Gabor mode enrichment in Large Eddy Simulation of turbulent flows

Aditya Ghate
Sanjiva Lele

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Precourt Institute for Energy
energy.stanford.edu

TomKatCenter FOR SUSTAINABLE ENERGY
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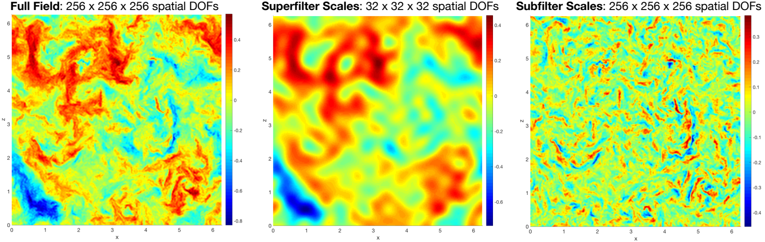
Turbulence Enrichment: Motivation

Consider a **2-scale representation** of isotropic turbulence using **spatial filtering**



The cost problem at high Reynolds numbers

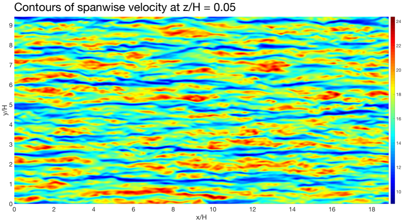
- Time resolved 256^3 simulation is > 4096 times more expensive than 32^3 simulation
- Wall-modeled LES is a promising solution to the $N \propto Re_L^{37/14}$ scaling in wall-bounded flows
- Subfilter scales play an important role in several applications
  - Engineering: Unsteady loading and structural vibrations
  - Geophysics: Scalar dispersion and mixing
  - Both: Particle-laden turbulence (Lagrangian statistics at subfilter scale)

Turbulence Enrichment: Motivation

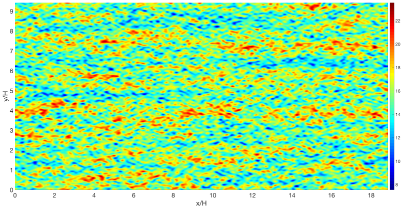
Conventional solutions to the cost problem:

- Use of minimally dissipative SGS modeling and numerical schemes, higher order numerics/basis representation, approximate deconvolution
- Improved representation of small scale turbulence: Wavelets

Half channel using 192 x 192 x 64 spatial DOFs



Wall-damped Smagorinsky SGS closure (Mason & Thomson, 1992)



Anisotropic minimum dissipation SGS closure (Rozema et al., 2015)

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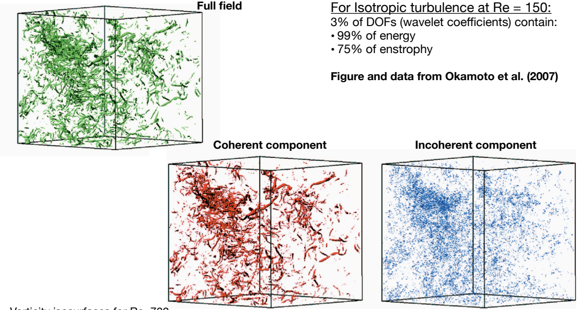


For Isotropic turbulence at Re = 150:
3% of DOFs (wavelet coefficients) contain:
• 99% of energy
• 75% of enstrophy

Figure and data from Okamoto et al. (2007)

Vorticity isosurfaces for Re=732.

What is Turbulence Enrichment?

Problem statement: One-way coupled enrichment

Given a large scale field $\boldsymbol{u}^r(\boldsymbol{x}, t)$, construct a subfilter scale field, $\boldsymbol{u}^{\tilde{s}}(\boldsymbol{x}, t)$ which:

1. Is discretely divergence free: $\nabla \cdot \boldsymbol{u}^{\tilde{s}} = 0$
2. Estimates the second order statistics of the true subfilter field, $\boldsymbol{u}^s$:

$$\left\langle u_i^{\tilde{s}}(\boldsymbol{x}+\boldsymbol{y}, t+\tau) u_j^{\tilde{s}}(\boldsymbol{y}, \tau) \right\rangle \approx \left\langle u_i^{s}(\boldsymbol{x}+\boldsymbol{y}, t+\tau) u_j^{s}(\boldsymbol{y}, \tau) \right\rangle$$

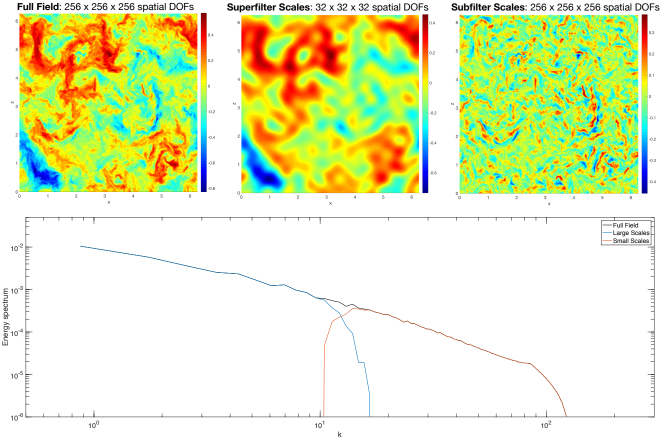
3. Is spectrally separated from $\boldsymbol{u}^r$, i.e. $\left\langle u_i^r u_j^{\tilde{s}} \right\rangle = 0$; Parseval's theorem implies spectrally sharp filtering used to obtain $\boldsymbol{u}^r$
4. *Enrichment* is not *reconstruction*: $\boldsymbol{u}^{\tilde{s}}(\boldsymbol{x}, t)$ is not an approximation to $\boldsymbol{u}^s(\boldsymbol{x}, t)$; it is merely *statistically equivalent*

Outline

Outline

1. 3 key requirements for enrichment: Why are existing models inadequate? (10 minutes)
2. The representation problem: "without the agonizing pain"
   - 2.1 Wavelets vs. Wavepackets
   - 2.2 Quasi-homogeneity and Gabor transform
   - 2.3 Gabor modes: velocity field synthesis
   - 2.4 Initialization: Sampling of Gabor modes for anisotropic turbulence
3. The temporal problem
   - 3.1 Evolution of Gabor modes in $(\boldsymbol{x} - \boldsymbol{k})$ frame
   - 3.2 Validation of the temporal problem: Half channel at $Re \to \infty$
4. Validation 1: Exact SGS closures
   - 4.1 Channel flow at $Re_\tau = 1000$ (skipped; refer to thesis)
   - 4.2 High latitude, stably stratified Atmospheric boundary layer (Ekman layer)
5. Validation 2: Non-exact/modeled SGS closures
   - 5.1 Forced isotropic turbulence at $Re \to \infty$
   - 5.2 Half-channel at $Re \to \infty$
6. Enrichment of a truncated POD representation: Actuator disk wake
7. Wrapup

Requirement #0: Spectral extrapolation



Correct inertial range scaling for global spectra

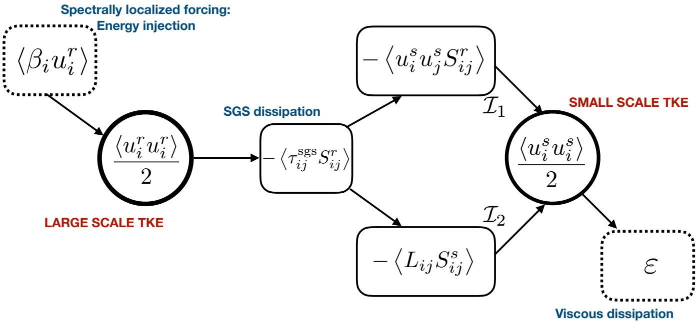
- $-5/3$rd scaling for power spectra of velocity components and relevant scalar field
- $-7/3$rd scaling for cross-spectra in wall-bounded (uw) and stratified ($w\theta$) flows
- Need modes with spectral localization

Requirement #1: Interscale energy exchange

Inter-scale kinetic energy transfers at large Reynolds numbers (Isotropic turbulence)

Assume that isotropic turbulence is forced such that kinetic energy is injected into large scales at the rate: $\left\langle \beta_i u_i^r \right\rangle$

- Loss in large scale TKE ($\frac{u_i^r u_i^r}{2}$) due to filtering, : $\left\langle \tau_{ij}^{SGS} S_{ij}^r \right\rangle$
- Gain of subfilter scale TKE ($\frac{u_i^s u_i^s}{2}$) from large scales: $\underbrace{\left[-\left\langle u_i^s u_j^s S_{ij}^r \right\rangle \right]}_{\mathcal{I}_1} + \underbrace{\left[-\left\langle L_{ij} S_{ij}^s \right\rangle \right]}_{\mathcal{I}_2}$



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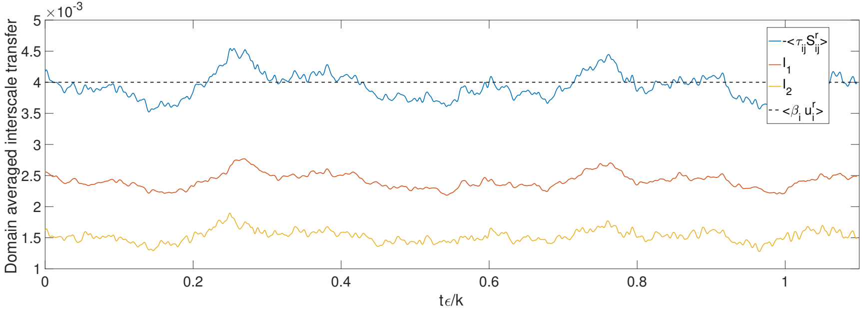
For a specific choice of the filtering wavenumber, $k_{co} = (32/3)$



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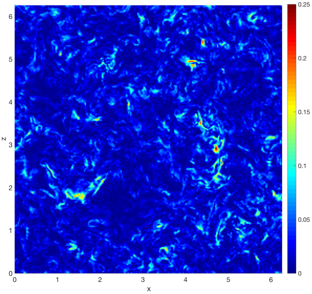
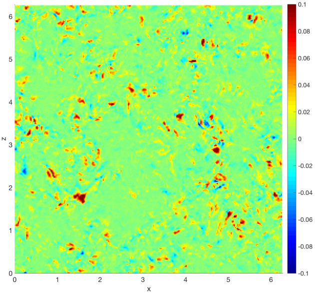


Small scale kinetic energy, $\frac{u_i^s u_i^s}{2}$



Inter-scale transfer, $\mathcal{I}_1 = -u_i^s u_j^s S_{ij}^r$

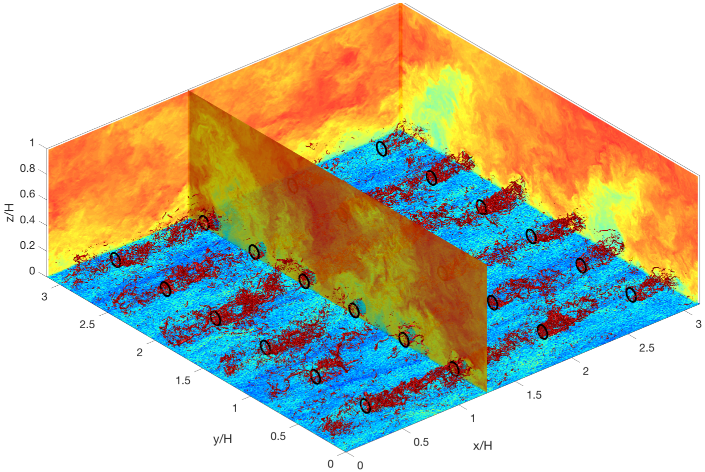
Consistency requirement for enriched subfilter scales

- $\mathcal{I}_1$ results in localized peaks in small scale kinetic energy
- Need modes with both spatial and spectral localization

Requirement #2: Spatial inhomogeneities

Wind farm - Atmospheric boundary layer (ABL) interactions

Contours of streamwise velocity, $\tilde{u}/u_\tau$:



Requirement #2: Spatial inhomogeneities

Spatially filtered down from a $384 \times 384 \times 256$ **grid to** $48 \times 48 \times 96$

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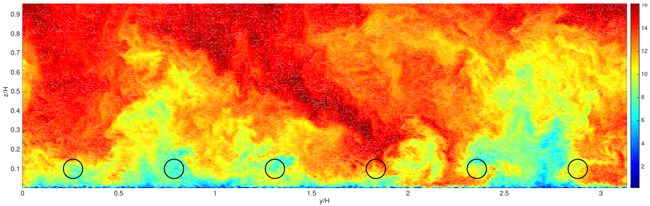
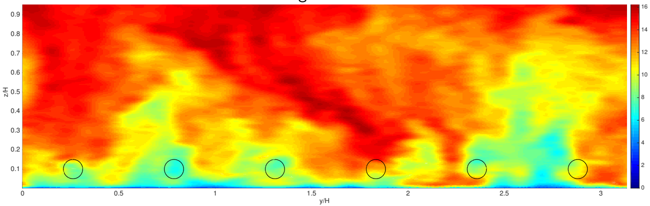


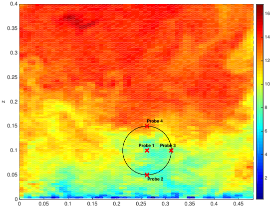
Original field



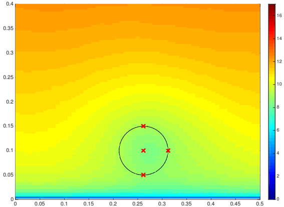
Spatially filtered field

Requirement #2: Spatial inhomogeneities

Statistics computed using probes and time averaging



Instantaneous



Time averaged

Probe	Large Scale, $< u^r w^r > / u_\tau^2$	Small Scale, $< u^s w^s > / u_\tau^2$
1	-0.41	-0.09
2	-0.22	**+0.05**
3	-0.49	-0.11
4	-0.82	-0.61

Single point cross-correlations

Consistency requirement for enriched subfilter scales

- Subfilter scales may contain turbulence generated by local secondary instabilities
- Need to account for spatial inhomogeneities and direct energy transfer between mean and subfilter scales

Requirement #3: Temporal evolution/decorrelation

Subfilter scale velocity evolves in time as:

$$\partial_t u_i^s + u_j^r \partial_j u_i^s + u_j^s \partial_j u_i^r + u_j^s \partial_j u_i^s = -\partial_i p^s + \frac{1}{\mathrm{Re}} \partial_j \partial_j u_i^s + \partial_j \tau_{ij}$$

Now project on to a divergence free basis:

$$\partial_t u_i^s = -\underbrace{\left[u_j^r \partial_j u_i^s - \left(u_j^r \partial_j u_i^s\right)^r\right]^\perp}_{\text{Term 1 (Sweeping)}} - \underbrace{\left[u_j^s \partial_j u_i^r - \left(u_j^s \partial_j u_i^r\right)^r\right]^\perp}_{\text{Term 2 (Straining)}} - \underbrace{\left[\partial_j \left(u_i^s u_j^s - (u_i^s u_j^s)^r\right)\right]^\perp}_{\text{Term 3 (Nonlinear relaxation)}}$$
$$+ \underbrace{[\partial_j L_{ij}]^\perp}_{\text{Term 4 (Leonard stress source/sink)}} + \underbrace{\frac{1}{Re} \partial_j \partial_j u_i^s}_{\text{Term 5 (viscous diffusion)}}$$

After some math ...

$$c_i(\tau) = \left\langle u_i^s(\boldsymbol{x}, t) u_i^s(\boldsymbol{x}, t+\tau) \right\rangle = \left\langle u_i^s(\boldsymbol{x}, t) u_i^s(\boldsymbol{x}, t) \right\rangle + \underbrace{\left\langle u_i^s(\boldsymbol{x}, t) \int_t^{t+\tau} \mathcal{T}_1 dt' \right\rangle}_{\mathcal{D}_1 \text{(Sweeping)}}$$
$$+ \underbrace{\left\langle u_i^s(\boldsymbol{x}, t) \int_t^{t+\tau} \mathcal{T}_2 dt' \right\rangle}_{\mathcal{D}_2 \text{(Straining)}} + \underbrace{\left\langle u_i^s(\boldsymbol{x}, t) \int_t^{t+\tau} \mathcal{T}_3 dt' \right\rangle}_{\mathcal{D}_3 \text{(Nonlinear relaxation)}} + \underbrace{\left\langle u_i^s(\boldsymbol{x}, t) \int_t^{t+\tau} \mathcal{T}_4 dt' \right\rangle}_{\mathcal{D}_4 \text{(Leonard stress source/sink)}} + \underbrace{\left\langle u_i^s(\boldsymbol{x}, t) \int_t^{t+\tau} \mathcal{T}_5 dt' \right\rangle}_{\mathcal{D}_5 \text{(viscous diffusion)}}$$

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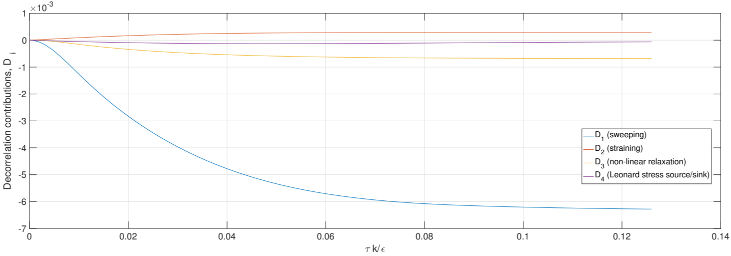
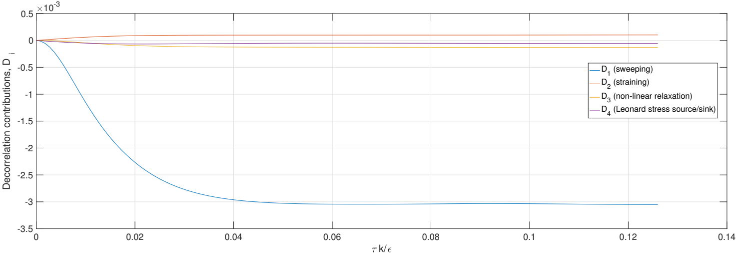


Filter wavenumber, $k_{co} = 32/3$



Filter wavenumber, $k_{co} = 64/3$

Turbulence enrichment: the main challenge

- Correct temporal decorrelation requires the *sweeping* of subfilter scales by superfilter scales
- Correct interscale energy transfer requires modeling of *straining* of subfilter scales by superfilter scales along with that injected by the *Leonard stress* term.
- Energy injected by large scales (vortex stretching) must cascade downscale (non-linear relaxation) for energy equilibrium (correct spectral extrapolation)
- Need a Lagrange multiplier (pressure) to enforce mass conservation (divergence free); require solution to spatially non-local Poisson equation

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Existing models/algorithms

Methods with increasing computational complexity

- **Spectral extrapolation emphasis**: Fractal interpolation (Scotti & Meneveau, 1997), Kinematic simulations (Kraichnan, 1970; Fung et al., 1992; Flohr & Vassilicos, 2000)
- **Energetics emphasis**: Domaradzki and collaborators (Kerr et al., 1996; Domaradzki & Saiki, 1997; Domaradzki & Loh, 1999) which assume *single-action* production of small scales; Bassenne et al. (2017) extended and applied to particle laden turbulence
- **Variational multiscale (VMS) and multilevel algorithms (TMA)**: Hughes et al. (1998); Terracol et al. (2001)
- **Wavelet thresholding**: CVS (Farge & Schneider, 2001); SCALES (Goldstein & Vasilyev, 2004)

Gabor mode enrichment

A new method developed for subfilter scale enrichment:

- Provides DOF compression similar to wavelet methods with computational complexity comparable with kinematic simulations
- Satisfies all 3 requirements and validated for wall bounded incompressible turbulence with density stratification and frame rotation (Coriolis)
- Entirely physics based with no coefficient tuning

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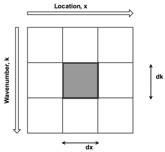
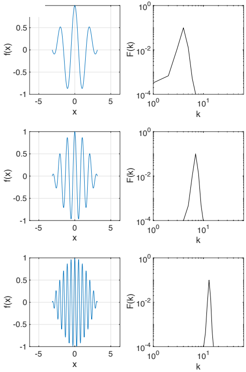
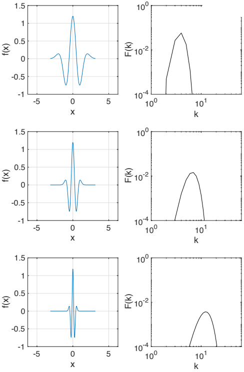
Wavepacket or Wavelet?

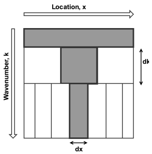


GABOR WAVEPACKET



MARR WAVELET

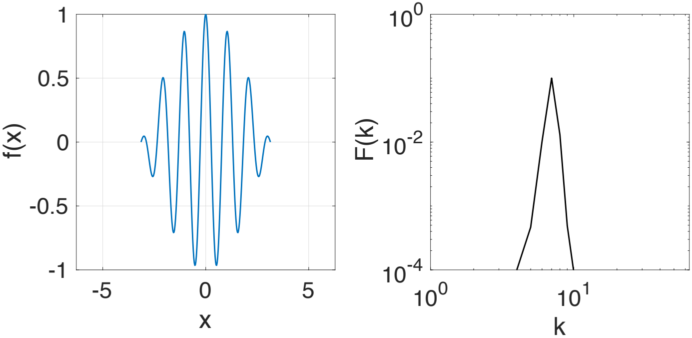




Uncertainty Principle: $(dx) \times (dk) \geq 2\pi$

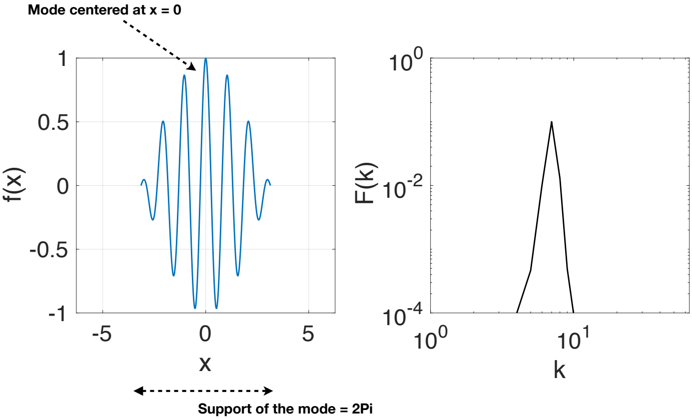
A single Gabor mode

Consider a single plane wave within the wavepacket. We will call this a **Gabor mode**.



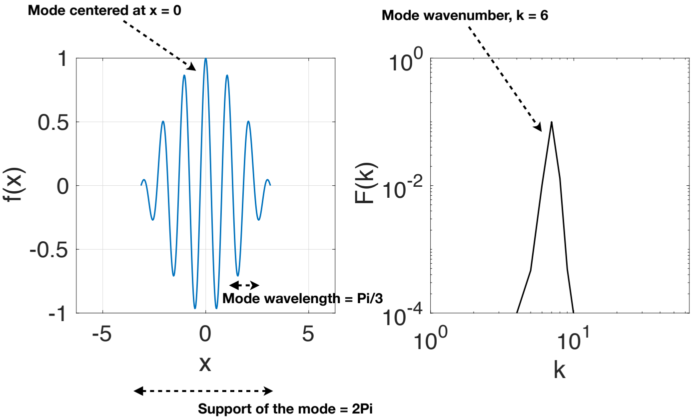
A single Gabor mode

The mode is centered at $\boldsymbol{x}_0$, and has a physical support, Δ (related to the LES filter width/grid scale).



A single Gabor mode

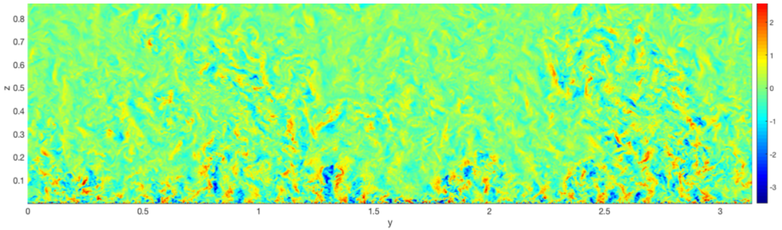
The mode also has a characteristic wavenumber, k and a wavelength, $\lambda = 2\pi/k$.



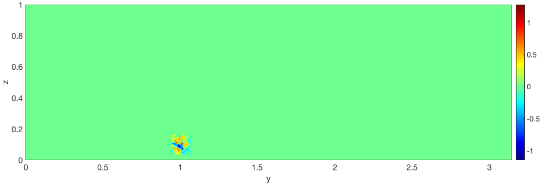
The scale separation parameter is defined as: $\varepsilon = \dfrac{\lambda}{\Delta}$

Quasi-homogeneity in turbulence

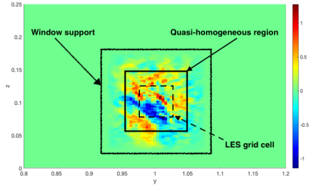
Consider the subfilter scales in the Windfarm - Atmospheric boundary layer problem



Subfilter streamwise (x) velocity on YZ plane



Windowed subfilter scale field; Window width $\Delta_w > \Delta_{QH} = 2\Delta_{LES}$



Zoomed view

Quasi-homogeneity in turbulence

Quasi-homogenenity assumption

Over the length scale, $\Delta_{QH} = 2\Delta_{LES}$, the subfilter field is *spatially homogeneous*.

- For a homogeneous field, both a) energy minimizing representation (POD) and b) optimal representation (minimum modes) is the Fourier representation (Lumley, 1970)
- A random homogeneous field can be realized as a Fourier-Stieltjes process :

$$\boldsymbol{u}^{\tilde{s}}(\boldsymbol{x}, \boldsymbol{x}_0, \Delta_w) = \int_{\boldsymbol{k} \in \mathcal{R}^3} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} d\boldsymbol{Z}(\boldsymbol{k}, \boldsymbol{x}_0, \Delta_w)$$

- Compatibility between random field, $\boldsymbol{u}^{\tilde{s}}$ and true subfilter field $\boldsymbol{u}^{s}$ is simply:

$$\frac{\left\langle dZ_i^\star(\boldsymbol{k}, \boldsymbol{x}_0) dZ_j(\boldsymbol{k}, \boldsymbol{x}_0) \right\rangle}{d\boldsymbol{k}} = \phi_{ij}(\boldsymbol{k}, \boldsymbol{x}_0) = \int_{\boldsymbol{r} \in \mathcal{R}^3} \langle u_i(\boldsymbol{x} - \boldsymbol{x}_0, \boldsymbol{x}_0) u_j(\boldsymbol{x} - \boldsymbol{x}_0 - \boldsymbol{r}, \boldsymbol{x}_0) \rangle \, e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} d\boldsymbol{r}$$

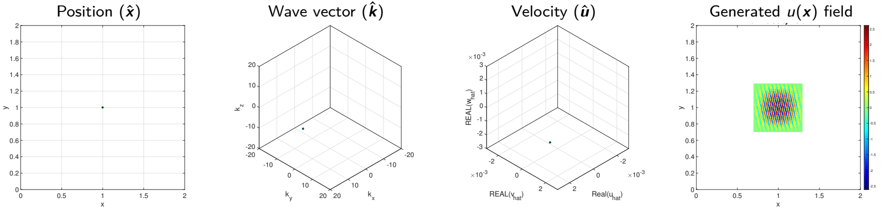
as $d\boldsymbol{k} \to 0$ (as the number of seeded modes increases to infinity)

Velocity synthesis from discrete Gabor modes

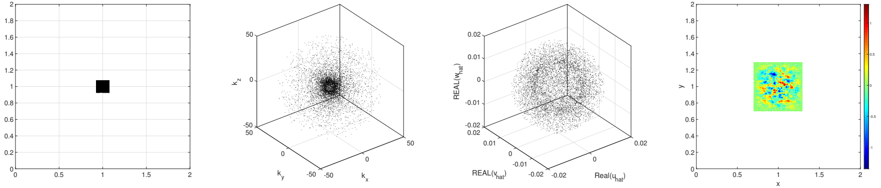
Gabor modes are wavepackets that carry:

1. Position, $\boldsymbol{x}(t)$
2. Wave vector, $\boldsymbol{k}(t)$
3. Velocity, $\hat{\boldsymbol{u}}(t)$
4. Scalar field(s), $\hat{\theta}(t)$

Single Mode



Multiple modes

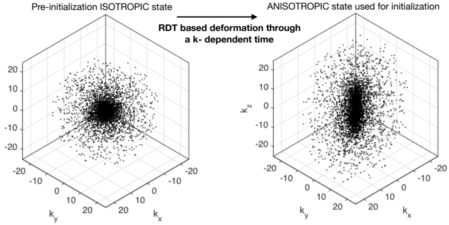


Initialization of Gabor modes

Eddy Lifetime Hypothesis: Panofsky, et al. (1982); Mann (1992)

A *realizable* local anisotropic state (quantified using the anisotropy tensor $b_{ij} = \frac{R_{ij}}{R_{kk}} - \frac{1}{3}\delta_{ij}$) can be obtained by straining the isotropic state ($b_{ij} = 0$) using Rapid distortion theory (RDT), through a $\boldsymbol{k}-$ dependent time scale. Mann's model for such a $k-$ dependent time:

$$\tau(k) \propto \frac{1}{k\sqrt{\int_k^\infty E(k)dk}} \implies \tau(k)S = c_\tau\,(kL)^{-2/3}\left[{}_2F_1\left(\frac{1}{3},\frac{17}{6};\frac{4}{3};-(kL)^{-2}\right)\right]^{-1/2}$$



Model constants (L, c_τ) can be determined using information (energy transfer rate, etc.) from the large scales via a least squares minimization.

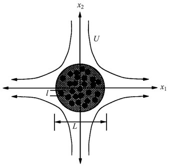
Gabor transform in fluid mechanics

Advantage of Gabor representation over wavelet representation

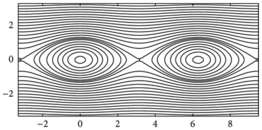
- The condition, $\Delta_w \gg \Delta_{LES}$ is equivalent to $\varepsilon \ll 1$. In practice, $\varepsilon \approx 1/4$.
- Evolution of Gabor modes can be simplified to the leading order in ε (WKB asymptotic expansion).

Method inspired by 2 applications

- **WKB-variant of Rapid distortion theory**: Nazarenko et al. (1999); Dubrulle et al. (2001); Laval et. al. (2004) extensions of RDT to inhomogeneous turbulence
- **Geometric optics and local stability analysis**: Bayly (1986); Lifshitz & Hameiri (1991); Cambon et al. (2998) developed *Zonal approaches* for stability analysis.



Taylor's "four-roller mill"
Distortion using WKB-RDT
(Nazarenko et al., 1999)

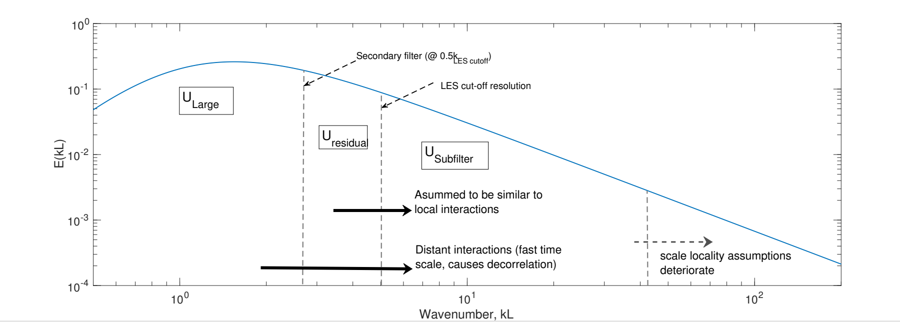


Stuart-Cells
Elliptic, Centrifugal and hyperbolic instability
(Godeferd, Cambon & Leblanc, 2001)

Outline

Temporal Evolution of Gabor modes

An idealized energy spectrum



Notation: U: Large scales, U^r: Residual scales, and u: Subfilter/Small scales

Subfilter scale equations

$$\partial_t u_i + U_j \partial_j u_i + u_j \partial_j U_i = -\partial_i p - \partial_j h_{ij} + \partial_j \tau_{ij}^d + \delta_{ij} g_j \frac{\theta}{\Theta_0} - 2\epsilon_{ijk}\Omega_j u_k + \nu \partial_j \partial_j u_i$$

$$h_{ij} = \partial_j(u_i u_j) + \partial_j(u_i U_j^r) + \partial_j(U_i^r u_j)$$

Modeling the small scale evolution

NEXT STEP: Take Gabor transform and keep leading order terms in ε (WKB asymptotic expansion)

Gabor transform of derivatives

To a leading order in scale separation parameter, ε the Gabor transform of derivatives can be expressed as:

$$\widehat{\partial_m u} = ik_m \hat{u} + \mathcal{O}(\varepsilon)$$

Furthermore, for a solenoidal field ($k_j \hat{u}_j = 0$), pressure non-linearity can be projected out using a Projection tensor

$$\left(\delta_{mj} - \frac{k_m k_j}{k^2}\right) \widehat{\partial_j p} = \left(\delta_{mj} - \frac{k_m k_j}{k^2}\right) (ik_j \hat{p}) = 0$$

The Quasi-homogeneity assumption

The large scale field, $U(\boldsymbol{x})$ can be expressed in its truncated Taylor series expansion within a neighborhood $||\boldsymbol{x} - \boldsymbol{x_0}|| < l \propto \Delta_{QH}$ where $\boldsymbol{x}, \boldsymbol{x}_0 \in \mathbb{R}^3$.

$$U(\boldsymbol{x}) = U^0 + (\boldsymbol{x} - \boldsymbol{x}_0) \cdot \nabla U|^0$$

Model for the local (in scale space) convective non-linearity

The action of the convective non-linearity due to local triadic interactions will be modeled using a spectral viscosity based on Renormalization Group Theory (RNG) (see Canuto & Dubovikov, PoF, 1996)

$$\widehat{\partial_j h_{ij}}^{\perp} = -\nu_t(k) k^2 \hat{u}_i \quad , \quad \nu_t(k) = \left(\nu^2 + c_\nu \int_k^\infty q^{-2} E(q) dq\right)^{1/2} - \nu$$

Modeling the small scale evolution

Governing equations for Gabor modes

1. Motion described in a *sweeping* frame:

$$\partial_t x_j = U_j^0$$

2. An Eikonal equation for evolution of a wavenumber:

$$\partial_t k_j = -k_m \partial_j U_m^0$$

3. A WKB-RDT approximation for evolution of complex amplitude

$$\partial_t \hat{u}_i = \left(\frac{2k_i k_m}{k^2} - \delta_{im} \right) \hat{u}_j \partial_j U_m^0 + \left(\frac{k_i k_j}{k^2} - \delta_{ij} \right) g_j \beta \hat{\theta} - (\nu + \nu_t) k^2 \hat{a}_i + \hat{f}_i^{\perp} - 2\epsilon_{ijk} \Omega_j \hat{u}_k$$

$$\partial_t \hat{\theta} = -\hat{u}_j \partial_j \Theta^0 - (\kappa + \kappa_t) k^2 \hat{\theta} + \hat{f}_\theta$$

where, $\hat{f}_i$ and $\hat{f}_\theta$ are Gabor projections of the Leonard stress terms $\partial_j L_{ij}$ and $\partial_j q_j$ respectively.

Important consideration: The ODEs governing evolution of the Gabor modes are only accurate up to leading order in ε; the proposed model is not a numerical method.

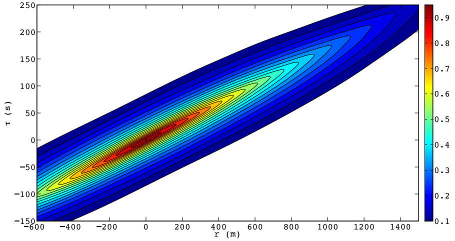
Validation of temporal dynamics

Half-channel at $Re \to \infty$

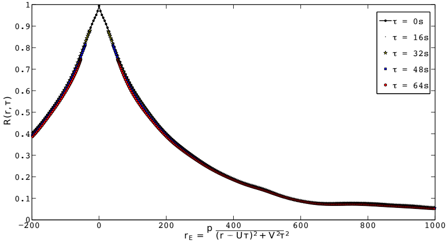
- Validate the space-time behavior by comparison with high-resolution LES data (Wilczek, Stevens & Meneveau, 2015, J. Fluid Mech.) at $z = 0.154H$.
- Gabor mode simulation uses a 3-scale decomposition: a) $kL < 0.6$ (frozen), b) $0.6 < kL < 6$ and c) $6 < kL$ where $L = 0.075H$ is determined from LES data.

Space-time correlations

$\langle u(x,t)u(x+r,t+\tau)\rangle$



Guowei He's (2006) transform



Validation of temporal dynamics

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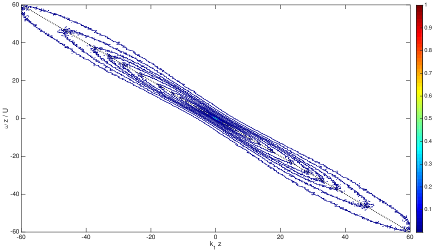
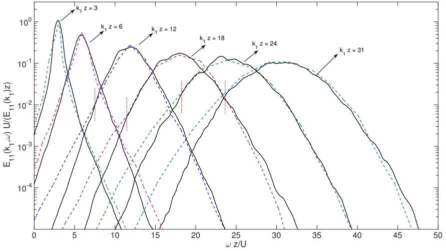
$k - \omega$ spectrum

Log-spaced contours of $k - \omega$ spectrum



colored dashes - LES, solid black lines - Gabor modes



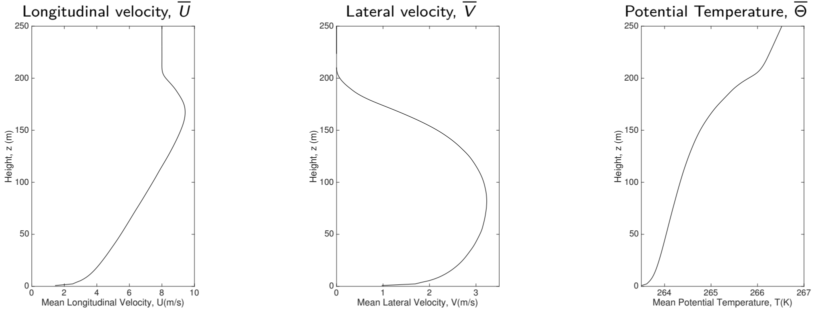
Outline

High latitude, stably stratified ABL

Validation methodology

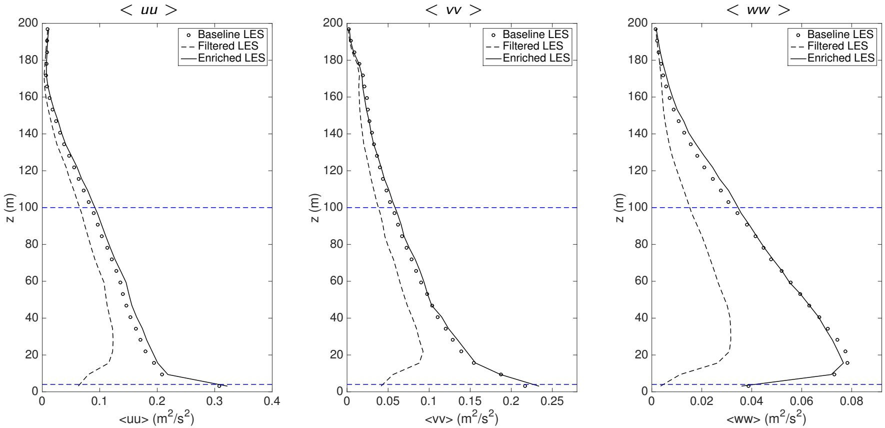
- Baseline LES: Conventional LES on a 512 × 512 × 512 numerical grid, $Re \to \infty$ (results validated using independent studies).
- Coarse LES: Baseline LES filtered on a 40 × 40 × 128 numerical grid.
- Enriched LES: 800 Gabor modes in each Coarse LES cell; resulting fields resolved on a 1440 × 1440 × 2304 numerical grid.
- Effective compression in Dofs is approximately 97%.

Mean profiles (averaged between hour 7 and hour 8 flow time past initialization)



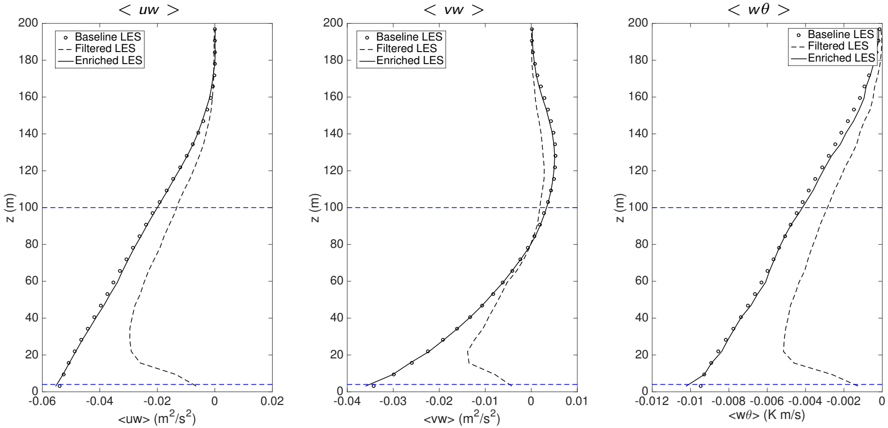
High latitude, stably stratified ABL

Single-point variances $< uu >$, $< vv >$ and $< ww >$ profiles in z



High latitude, stably stratified ABL

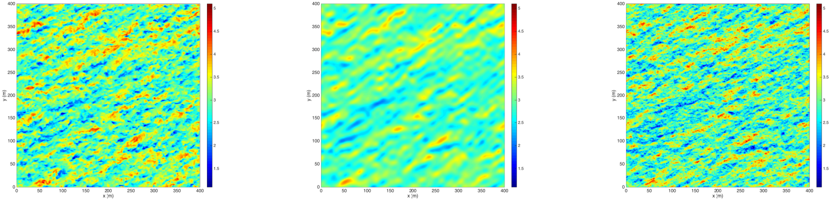
Single point cross-correlations, $< uw >$, $< vw >$ and $< w\theta >$ profiles in z



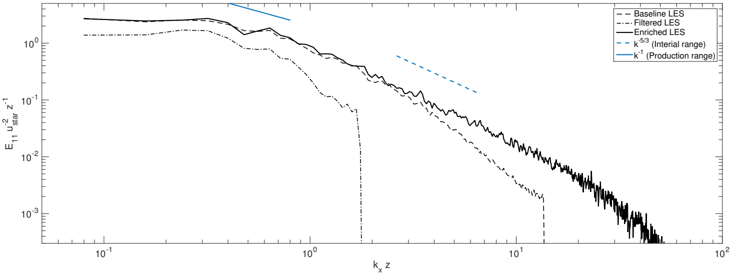
High latitude, stably stratified ABL

Contours of u on $z = 6$m plane

Baseline LES | Coarse LES | Enriched LES



k_x spectra of u on $z = 6$m plane



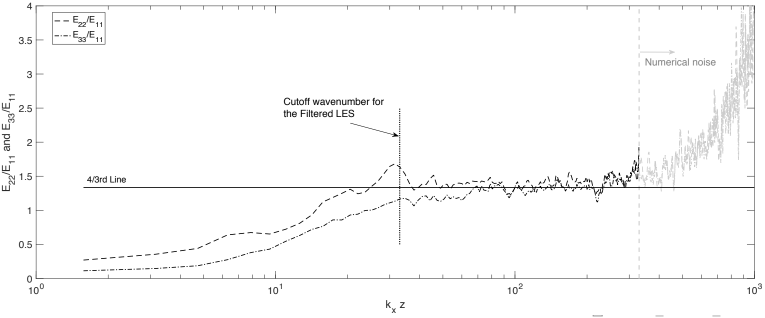
High latitude, stably stratified ABL

Local isotropy hypothesis

For an idealized energy spectrum, $E(k) = C_K \varepsilon^{2/3} k^{-5/3}$, we have

- $E_{11}(k_1) = \frac{18}{55} C_K \varepsilon^{2/3} k_1^{-5/3}$; $E_{22}(k_1) = E_{33}(k_1) = \frac{24}{55} C_K \varepsilon^{2/3} k_1^{-5/3}$
- $E_{22}/E_{11} = E_{33}/E_{11} = 4/3$
- In sheared turbulence, for some $k > k_{iso}$ local isotropy should exist.
- Kaimal's (1973) measurements for a stably stratified surface layer shows that E_{22}/E_{11} asymptotes before E_{33}/E_{11}

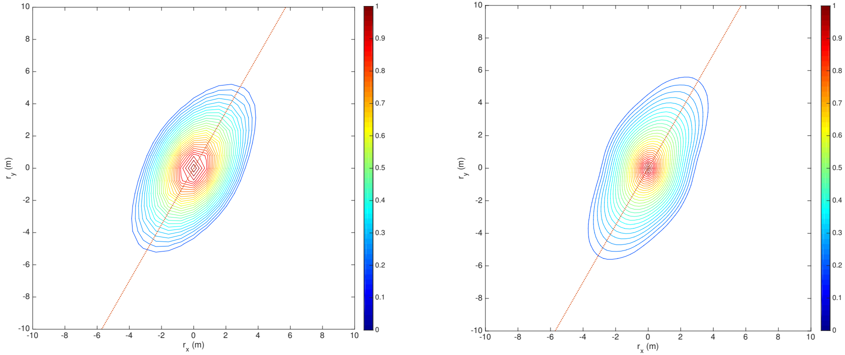
Spectra taken at $z = 100m$



High latitude, stably stratified ABL

Two-point correlations, $\langle v(x + r_x, y + r_y) v(x, y) \rangle$ at $z = 6$m)

Exact small scales **Enriched small scales**

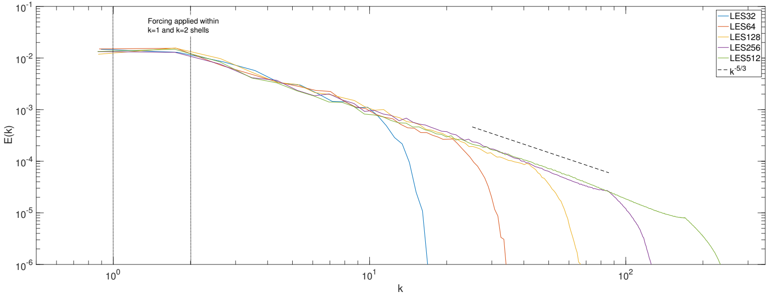


Outline

Forced, homogeneous isotropic turbulence

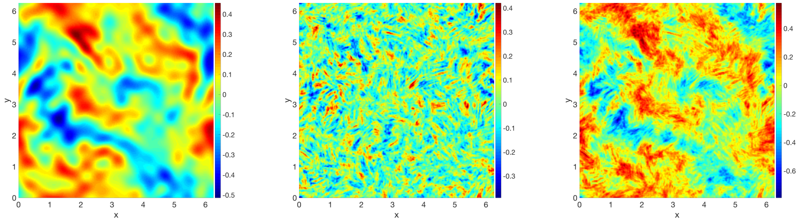
Problem description

- Forcing restricted within $1 < |k| < 2$ (Carati, et al., 1995) which sets the dissipation rate
- Simulation performed within the $Re \rightarrow \infty$ limit; inertial range extends to grid Nyquist limit regardless of resolution
- Sigma model (Nicoud et al., 2011) used for SGS dissipation
- Grid sizes ranging from 32^3 through 512^3 used; $2/3rd$ dealiasing used in all cases
- All interpolations from coarse to fine resolution performed using zero-padding in Fourier space



Forced, homogeneous isotropic turbulence

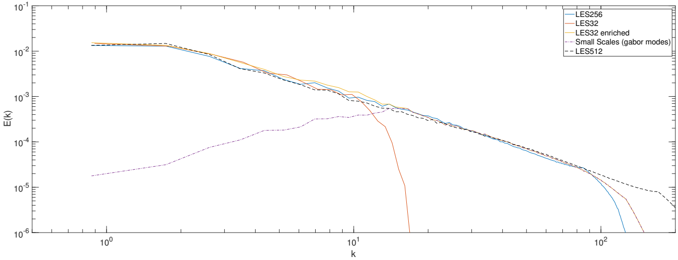
v component on XZ plane



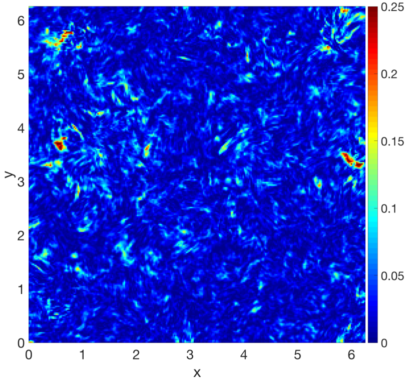
32^3 simulation (upsampled)

Gabor mode induced fields

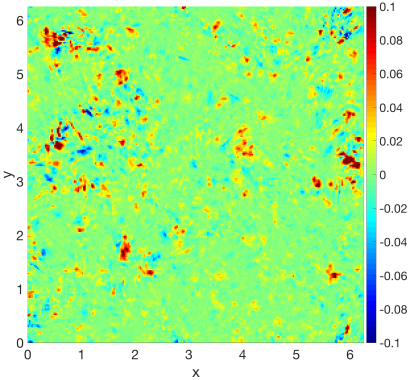
Enriched 32^3 LES



Forced, homogeneous isotropic turbulence



Small-scale Kinetic energy, $u_i^r u_i^r$



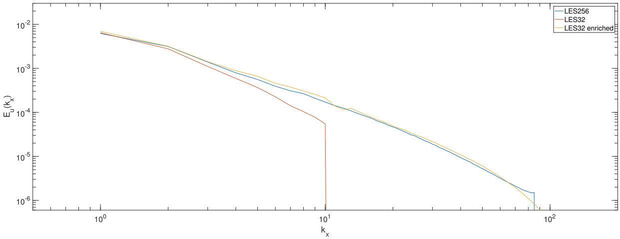
Inter-scale transfer, $\mathcal{I}_1 = -u_i^s u_j^s S_{ij}^r$

Global inter-scale energy transfers

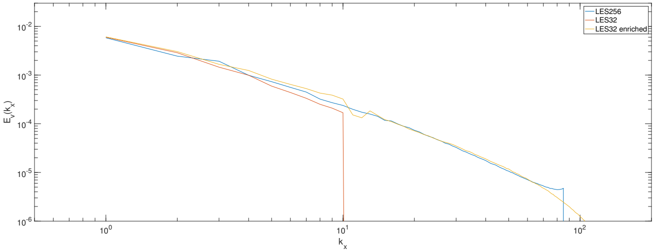
1. $\mathcal{I}_1 = -\left\langle u_i^s u_j^s S_{ij}^r \right\rangle$: 0.025 (Gabor mode enrichment); 0.0255 (exact)
2. $\mathcal{I}_2 = -\left\langle L_{ij} S_{ij}^s \right\rangle$: 0.0018 (Gabor mode enrichment); 0.0016 (exact)

Forced, homogeneous isotropic turbulence

1D spectra in k_x



u component

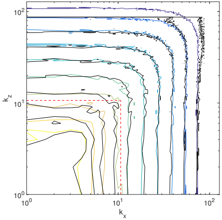


v component

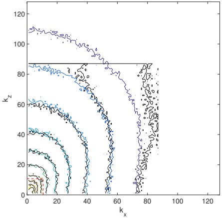
Forced, homogeneous isotropic turbulence

2D spectra in $k_x \times k_y$

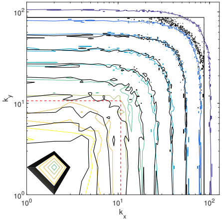
Colored lines: 32^3 Enriched LES; Black lines: 256^3 LES



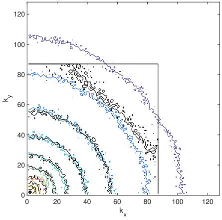
u component (log spacing)



u component (linear spacing)

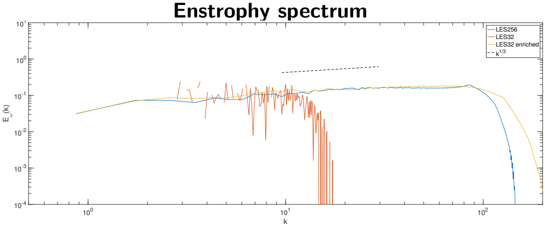


w component (log spacing)

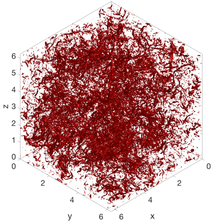


w component (linear spacing)

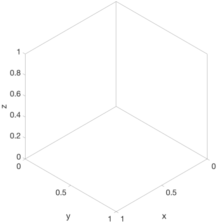
Forced, homogeneous isotropic turbulence



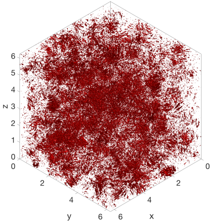
Vorticity contours at level: $|\omega| = 150\varepsilon/q$



256^3 LES



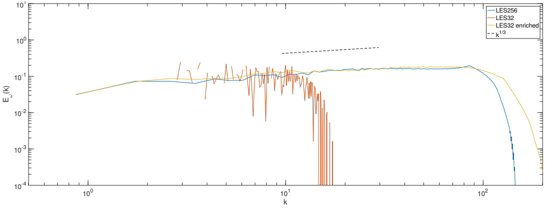
32^3 LES



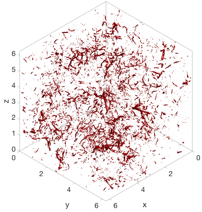
32^3 LES (Enriched)

Forced, homogeneous isotropic turbulence

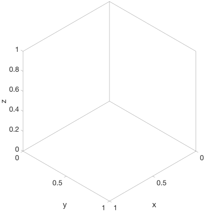
Enstrophy spectrum



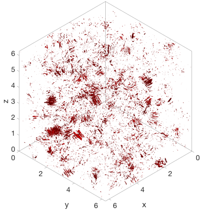
Vorticity contours at level: $|\omega| = 210\varepsilon/q$



256^3 LES



32^3 LES



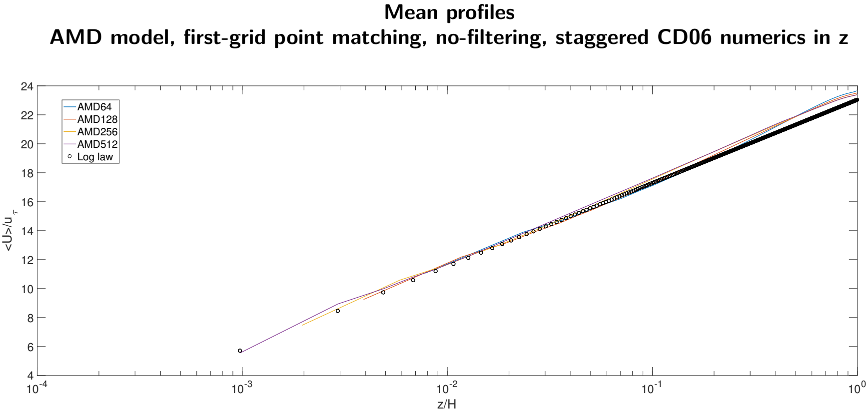
32^3 LES (Enriched)

Half-channel

Problem description

- WMLES in a half-channel configuration (Stevens, et al., 2015) with wall roughness, $z_0/H = 10^{-4}$ (Yang, et al., 2017)
- Three SGS models used:
  - **Anisotropic Minimum Dissipation (AMD)**: Rozema, et al. (2015); Abkar, et al. (2016); scheme dependent choice of model coefficients (Bae, 2018)
  - **Sigma model**: Nicoud et al. (2011); previously validated in Ghaisas et al. (2017)
  - **Wall-damped Smagorinsky**: Mason & Thomson (1992); Goit & Meyers (2015); Onder & Meyers (2018)
- Wall model: Equilibrium wall-model with first grid-point matching; no-filtering; no grid-stretching
- Several aspects investigated; matching location, choice of numerics (FD02 vs CD06), advection formulation (rotational vs. skew-symmetric)
- Domain size: $(6\pi \times 3\pi \times 1)H$; grid resolution stepped up from $192 \times 192 \times 64$ through $1536 \times 1536 \times 512$
- $192 \times 192 \times 64$ simulations enriched; $1536 \times 1536 \times 512$ simulation used as reference

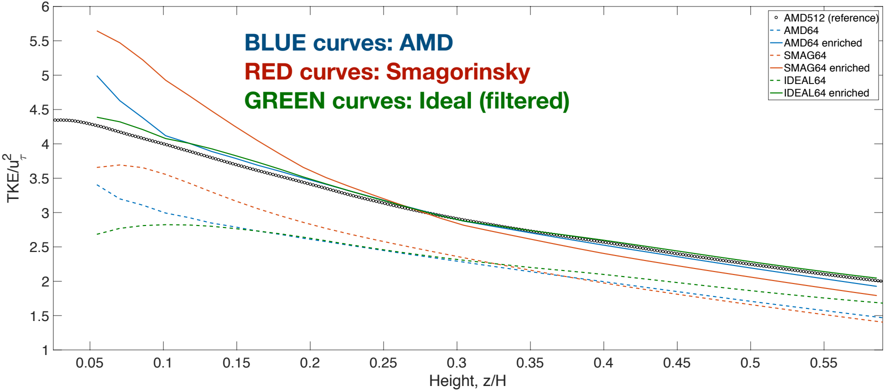
Half-channel



No perceivable log-layer mismatch.

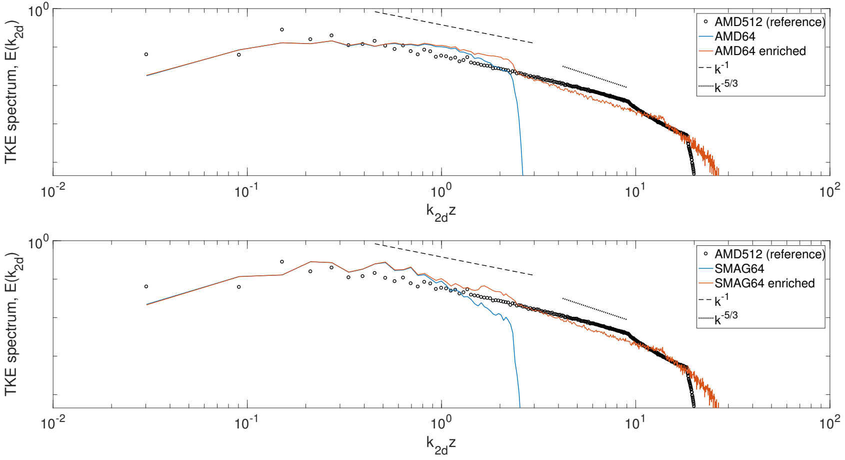
Half-channel

Kinetic energy perspective: Single point correlation



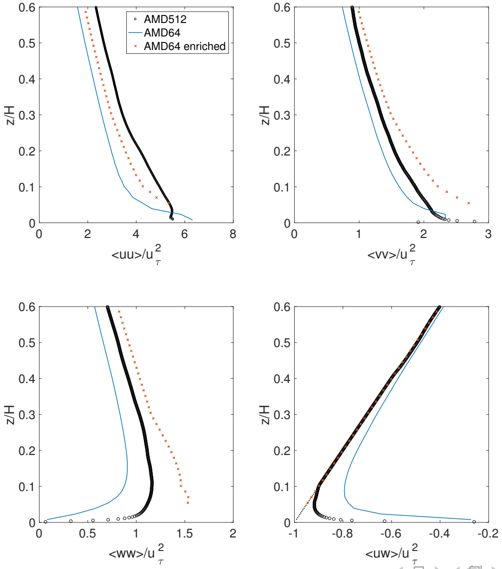
Half-channel

Kinetic energy perspective: TKE spectrum (2d polar wavenumber) **at** $z = 0.054H$



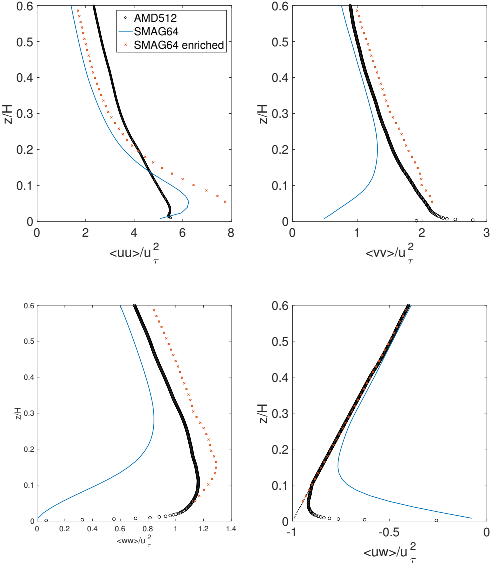
Half-channel

Anisotropy perspective: Single point correlations (AMD64 enrichment)



Half-channel

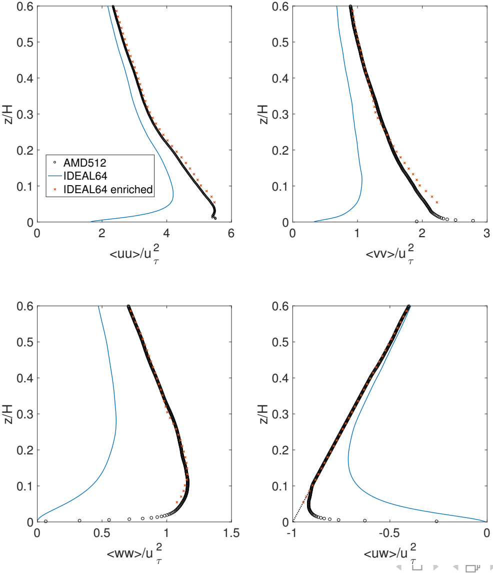
Anisotropy perspective: Single point correlations (SMAG64 enrichment)



Half-channel

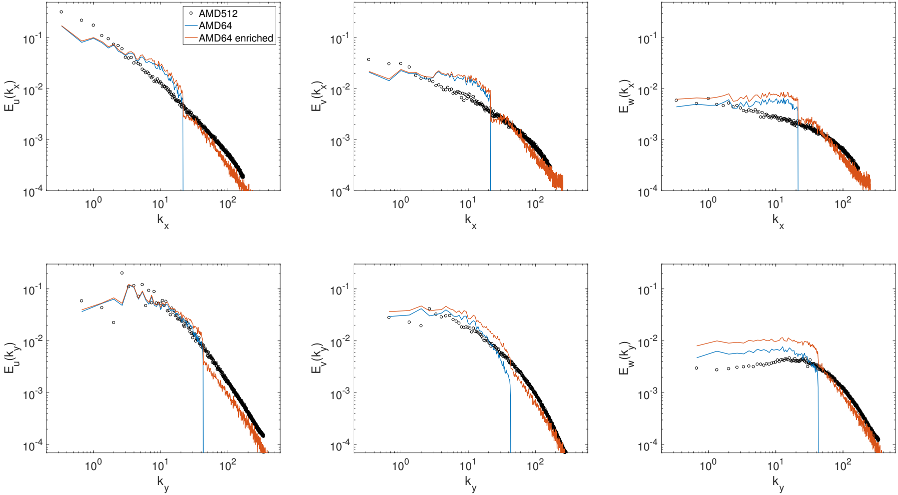
Anisotropy perspective: Single point correlations (IDEAL64 enrichment)

Spectrally sharp filter in X-Y plane, Pade-leastsquares filter in Z



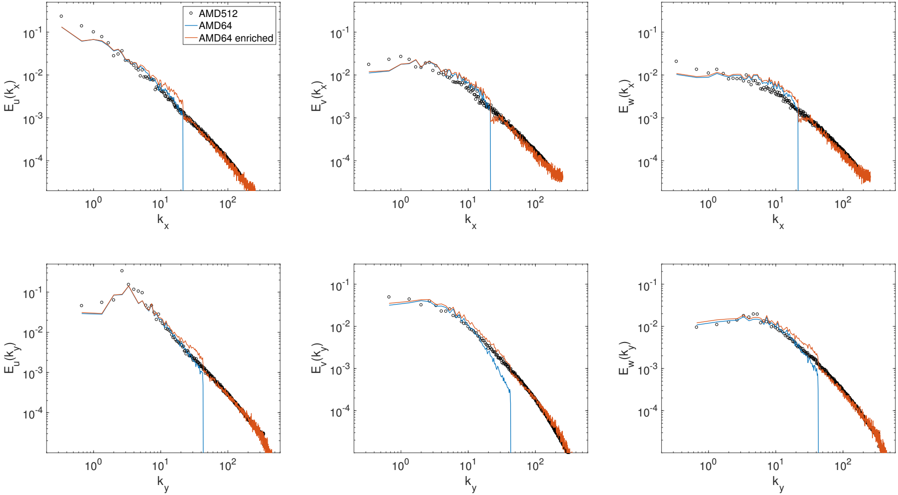
Half-channel

Anisotropy perspective: Two-point correlations (spectra) for AMD64 at **z=0.054H**



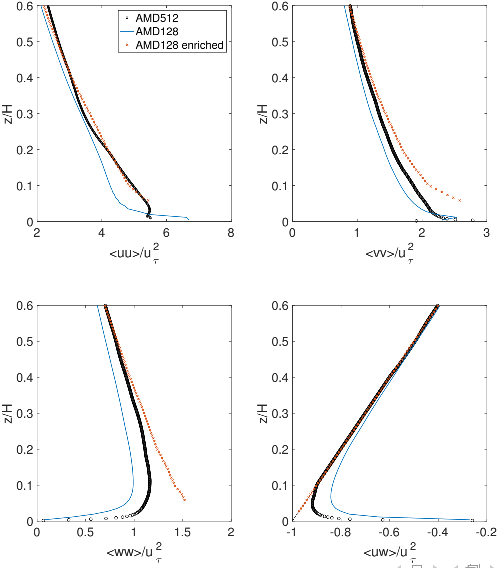
Half-channel

Anisotropy perspective: Two-point correlations (spectra) for AMD64 at **z=0.40H**



Half-channel

Revisit anisotropy: Using AMD128 (enrichment of $384 \times 384 \times 128$ simulation)

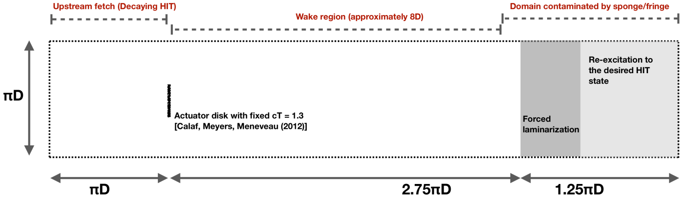


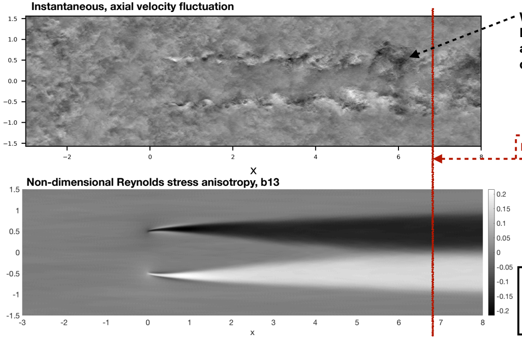
Summary: enrichment of very coarse wall-bounded simulations

- Eddy viscosity based closures such as AMD and Sigma models are good at turbulent kinetic energy and overall TKE dissipation rate; results in good enrichment beyond Nyquist limit.
- But, in $Re_\tau \to \infty$ limit with no near-wall grid-stretching, all 3 models severely over-predict Reynolds-stress anisotropy near the wall; some evidence of aliasing in the wall-normal component for AMD64 and SIGMA64
- Away from the wall at 64 resolution, none of the models predict correct energy in the $kx = 1, ky = 4$ superstructure
- Over-prediction of Reynolds stress anisotropy results in Gabor mode enrichment of overly isotropic subfilter scales, making matters worse in terms of normal stress anisotropy
- Successful Gabor mode enrichment requires careful prediction of both: a) physical anisotropy (shear induced), and b) geometric anisotropy (anisotropic filtering due to non-unity grid aspect ratios)
- The results shown are quite encouraging; models for SGS closure such as Modulated Gradient model (Lu & Porte-Agel, 2010), or deconvolution based closures like DRM (Chow, et al., 2004) might be more appropriate for enrichment at very coarse resolutions

Outline

HIT-Actuator disk interaction





Wake turbulence consists of Kelvin-Helmholtz wavepackets along with entrained and distorted isotropic turbulence

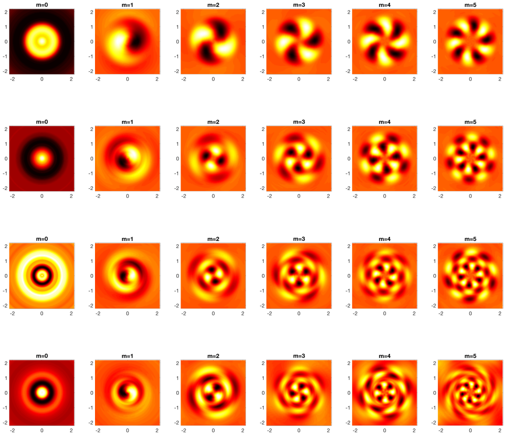
Inflow plane, sampled at x = 6.66D

Problem size: 800M grid cells
SGS closure: Sigma model
Numerics: Fourier collocation

HIT-Actuator disk interaction

A dictionary of data-driven or resolvent modes.

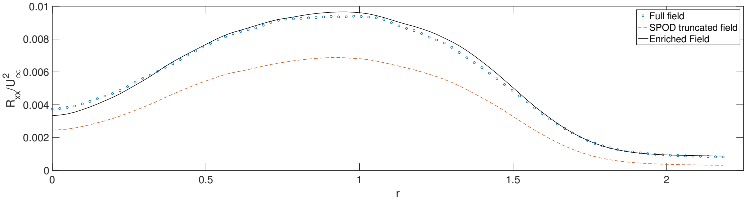
Mode shapes for axial velocity, u_x at Strouhal number, $f = 0.39$ (Mode number, j increasing downwards)



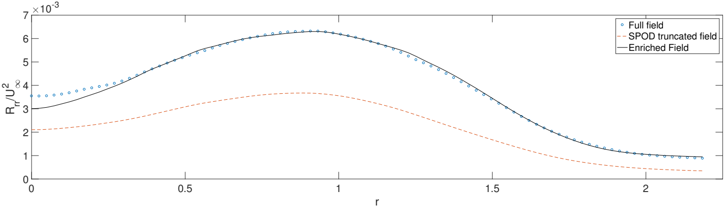
A lot of uncertainty in higher order modes ($m \gg 1, j \gg 1$) even at low Strouhal numbers. Very slow rate of convergence ($\sqrt{N_{\text{ensembles}}}$).

HIT-Actuator disk interaction

Comparison of single point correlations



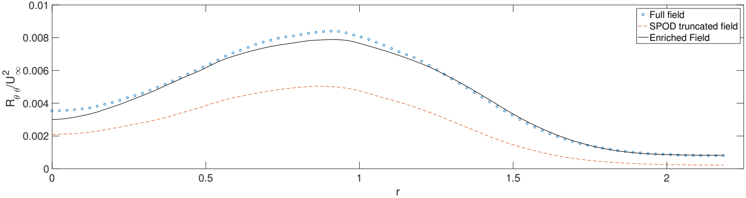
$\langle u_x u_x \rangle (r)$



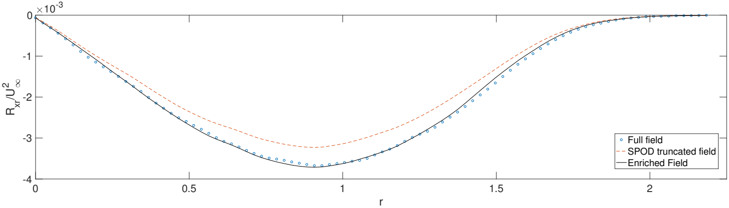
$\langle u_r u_r \rangle (r)$

HIT-Actuator disk interaction

Comparison of single point correlations.



$\langle u_\theta u_\theta \rangle (r)$

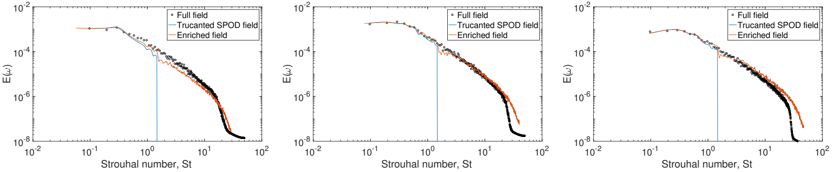


$\langle u_x u_r \rangle (r)$

HIT-Actuator disk interaction

We can also compare spectra in time.

Frequency spectrum for axial component, u_x



$r/D = 0.125$ $r/D = 0.50$ $r/D = 0.75$

Relevance of Gabor mode enrichment in traditional ROMs

- Problems involving wakes with turbulence co-flow/freestream do not have a low-rank character
- Higher order modes correspond to free stream turbulence distorted by the wake which entrains it
- Lower order modes can be constructed using lower resolution simulations and converge with fewer ensembles
- Possible to construct an arbitrary number of inflow realizations using truncated POD representation enriched with Gabor modes
- Excellent potential for use in mid-fidelity modeling tools for fatigue and uncertainty quantification

Conclusions

1. A new **physics based** method to enrich low resolution (incompressible flow) LES was developed using spatially and spectrally localized *Gabor modes*. The enriched turbulence:
    - Provides spectral extrapolation (consistent two-point correlations)
    - Captures correct inter-scale energy transfer from large scales
    - Can accommodate spatially inhomogeneous turbulence
    - Has correct temporal decorrelation
    - Is divergence free
    - Can satisfy simple boundary conditions: No-penetration/kinematic blocking, periodicity, etc.
2. The method was tested for homogeneous turbulence and wall-bounded flows with density stratification and frame rotation
3. The computational cost of the method is very promising
    - Temporal evolution is entirely governed by a set of ODEs
    - Transform to real-space can be computed efficiently using non-uniform FFTs (NUFFTs)
    - Operationally very localized, low communication and high arithmetic intensity (convolutions)
4. Bottom line: Successful enrichment requires good quality LES; enrichment does not improve a bad LES; better the LES, better the enrichment

Questions?

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Contact information:

Aditya Ghate
aditya.s.ghate@nasa.gov