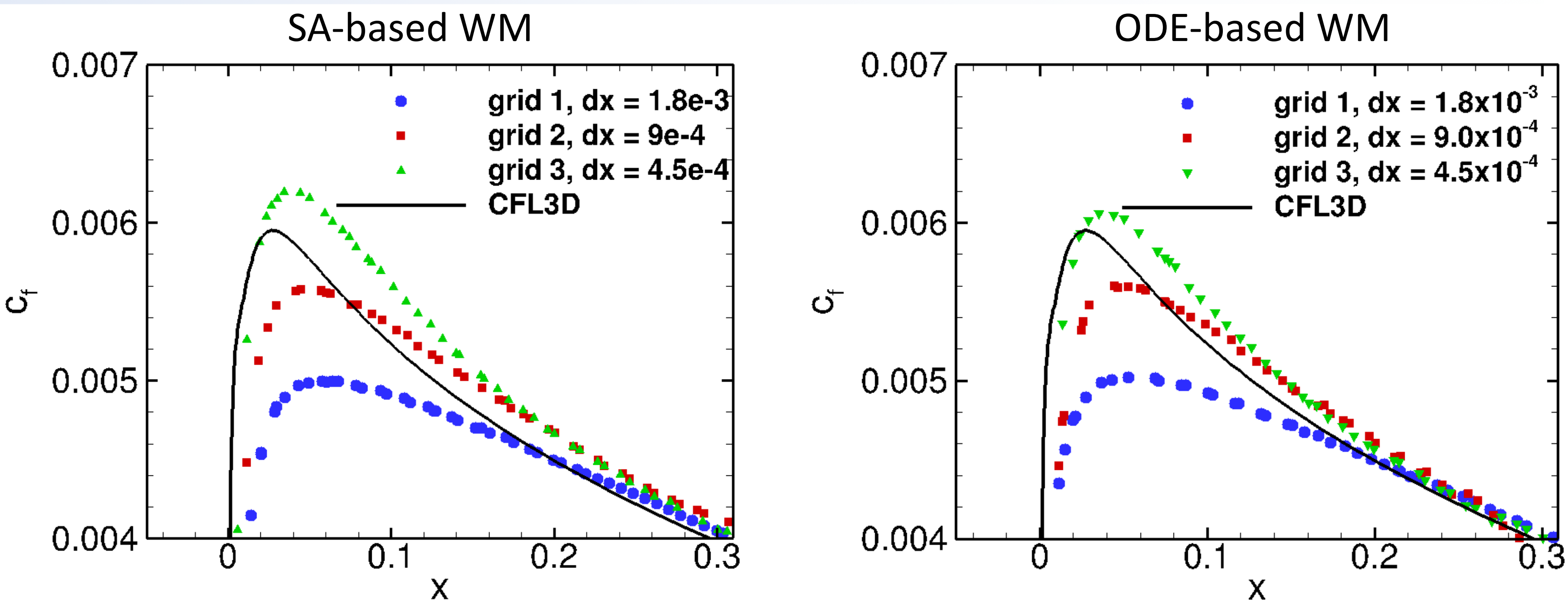



Pressure distribution



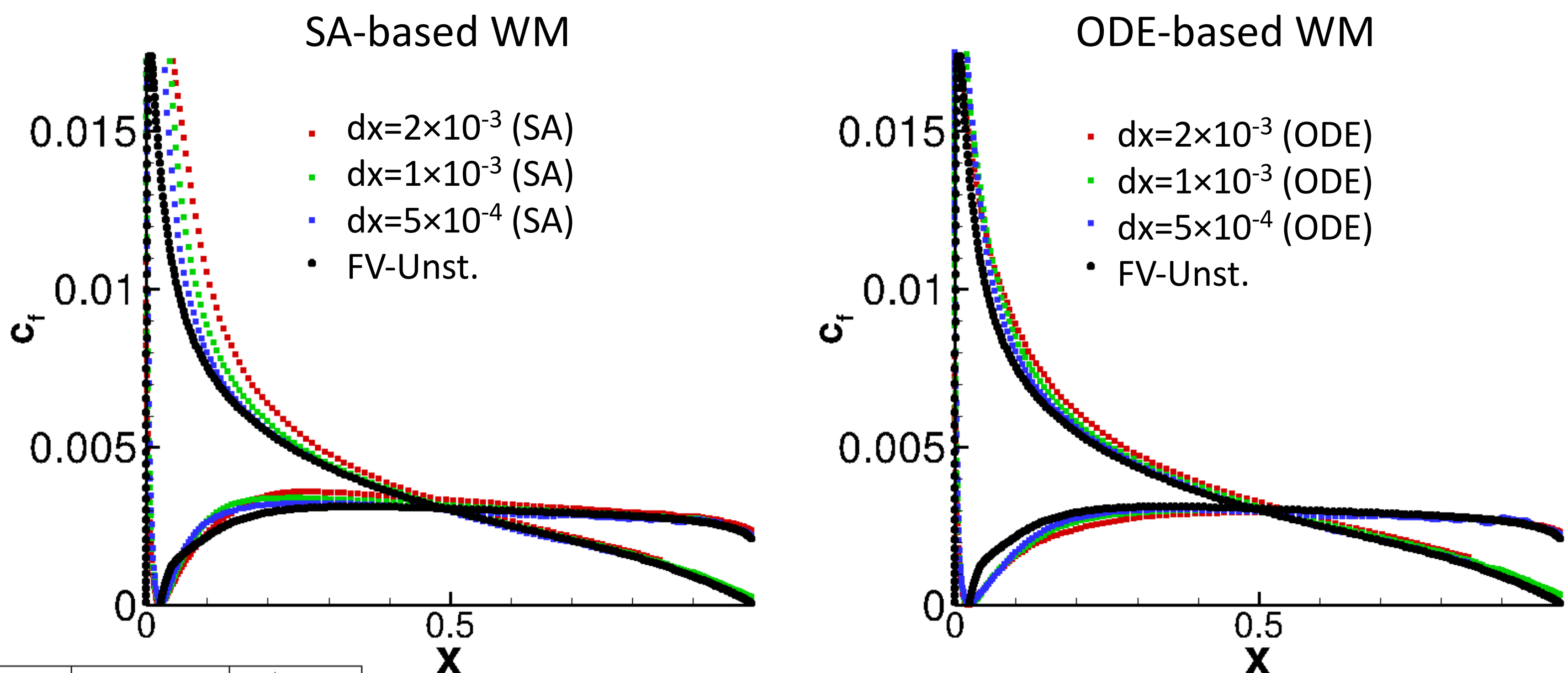

- ❑ Excellent agreement for $C_p$ , more challenging for $C_f$ but agreement is promising

# Validation Results




- ❑ $c_f$ in the back is well captured on coarsest mesh
- ❑ $c_f$ peak increases progressively with increasing grid resolution
- ❑ ODE-based wall model seems to align more closely (in agreement with standalone results)

# NACA0012 at 10 AoA

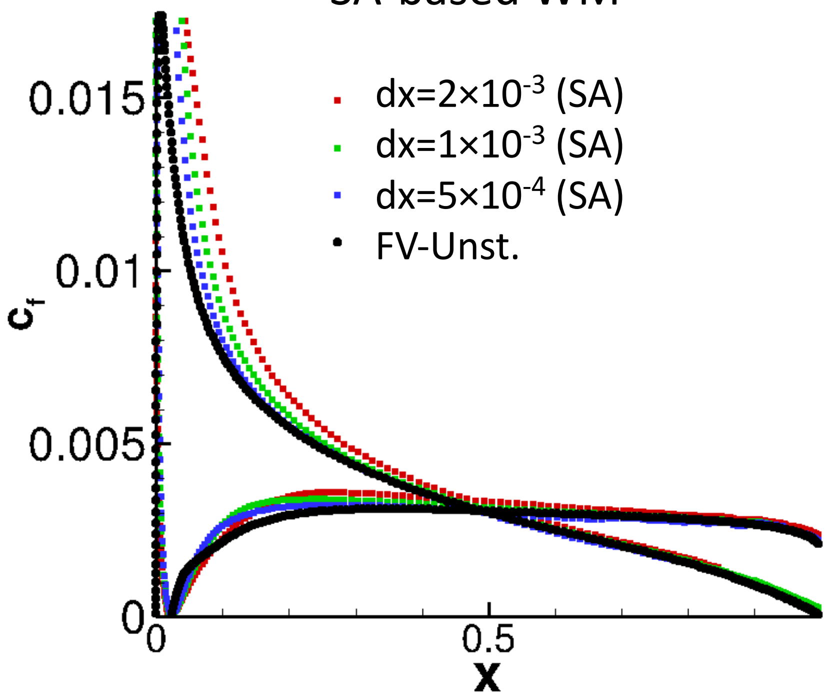



| Plane | $d$ | $y^+_{x=0.3}$ |
|---|---|---|
| 1 | $2 \times 10^{-3}$ | 572 |
| 2 | $1 \times 10^{-3}$ | 286 |
| 3 | $5 \times 10^{-4}$ | 143 |

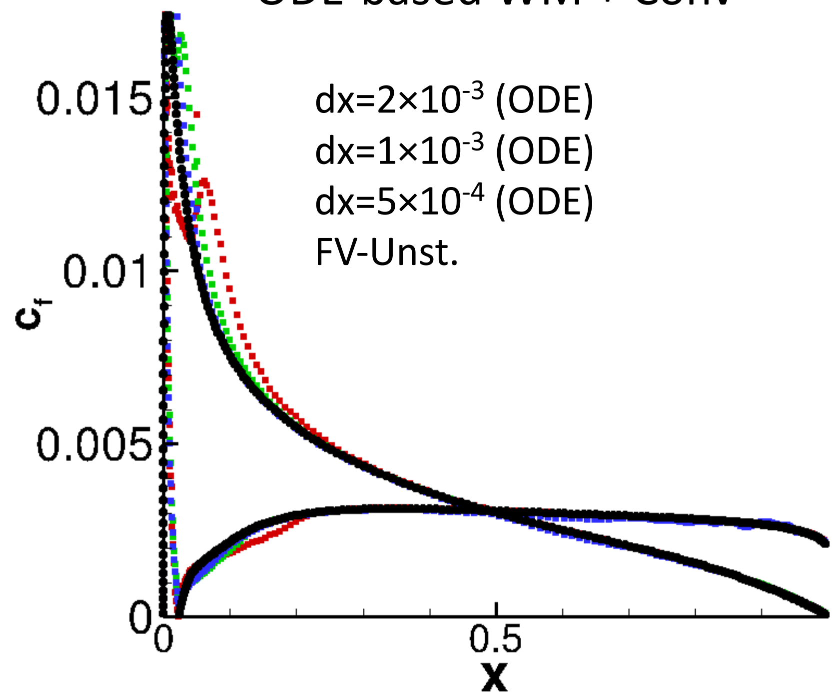
- Run at 10 AoA to obtain greater pressure gradient effects
- Clear improvement for ODE model – less sensitivity to the effective $y^+$ location

# NACA0012 at 10 AoA

SA-based WM



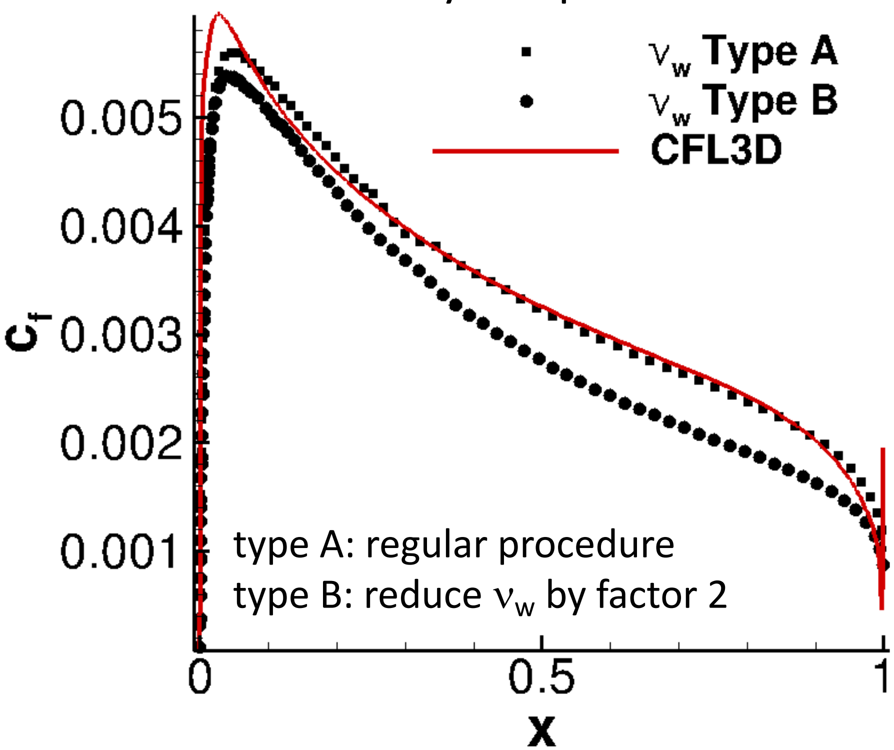

ODE-based WM + Conv



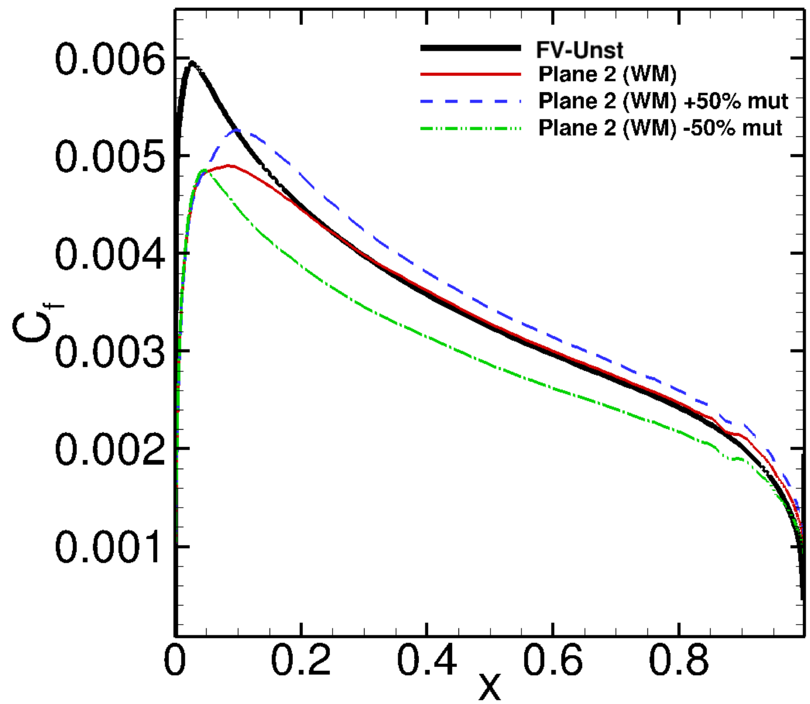

- ❑ Including convective and pressure gradients brings the agreement much closer - expectation to see even stronger dependency for more complex flows with strong pressure gradient and convective terms

# Sensitivity to Incorrect Input Data

Fully-Coupled



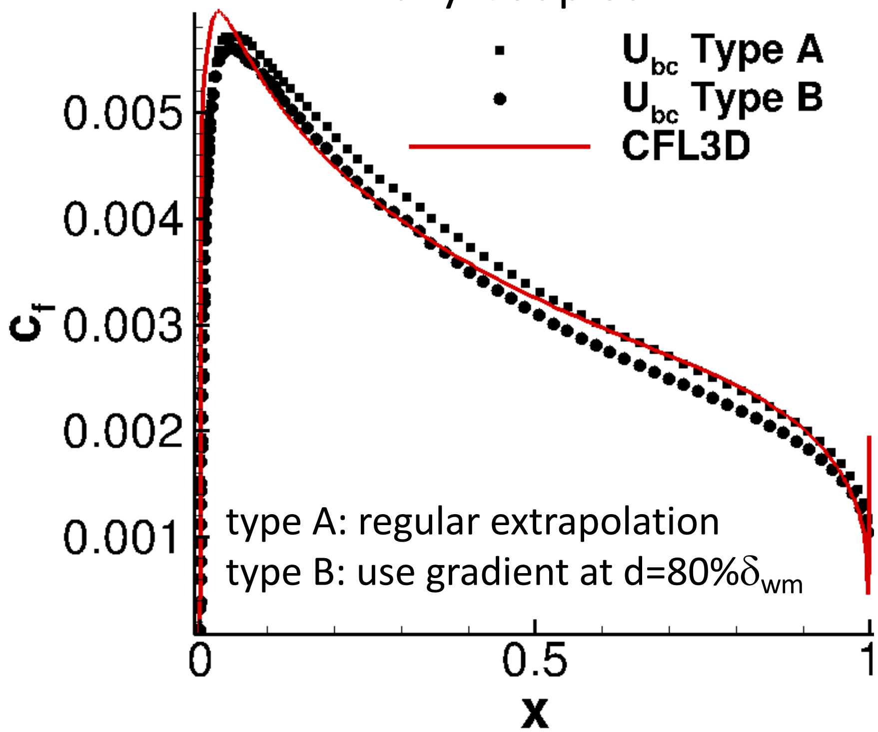

Standalone



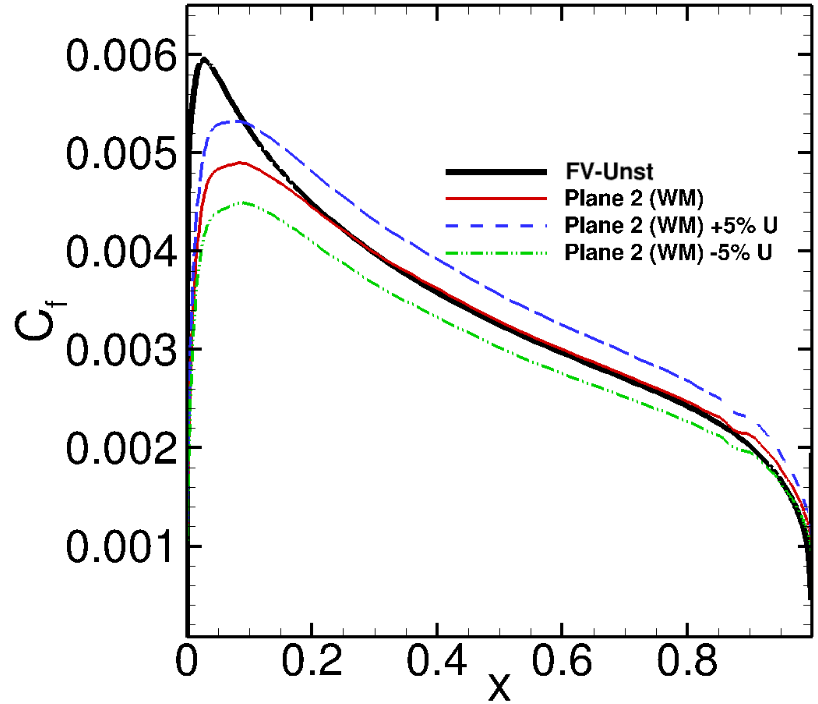

- Effect of an error in the SA field variable $\nu$ at forcing location
- Significant effect of $\nu_w$ on skin friction data ($\nu_w/2$ reduces $c_f$)
- standalone simulation reproduces this effect on skin-friction

# Coupling Effects

Fully-Coupled




Standalone

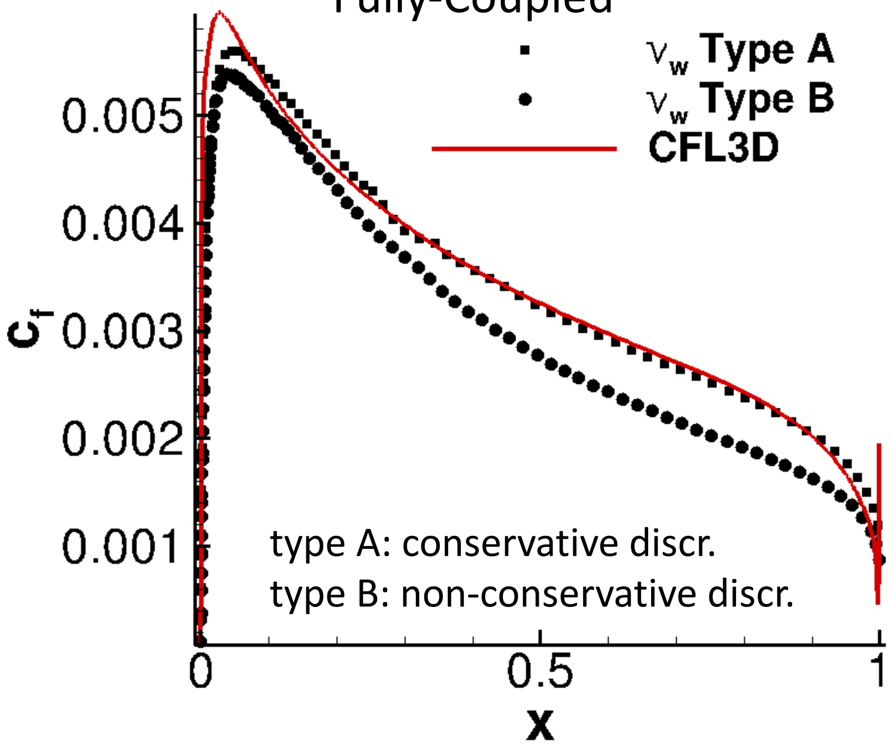



- Sensitivity of $c_f$ w.r.t. an error in the velocity
- type B boundary condition reduces slip velocity and, thus, skin friction coefficient
- standalone simulation reproduces this effect on skin-friction

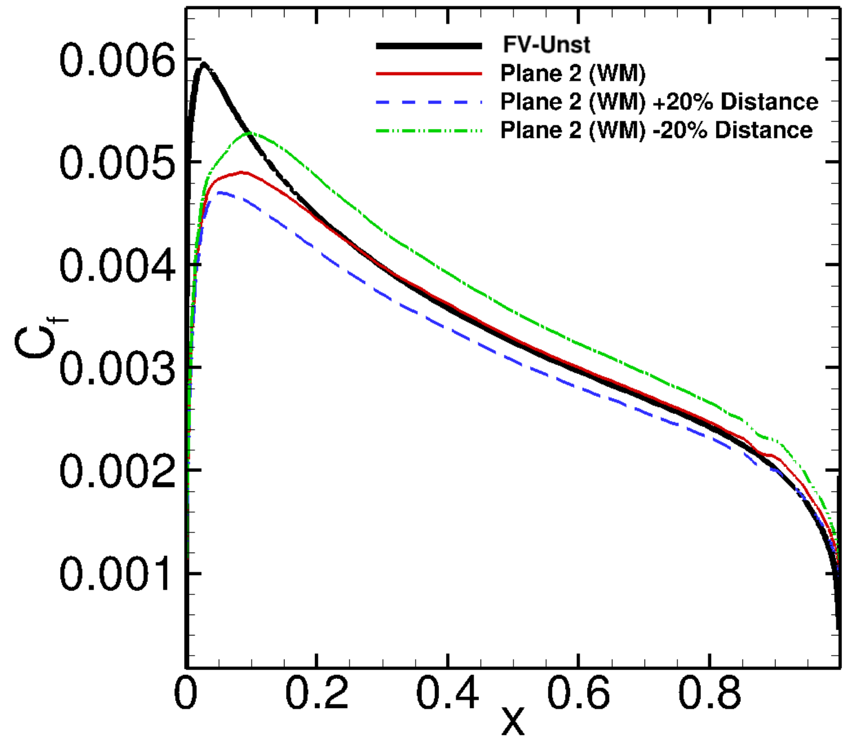
Note: Uses slightly different viscous discretization scheme.

# Coupling Effects

Fully-Coupled




Standalone




- Shows effect of different viscous flux implementations
- Problem with viscous discretization type B is that skin friction at the wall is enforced indirectly
  - Utilizing shear stress from wall model following constant shear assumption
- Change of wall distance by 20% of its true value has significant effect on skin-friction

# Outline

## Motivation/Introduction
Current state and challenges for IBM.

## Immersed Boundary Method (IBM)
*Introducing conservative FD IB scheme.*

## Viscous Wall Model (VWM)
*Discussion of different viscous wall modeling approaches.*

## IBM & VWM Coupling
*Introduces basic idea of immersed boundary method.*

## Validation Study
*Validation of newly developed method.*

## Final Discussion and Conclusion
What is the current state and what is next? Additional Challenges.

# Summary and Conclusions

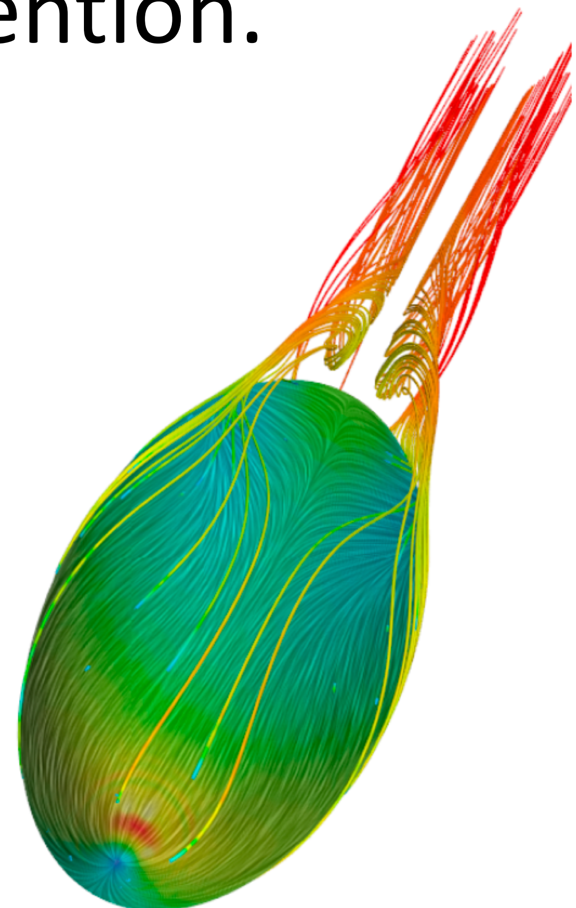
- Clear sensitivity of IBM-VWM results to WM formulation (even for attached flows)
  - Inclusion of pressure term
  - Improvement of modeling eddy viscosity
- Coupling influences can be significant
  - High sensitivity to errors in input data – still not fully understood at this point
  - depend on details of the underlying numerical schemes
- Conservative viscous discretization scheme significantly improves IBM-VWM results
  - this is essentially a new IBM
- Good agreement for test cases up to around $y^+ \approx 200$ (significant step up from before)
- High-lift common research model (wing area=197m$^2$, $\Delta x_{y+=1}$=$2\times10^{-5}$m)
  - Body-fitted mesh 80−240×$10^6$ with (180×$10^6$ in prism layers) Ashton et al. (AIAA 2018)
  - Rough estimate for $y^+$=200 gives 120-200×$10^6$ (where $y^+$=100 gives 4×, or y+=1 $\mathcal{O}(10^{12})$)
  - ➢ IBM does not require grid generation and reduces operation count

# Thank you for your attention.

# Any questions?

Thanks to LAVA team members of NASA Ames Research Center, in particular C. Kiris, M. Barad, and J. Housman; interaction on development of Cartesian Grid Methods has been invaluable

C. Brehm and N. Ashton, “Progress in the development of an immersed boundary method with viscous-wall model for 3D flows”, ICCFD-10, 9-13 July, 2018



3D Prolate Spheroid