Extension of d'Alembert's Paradox for Elongated Bodies

P. Spalart
Boeing Commercial Airplanes

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D’Alembert’s Ancestral Paradox

- “The inviscid equations predict a 3D body has no drag!”
  - Sacrebleu… (old French for “OMG!”)
  - It was 1752, and d’Alembert worked with Euler
  - Initially, he showed it only with fore-and-aft symmetry (football versus egg-plant)
- It turns out the body also has no lift!
  - Textbooks show the full result, but don’t harp on it
  - They rush to the 2D case, which can have lift

New Theorems for Elongated Bodies

- The body has an arbitrary cross-section, of diameter d, and length l
- The nose and tail shapes are unspecified
  - They merely need to preserve attached flow

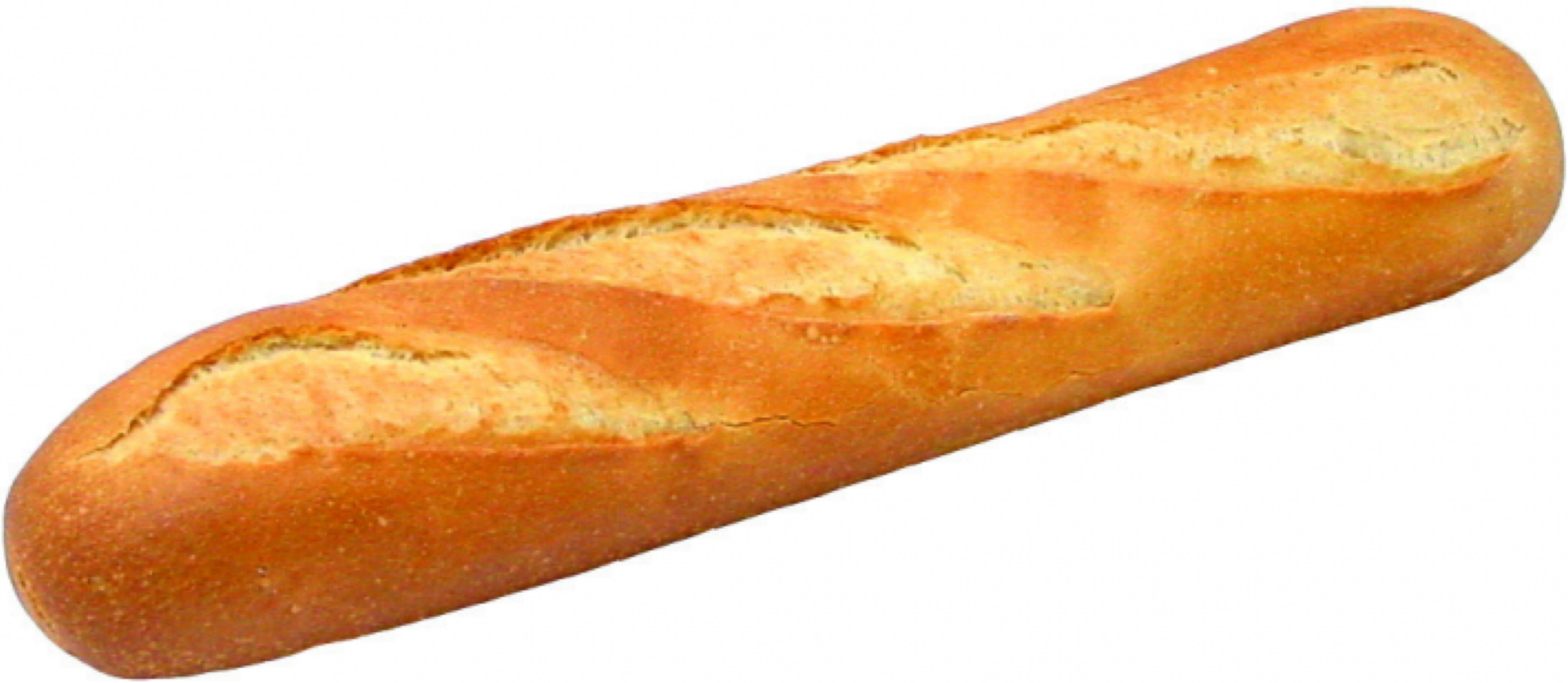
Not all our Flows are Attached…

European Train Cab Shapes

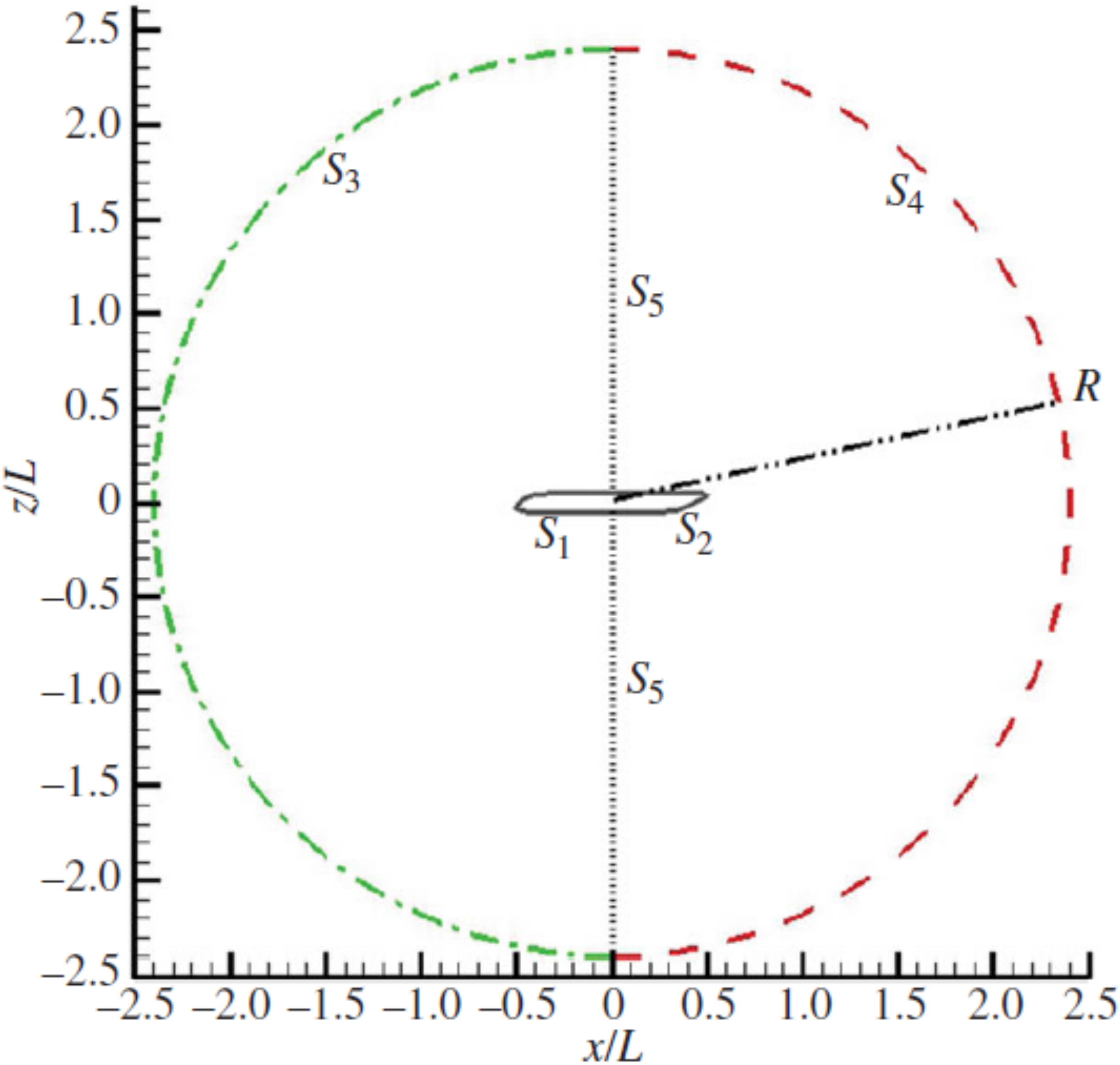
Shinkansen Cab Shapes

Theorem does not apply for lift, because of the ground plane, but applies for side-force in cross-wind

The REAL Body of Interest



Control Volumes



Force (on Nose) and Moment Coefficients

- Coefficients based on:
  - area equivalent to added mass of the 2D cross-section.
    - For a circle it's $\pi d^2 / 4$.
  - length of body, l (needed for the moment only)
  - angle of attack, α

$$C_{d1} = \cos(\alpha)\sin^2(\alpha) + O\left(\frac{d^2}{l^2}\right)$$

$$C_{l1} = (1 + \cos^2(\alpha))sin(\alpha) + O\left(\frac{d}{l}\right)$$

$$C_m = \sin(2\alpha) + O\left(\frac{d}{l}\right)$$

- The part with constant cross-section has no force along it
- When $\alpha = 0$, the nose and the tail forces are zero, ***separately***
- The nose and tail shapes have no effect (to leading order in d/l)

Publication Woes

- JFM and then TCFD reviewers thought there was too much math!
  - (and it was hard to read [I've been told that before…])

$$\Delta u(0,y,z) = \frac{Al}{4\pi(l^2/4+y^2+z^2)^{3/2}}U_\infty + O\left(\frac{d^3U_\infty}{l^3};l^2\right) + O\left(\frac{A''\,\mathrm{d}W_\infty}{l^3};l^2\right), \tag{2.7}$$

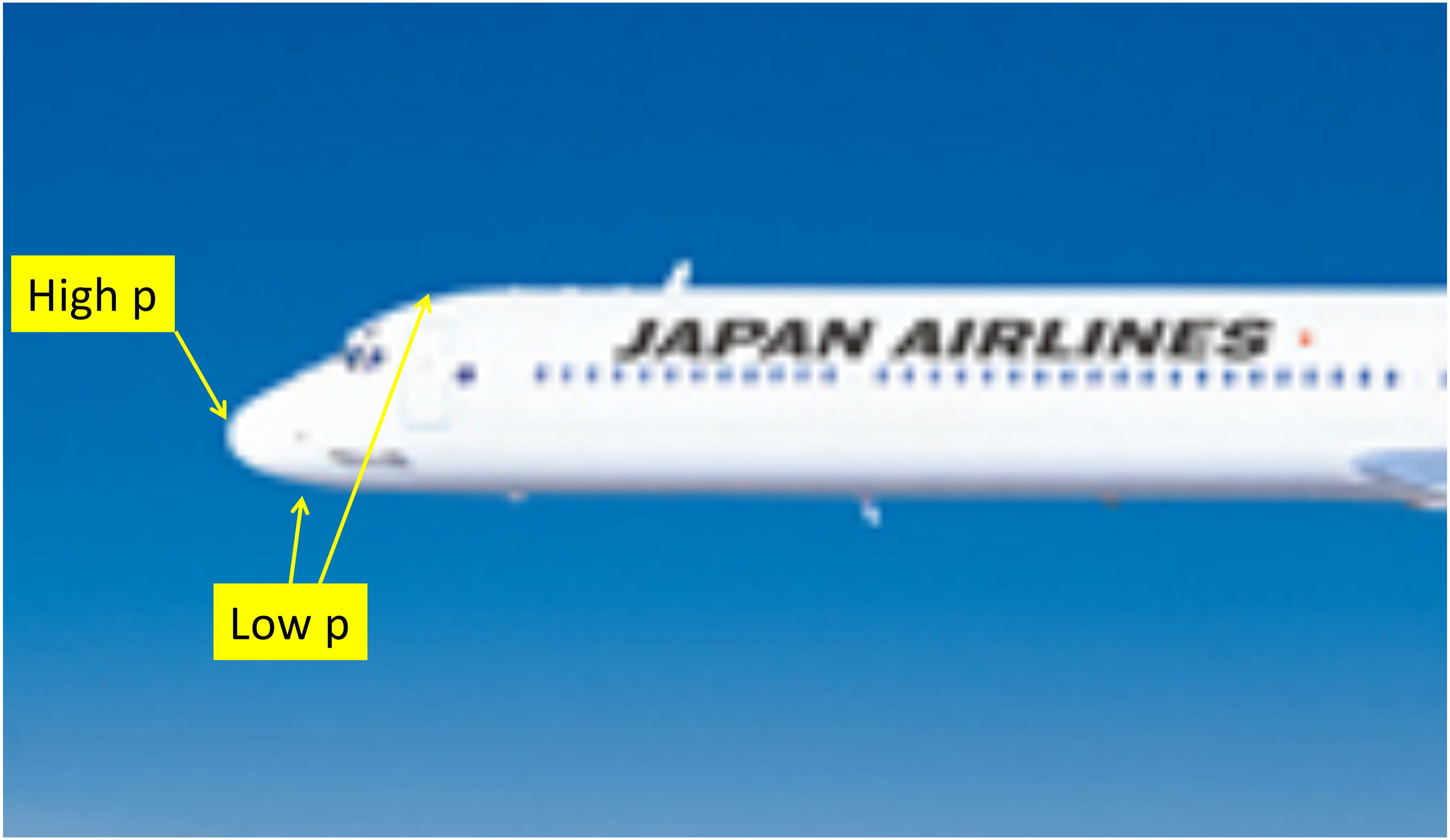
$$\begin{aligned}\Delta v(0,y,z) = \Delta V_{\mathrm{tr}} &+ \frac{A''W_\infty}{4\pi}\left[2-\frac{l}{\sqrt{(l/2)^2+y^2+z^2}}\right]\frac{2yz}{(y^2+z^2)^2}\\ &- \frac{A''W_\infty}{4\pi}\frac{l}{((l/2)^2+y^2+z^2)^{3/2}}\frac{yz}{y^2+z^2} + O\left(\frac{d^3U_\infty}{l^3};l^2\right) + O\left(\frac{A''W_\infty}{l^2};d^2\right)\end{aligned} \tag{2.8}$$

and

$$\begin{aligned}\Delta w(0,y,z) = \Delta W_{\mathrm{tr}} &+ \frac{A''W_\infty}{4\pi}\left[2-\frac{l}{\sqrt{(l/2)^2+y^2+z^2}}\right]\frac{z^2-y^2}{(y^2+z^2)^2}\\ &- \frac{A''W_\infty}{4\pi}\frac{l}{((l/2)^2+y^2+z^2)^{3/2}}\frac{z^2}{y^2+z^2} + O\left(\frac{d^3U_\infty}{l^3};l^2\right) + O\left(\frac{A''W_\infty}{l^2};d^2\right).\end{aligned} \tag{2.9}$$

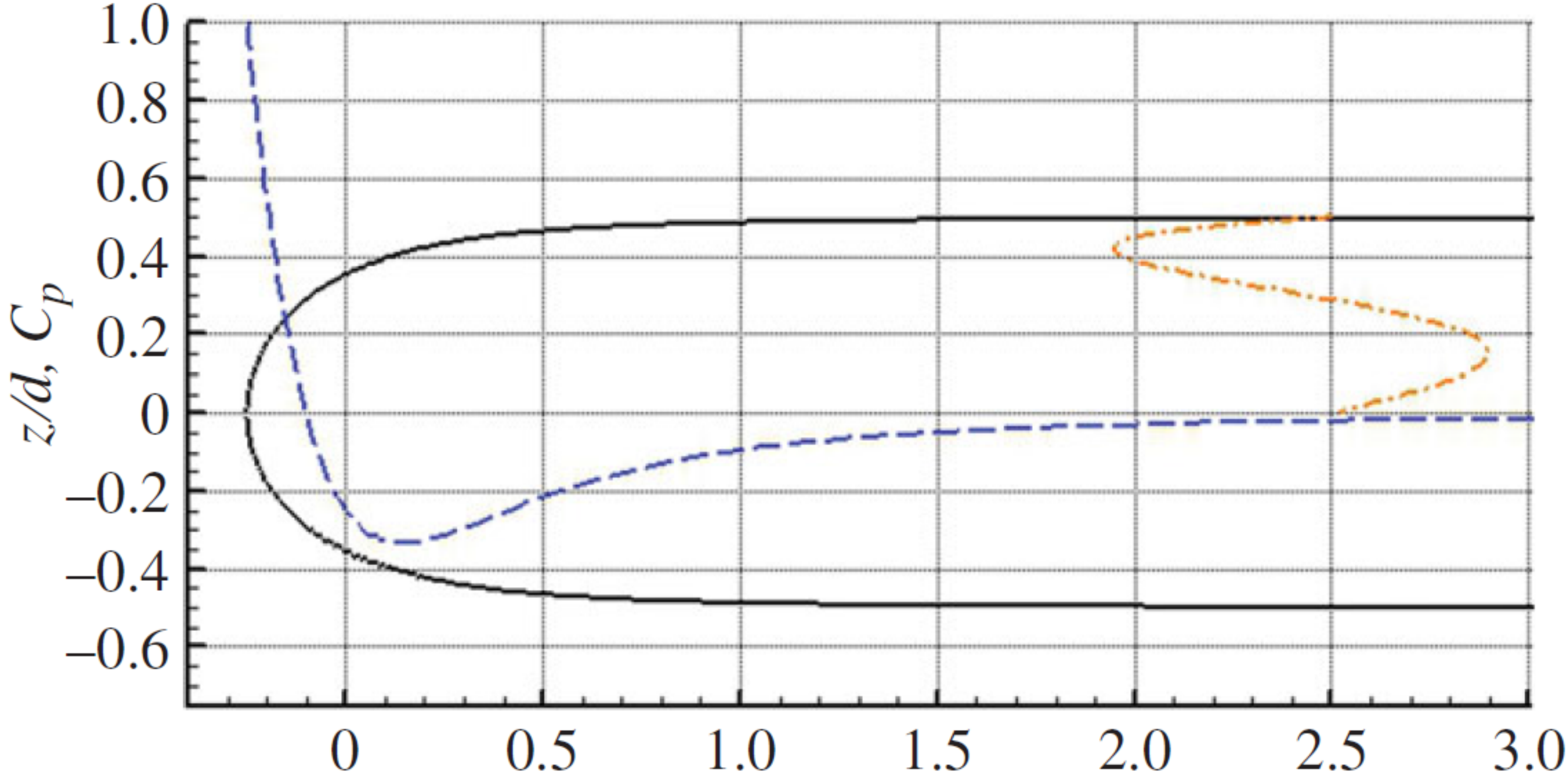
- Proc. Royal Society took it

How the Nose Forces End up Zero



Pressure Coefficient

Rankine Body



What Subsonic Planes Would Look Like Without This Theorem

Not easy on the airport gates…

Aft-Body

Theorem applies, but not as closely because of viscous effects, wing, and tail

Summary

- For a body with a long constant cross-section:
  - (and no separation, no vortices, no wing)
  - The long part itself has no lift or drag “per unit length”
  - At zero α, the nose and the tail each have zero forces
  - At any α, the nose and tail forces depend only on the cross-section (its added-mass equivalent area) and not on the nose/tail shapes
- This is not Slender-Body Theory, but it agrees with it when they overlap
- If you have a wing and viscosity, I believe the approximation still applies to the nose region
  - *You cannot improve, or degrade, performance by altering the nose shape!*

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  - *You cannot improve, or degrade, performance by altering the nose shape (unlike for movie stars)!*