Prediction of Ionospheric Activity from the AL Index Time Series Using the Ensemble Transform Kalman Filter

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Outline

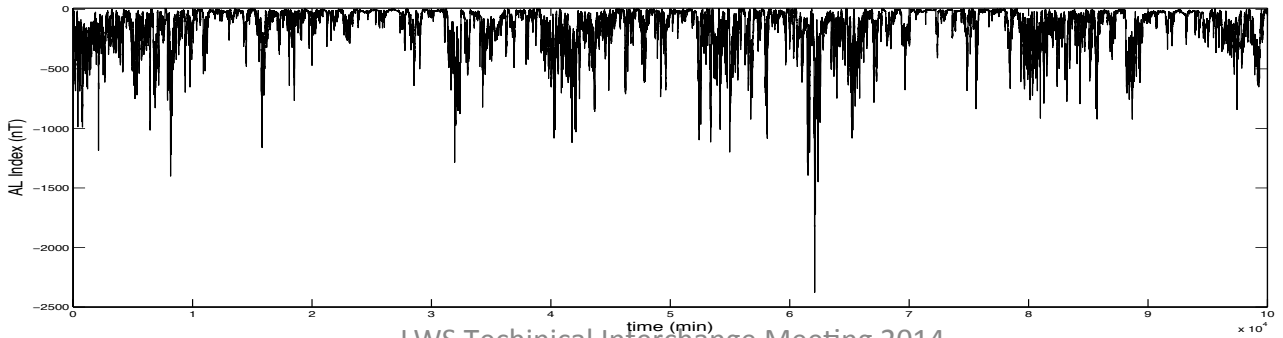
- Motivation
- Methodology
  - Phase Space Reconstruction
  - Singular Spectrum Analysis
  - Ensemble Transform Kalman Filter
- Results of AL Forecasting
- Concluding Remarks

Motivation

- Many natural systems exhibit nonlinear behavior:
  - In some cases, underlying dynamics may be well understood
    - Systems can be modeled as dynamical systems
    - For systems that can be modeled numerically and observed, applying data assimilation techniques to improve forecast has yielded great success
  - Some systems lack models and observational data
    - Only available information may time series single state variable
    - Many degrees of freedom that cannot be simultaneously measured
  - Time series may exhibit low-dimensional dynamics, i.e., attractor

Motivation

- Magnetosphere is a low-dimensional dynamical system
- Substorms are a magnetosperic response to solar wind driver
- Magnetic field in the auroral region is measured in several locations
- Indices constructed the envelope of the horizontal component resulting in scalar time series
- AL index is a measure of substorm dynamics



Objective

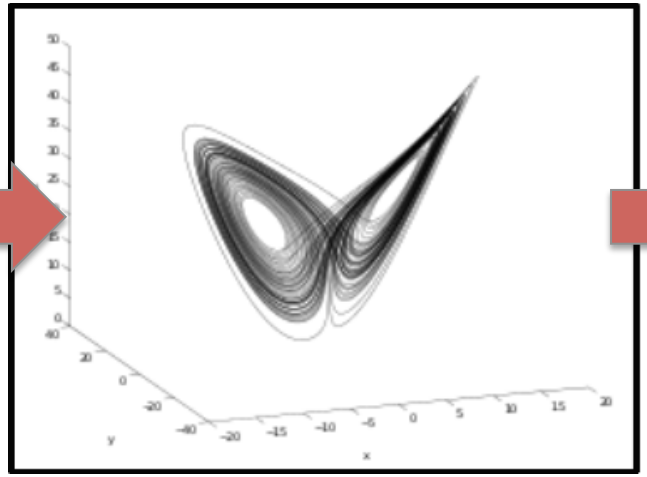
- Develop techniques for prediction based on limited data
  - Approach: Integration of
    - Phase space reconstruction
    - Singular Spectrum Analysis
    - Ensemble Kalman Filter (EnKF)
  - Apply this technique to the AL index to predict future values

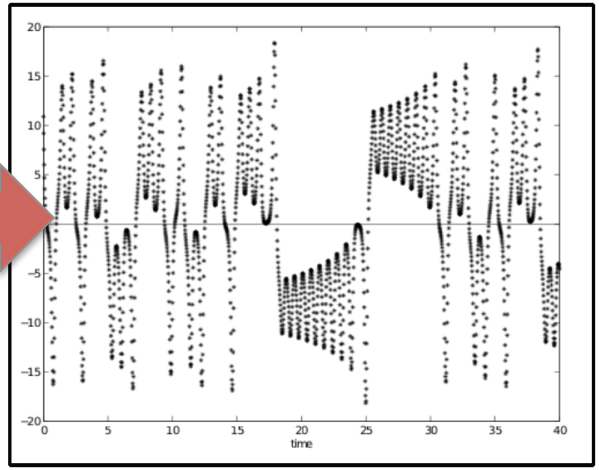
Test Case: Lorenz Attractor

$$\frac{dx}{dt} = \sigma(x - y) \qquad \sigma = 10$$

$$\frac{dy}{dt} = x(r - z) - y \qquad r = 28$$

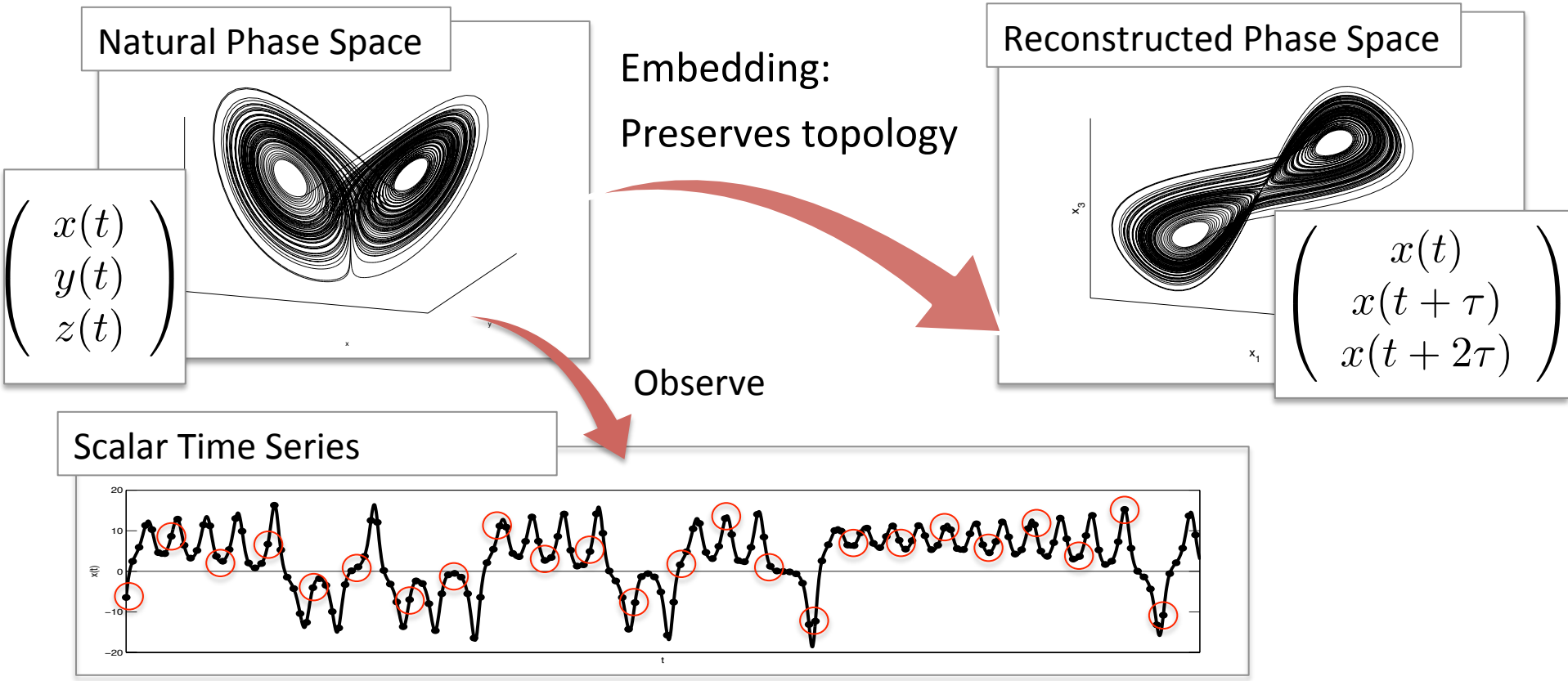
$$\frac{dz}{dt} = xy - bz \qquad b = 8/3$$





- Generate points on the Lorenz attractor – Only available information is a time series of the x coordinate at discrete times t
- Use this dataset to serve as a model with no error, mimicking measurements of a high accuracy instrument

Phase Space Reconstruction

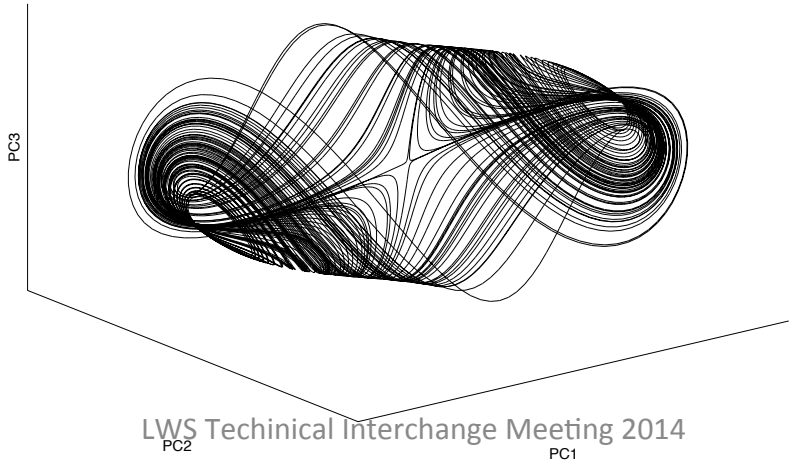


Time Delay Coordinates

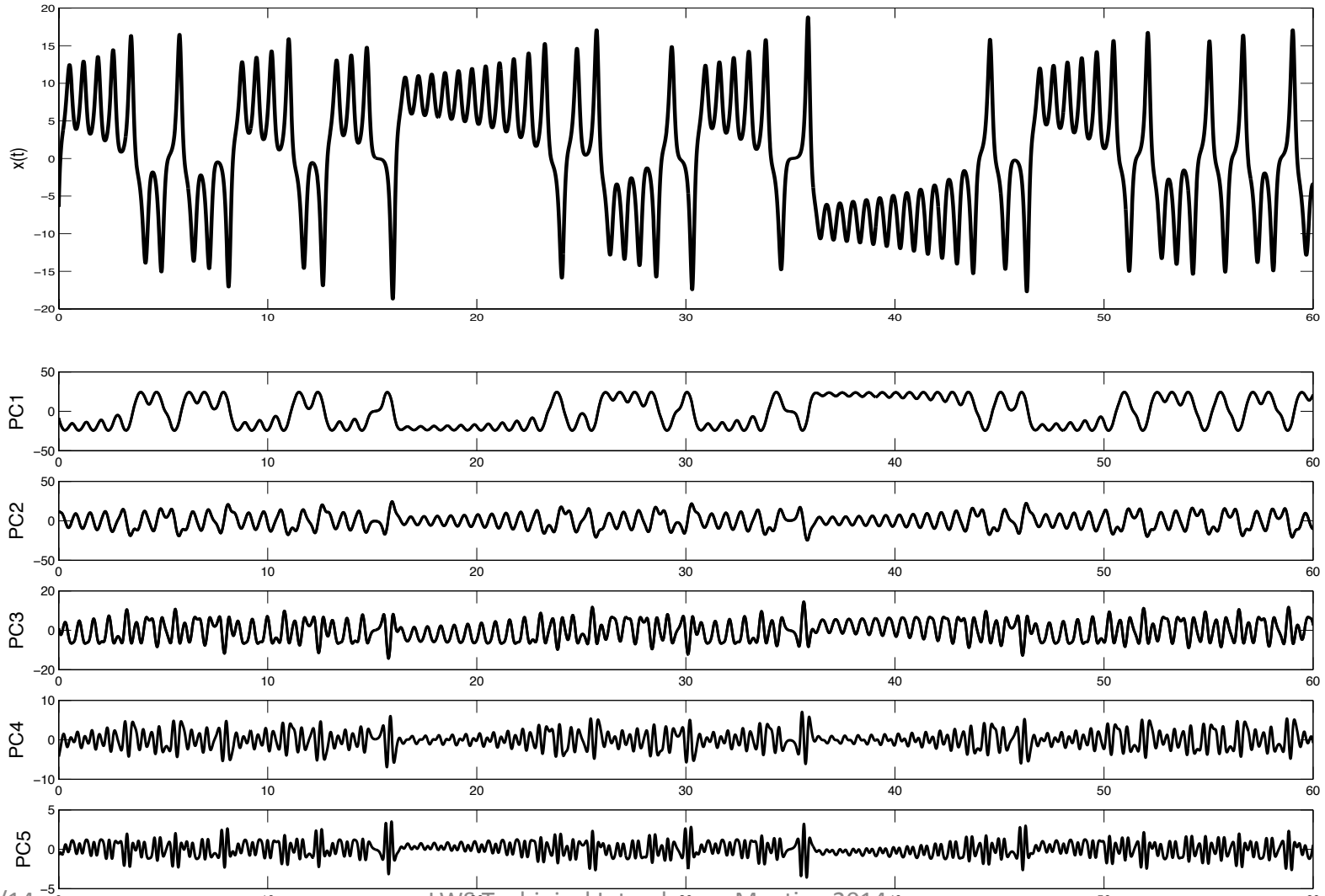
- Construct multivariate state vectors from scalar time series by sampling series at regular time intervals

Singular Spectrum Analysis (SSA)

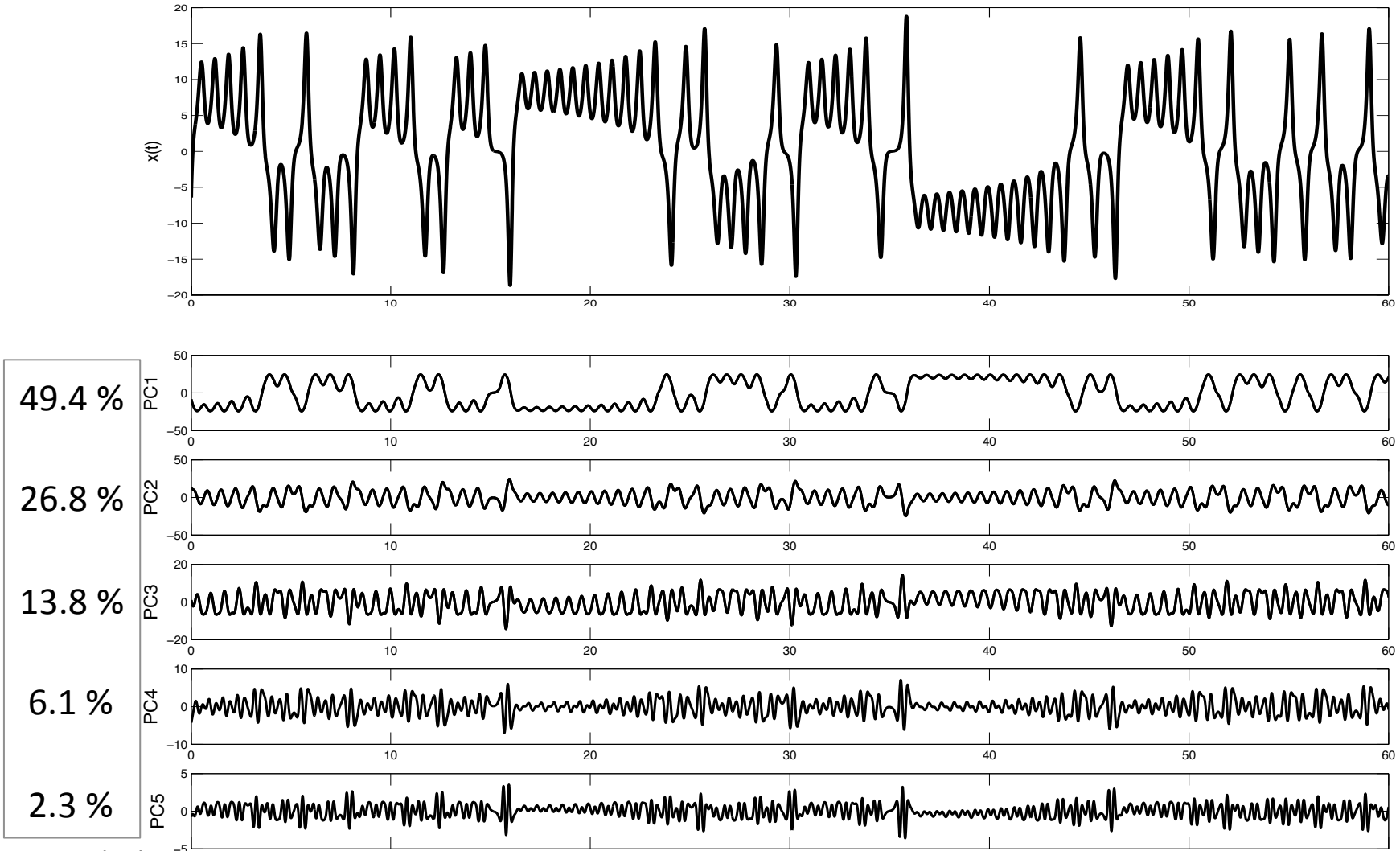
- Identify modes that explain most variability in the time domain
- Estimate an upper bound of the number of degrees of freedom necessary to represent dynamics
- Construct and orthogonal basis to represent dynamics



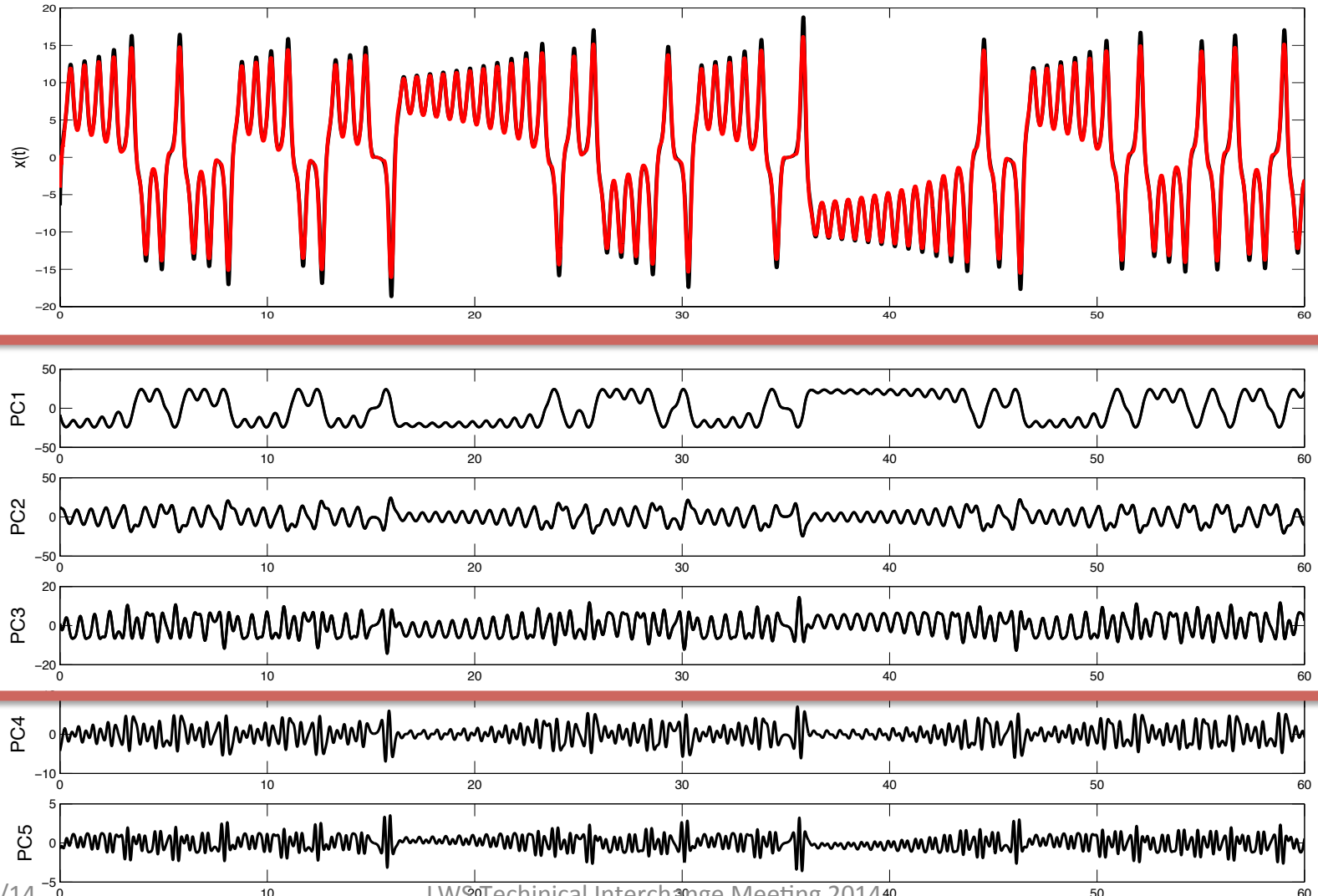
Principal Components



Principal Components

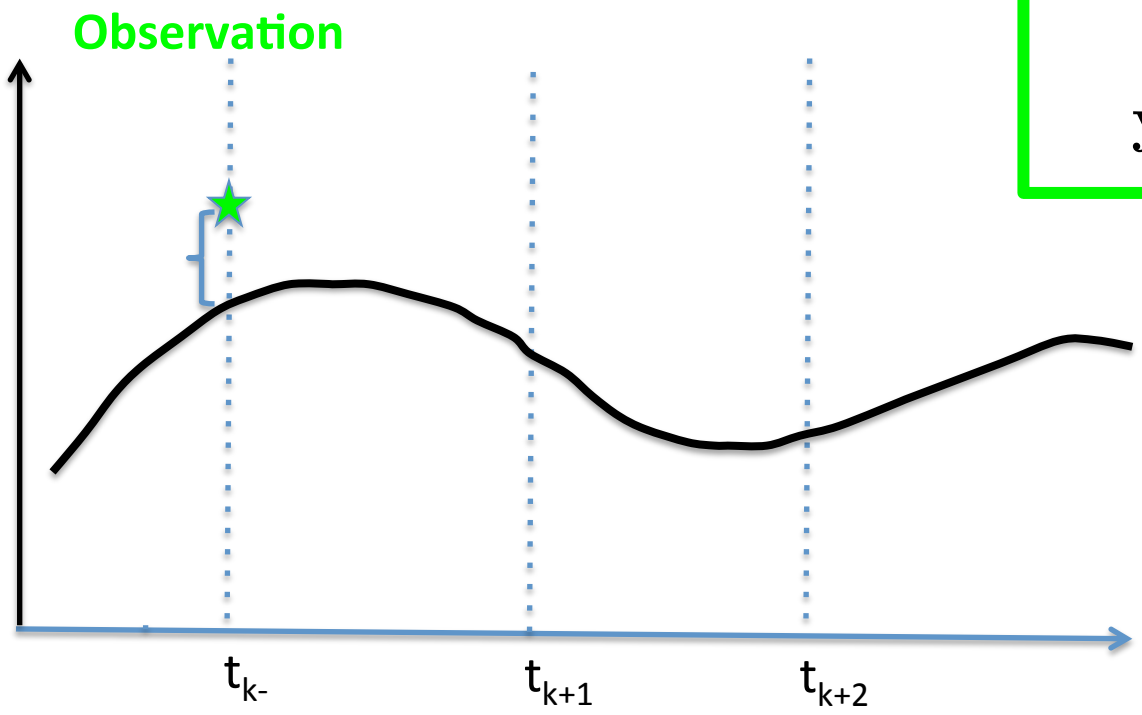


Reconstruction from Principal Components



Data Assimilation: Kalman Filter

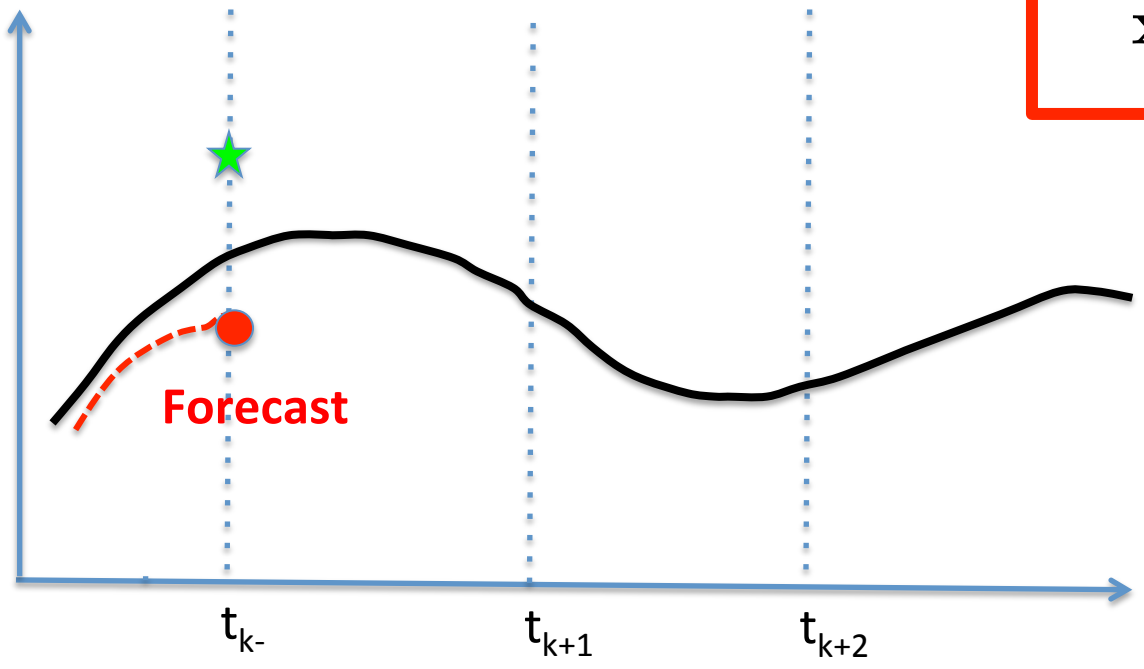
- Forecasts of nonlinear systems have strong dependence on initial conditions
- Observations contain errors and as do forecasts based on models
- Combining observations and previous forecasts yields a better estimate of true state of system



Observation

$$\mathbf{y}_k^o = \mathbf{H}\mathbf{x}_k^t + \epsilon_k^{ot}$$

Data Assimilation: Kalman Filter



Model Forecast

$$\mathbf{x}_k^b = m(\mathbf{x}_{k-1}^a)$$

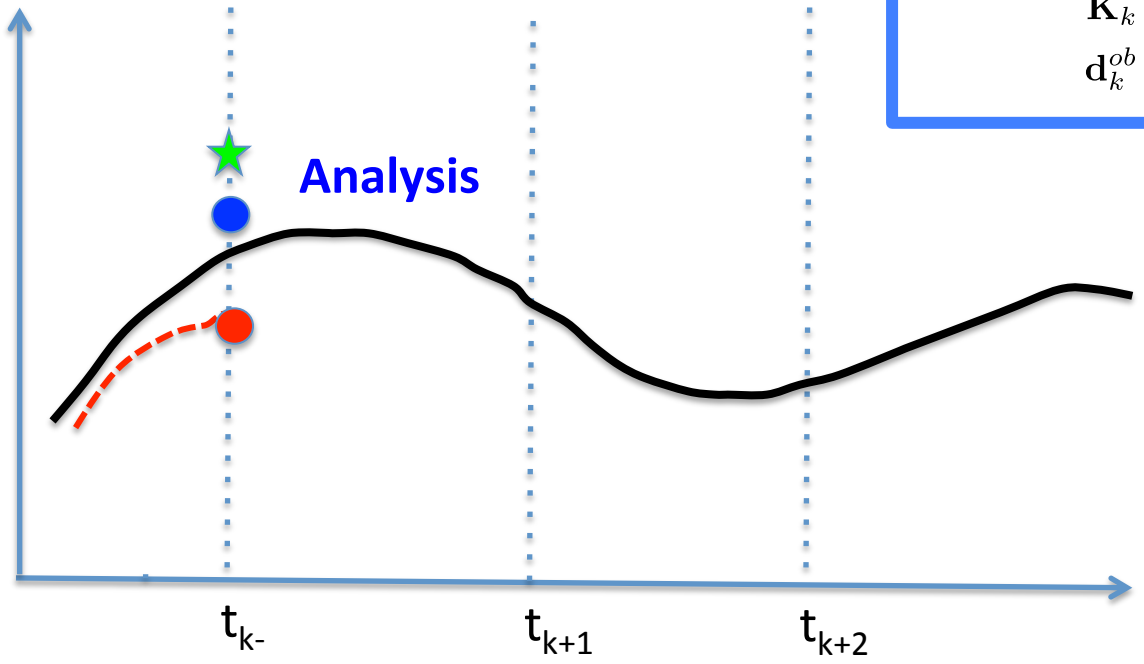
Data Assimilation: Kalman Filter

Analysis

$$\mathbf{x}_k^a = \mathbf{x}_k^b + \mathbf{K}_k \mathbf{d}_k^{ob}$$

$$\mathbf{K}_k = \mathbf{P}_k^b (\mathbf{H}\mathbf{P}_k^b \mathbf{H}^T + \mathbf{R}^o)^{-1}$$

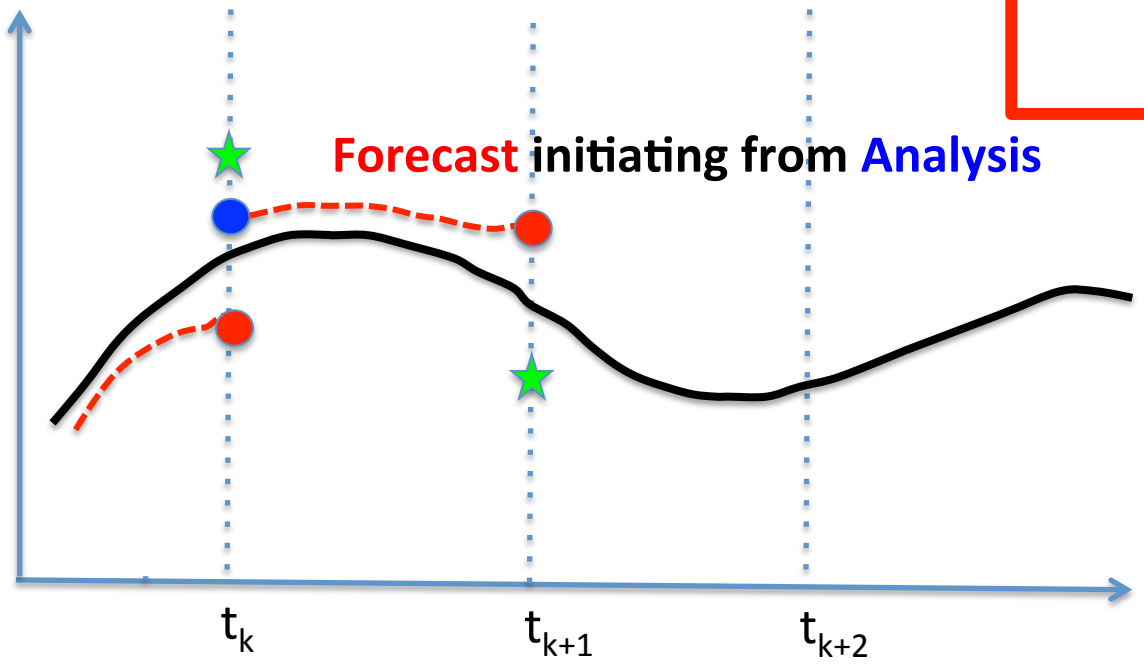
$$\mathbf{d}_k^{ob} = \mathbf{y}_k^o - \mathbf{H}\mathbf{x}_k^b$$



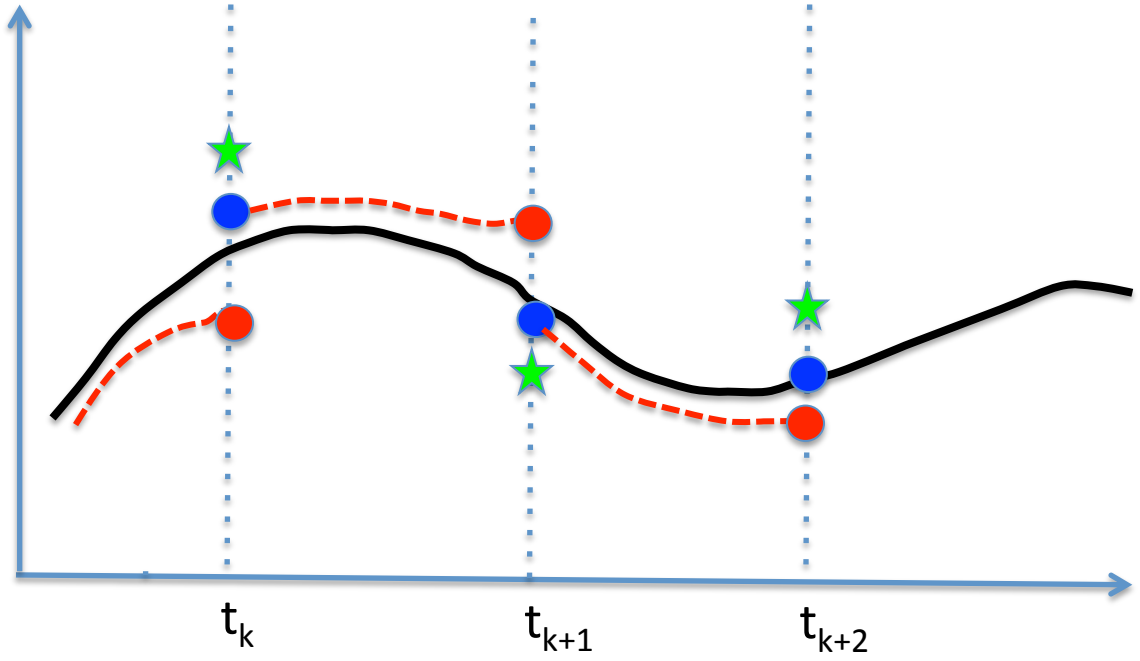
Data Assimilation: Kalman Filter

Model Forecast

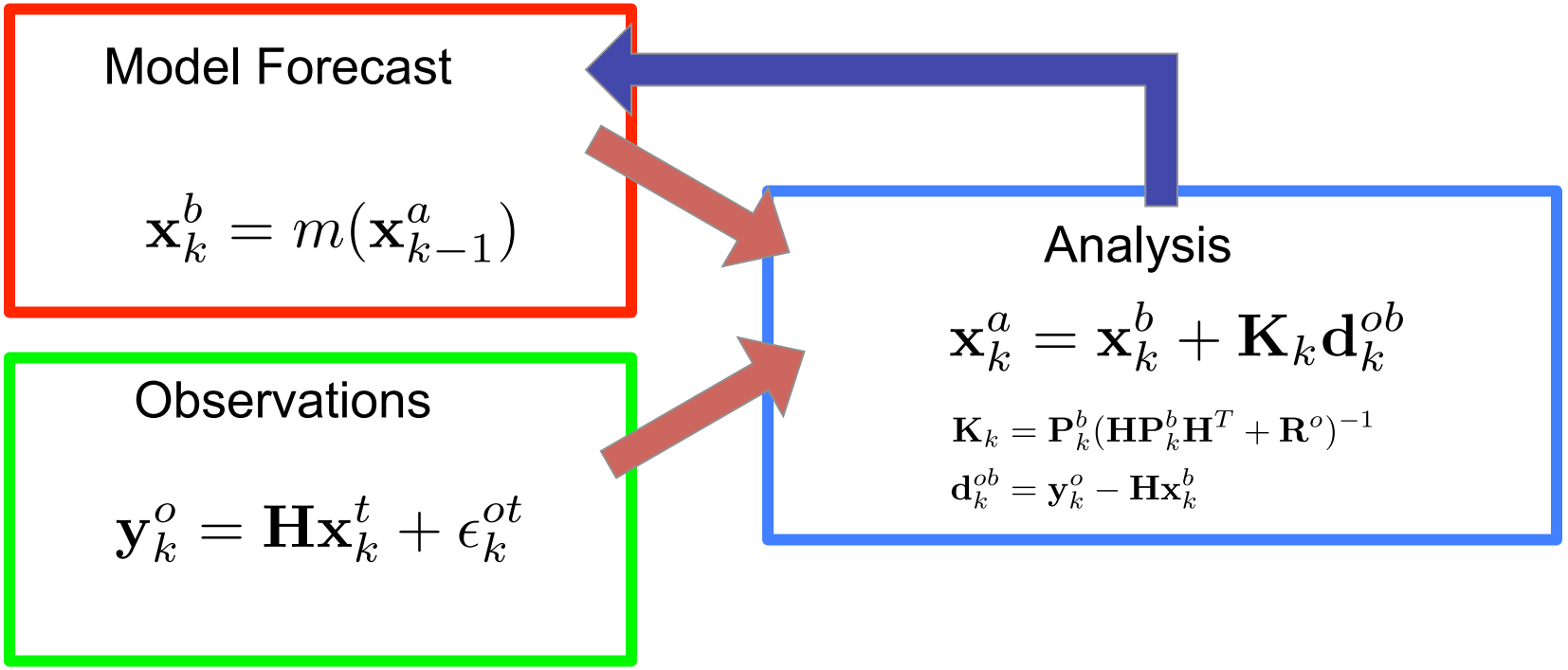
$$\mathbf{x}_{k+1}^{b} = m(\mathbf{x}_{k}^{a})$$



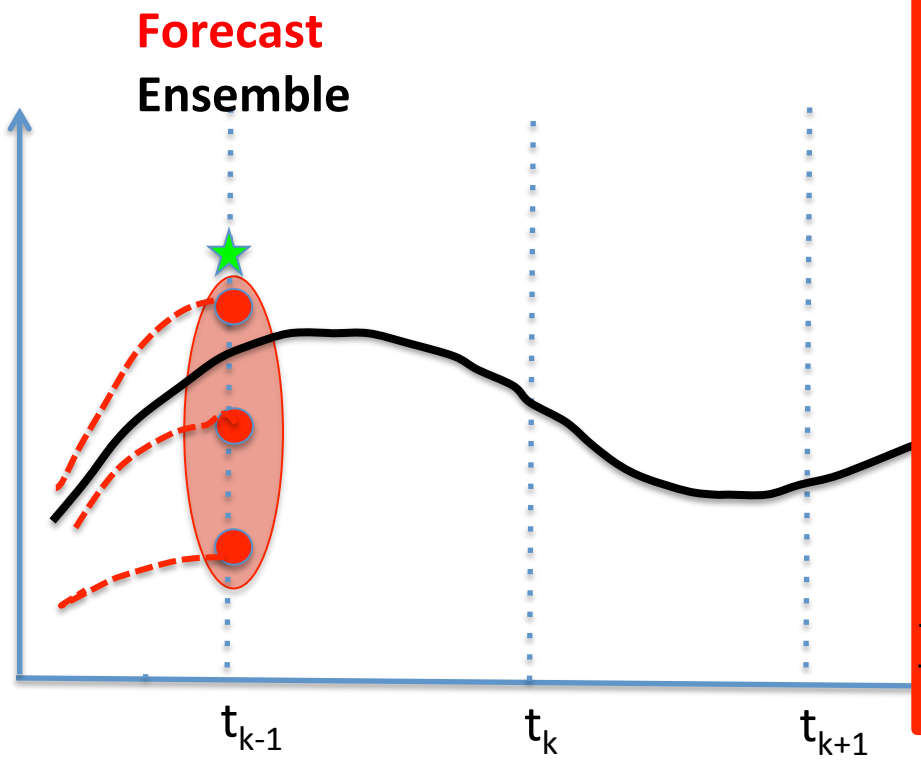
Data Assimilation: Kalman Filter



Data Assimilation: Kalman Filter



Data Assimilation: Ensemble Transform Kalman Filter



Forecast Ensemble

$$\mathbf{X}^b = \{\mathbf{x}_1^b, \ldots, \mathbf{x}_M^b\}$$

Ensemble mean

$$\bar{\mathbf{x}}^b = M^{-1} \sum_{i=1}^{M} \mathbf{x}_i^b$$

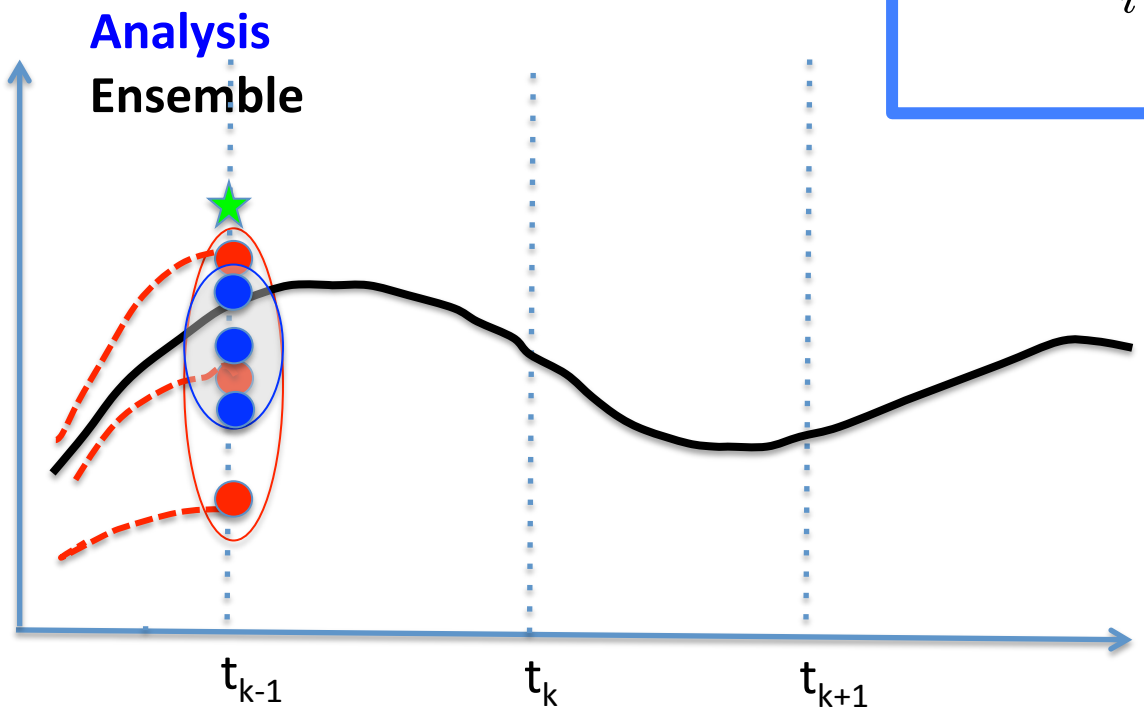
Ensemble spread

$$\hat{\mathbf{X}}^b = \{\mathbf{x}_1^b - \bar{\mathbf{x}}^b, \ldots, \mathbf{x}_M^b - \bar{\mathbf{x}}^b\}$$

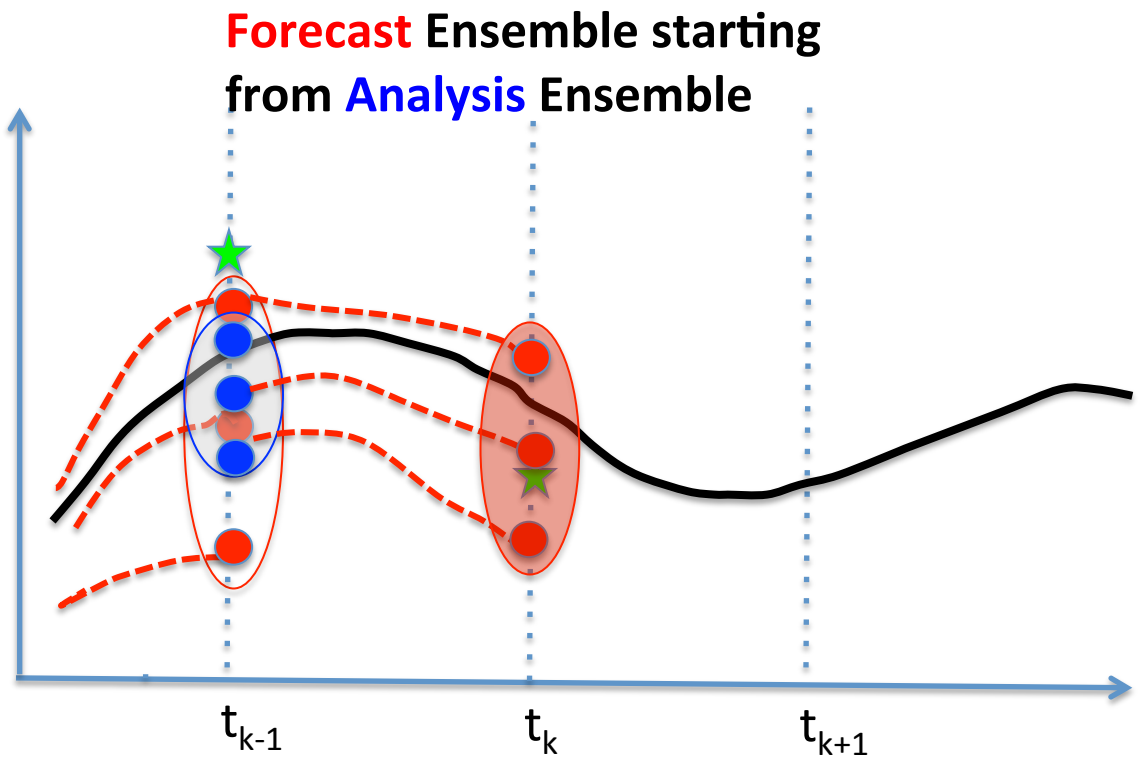
Data Assimilation: ETKF

Analysis Ensemble

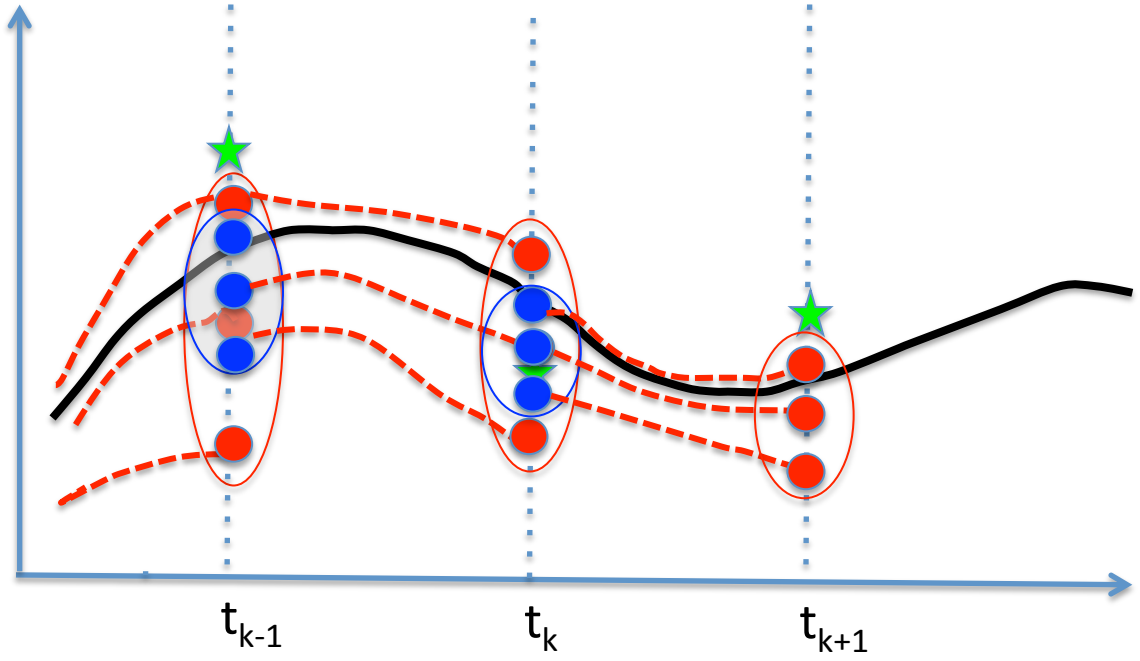
$$\mathbf{x}_i^a = \bar{\mathbf{x}}^b + \hat{\mathbf{X}}^b \mathbf{w}_i$$



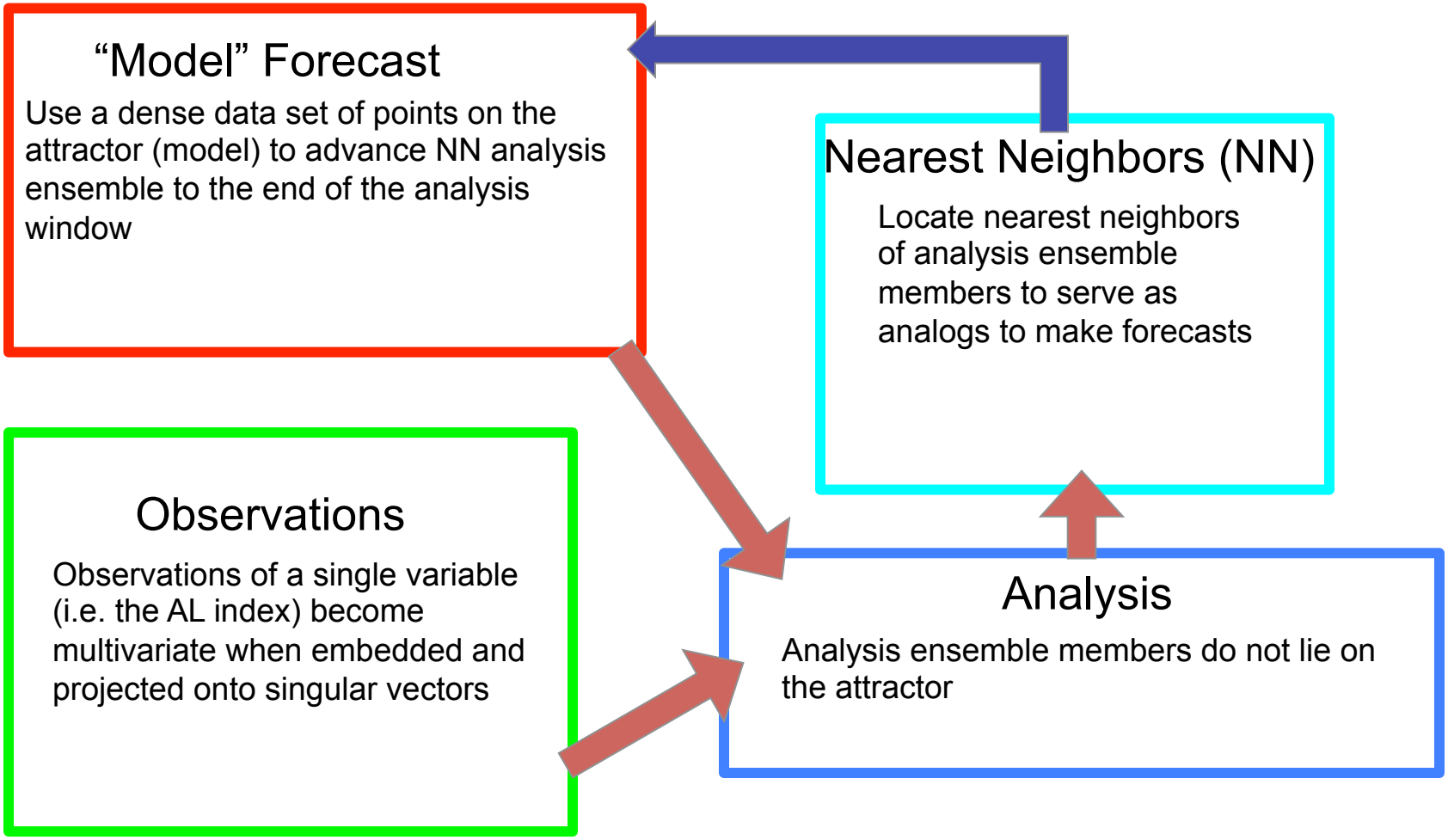
Data Assimilation: ETKF



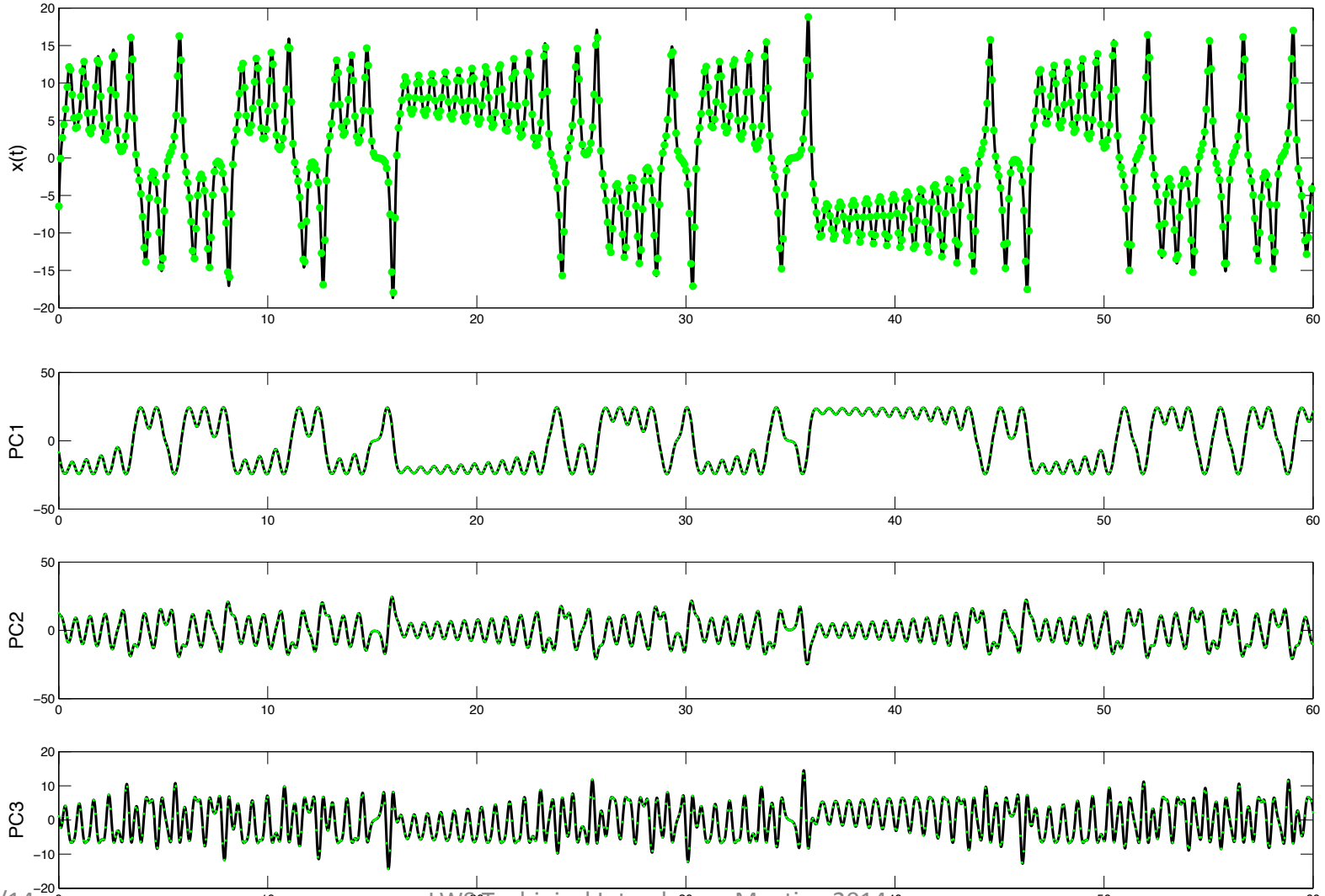
Data Assimilation: ETKF



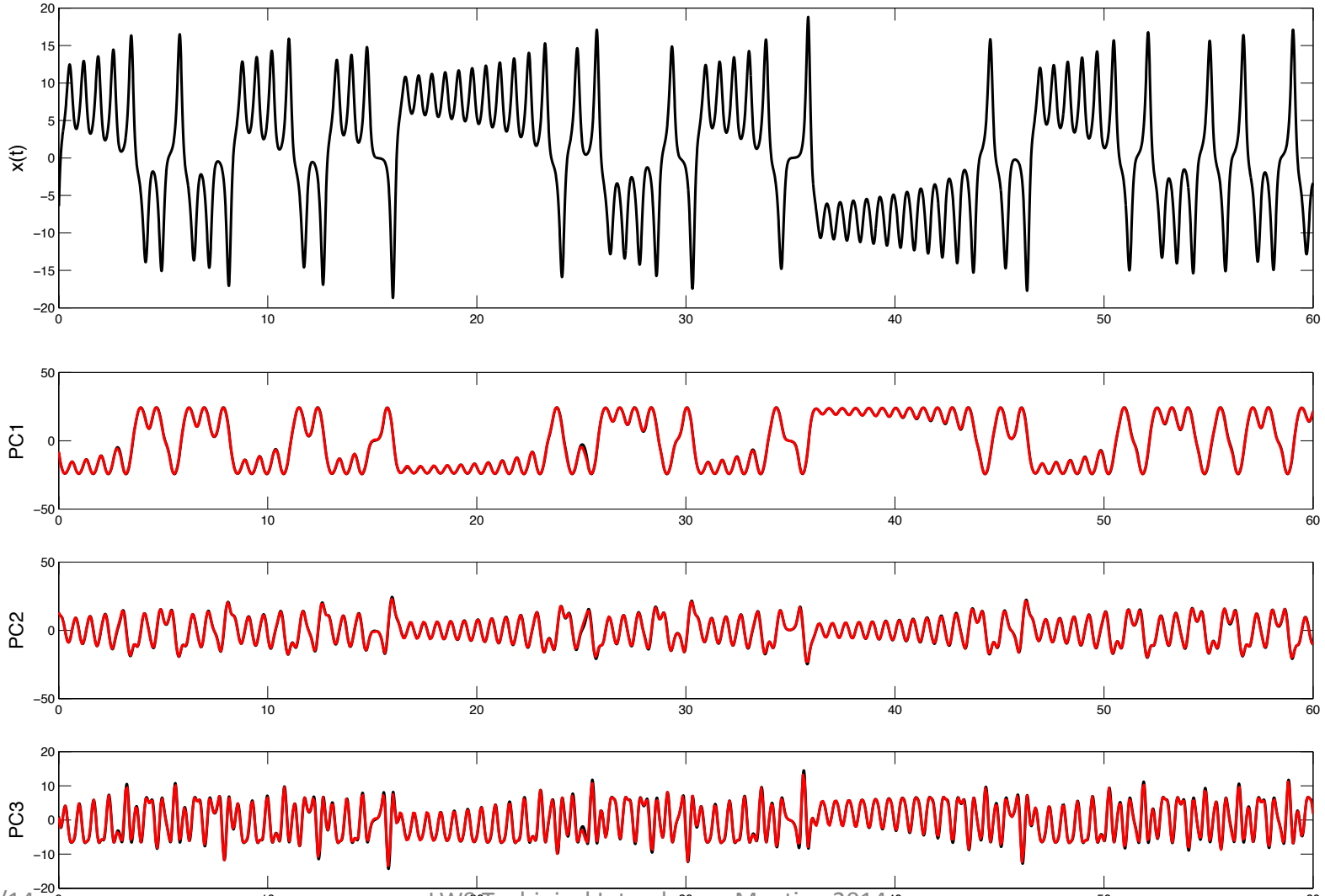
ETKF in Reconstructed Space



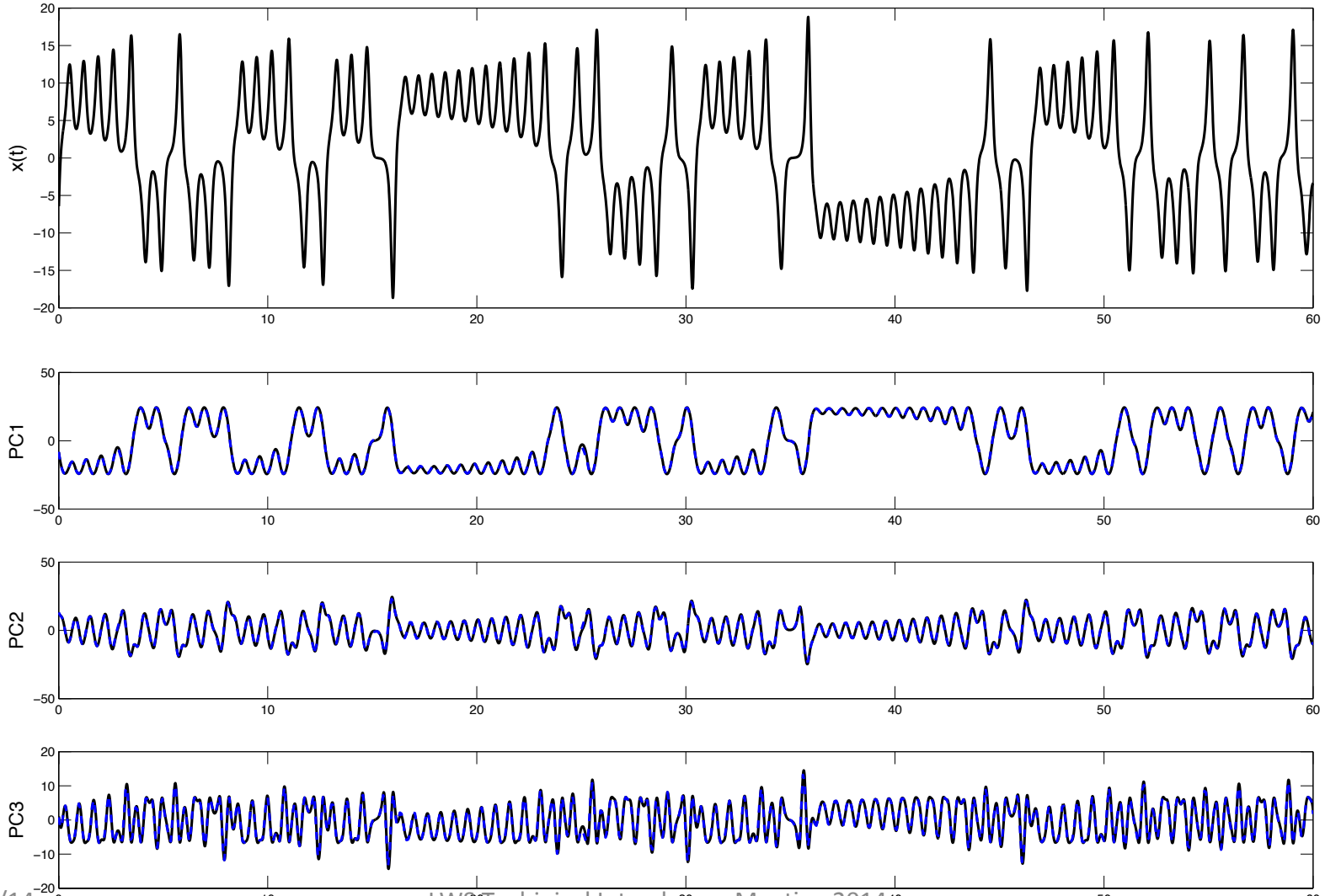
Observations of the Principal Components



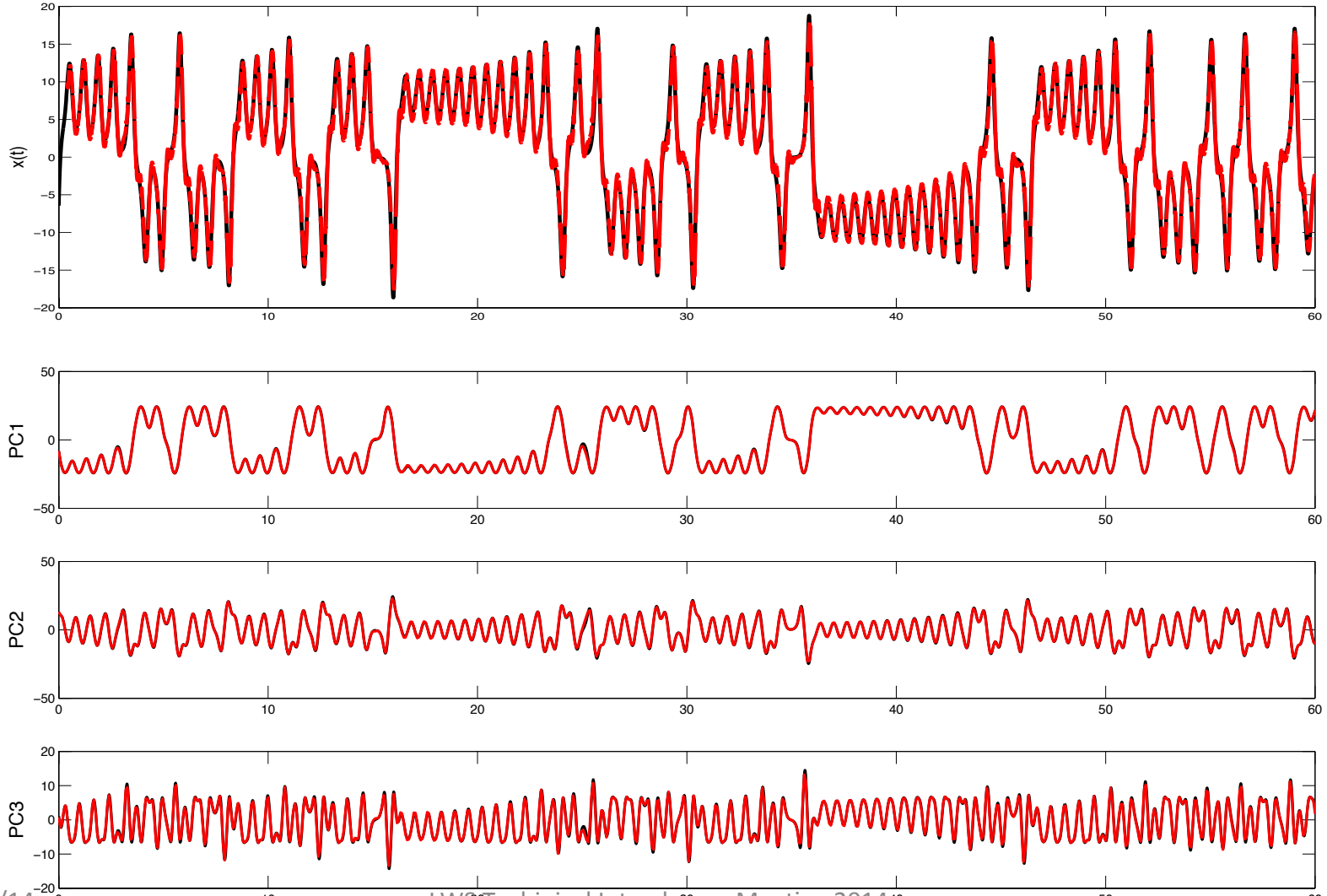
Forecasts of the Principal Components



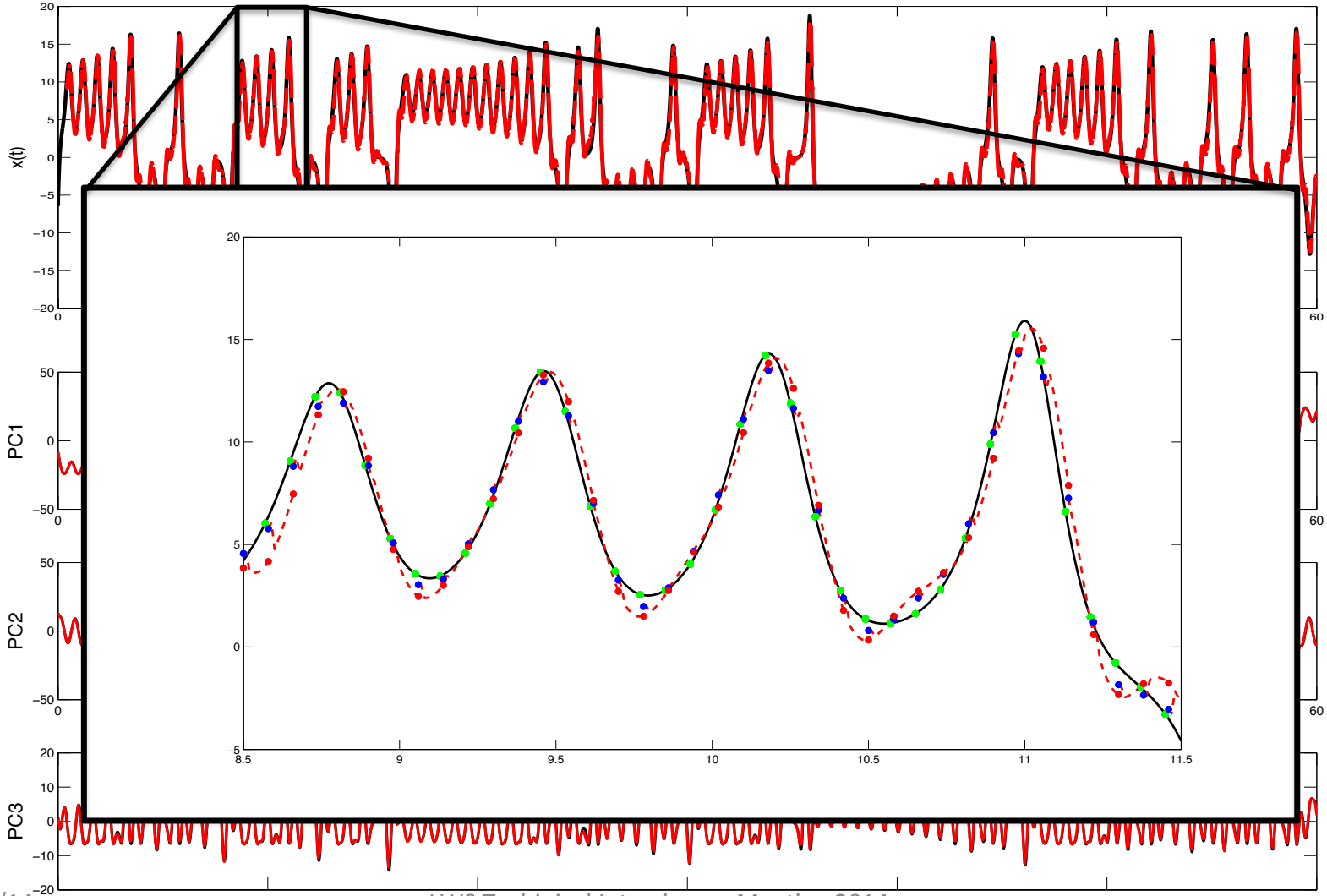
Analysis of the Principal Components



Reconstruction of the Lorenz x(t) from Forecasts

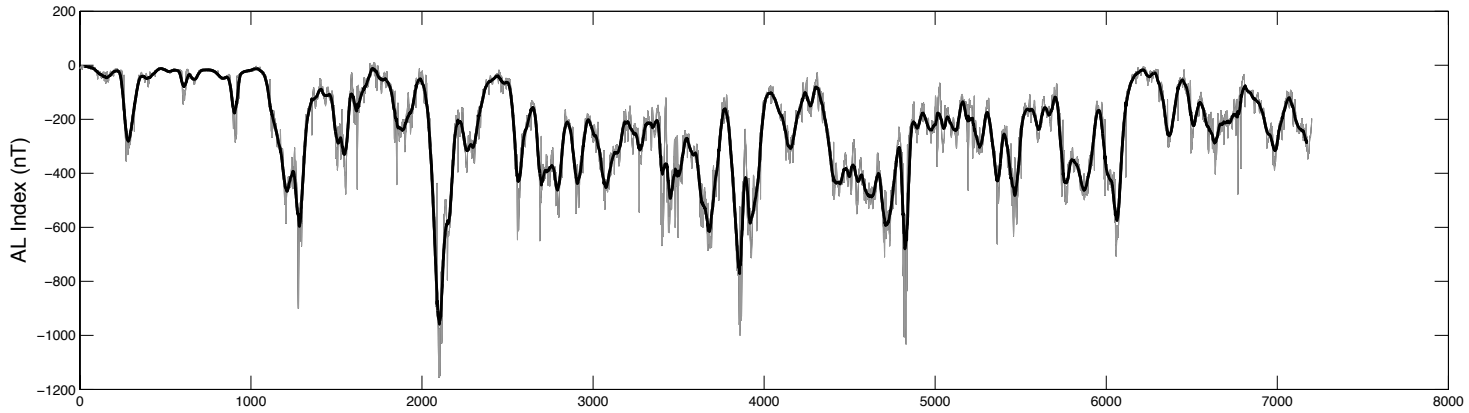


Reconstruction of the Lorenz x(t) from Forecasts

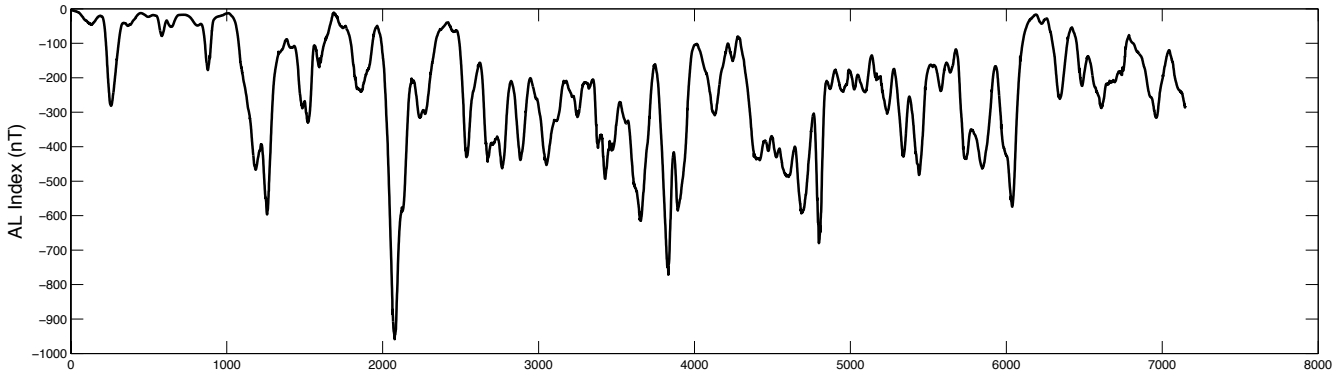


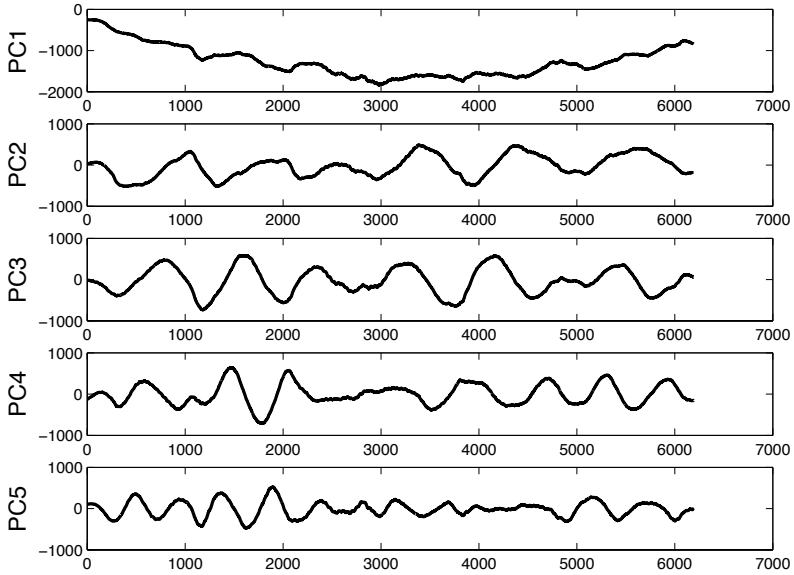
Application to AL Index

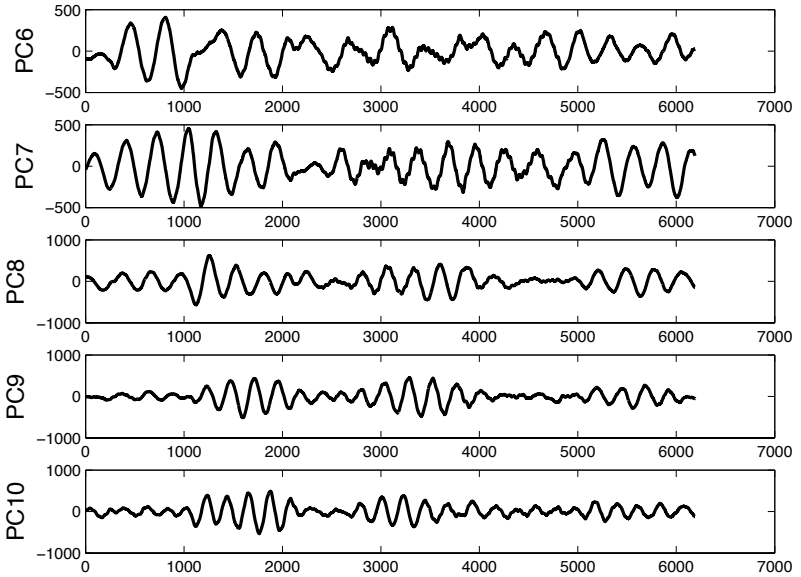
- First the Index is filtered using a low pass Lanczos filter to remove high frequency noise
- Time delay of 40 minutes is used
- For a good reconstruction, 10 out of 25 principal components are necessary
- Model data: one year of one minute resolution data from 2000
  - Singular vectors obtained from model data
- Truth data: High speed stream event on April 29-May 2, 2011
  - Principal components obtained by projecting time delay vectors onto model singular vectors



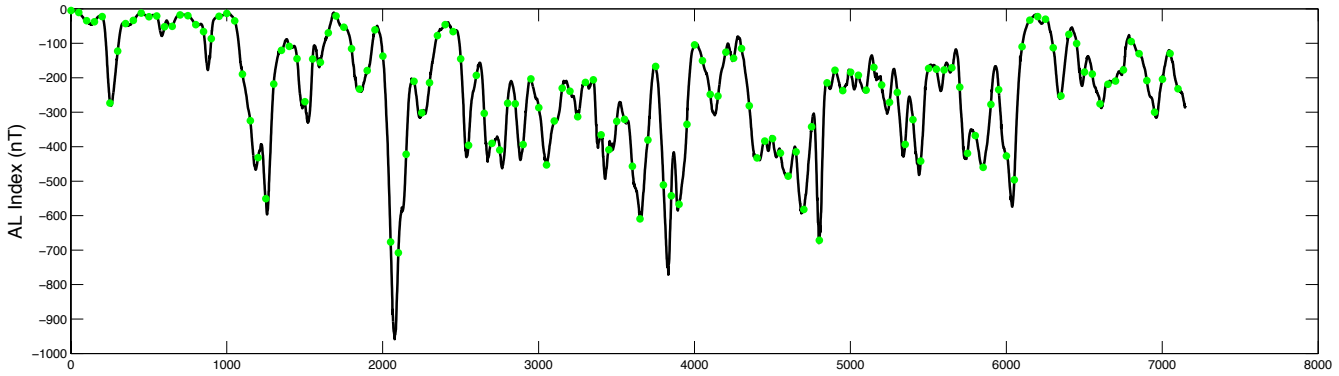
Principal Components of the AL Index

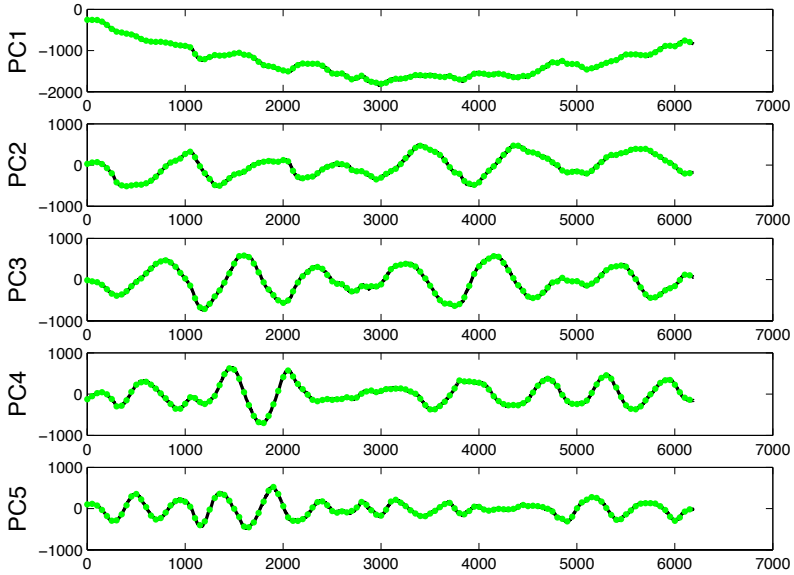


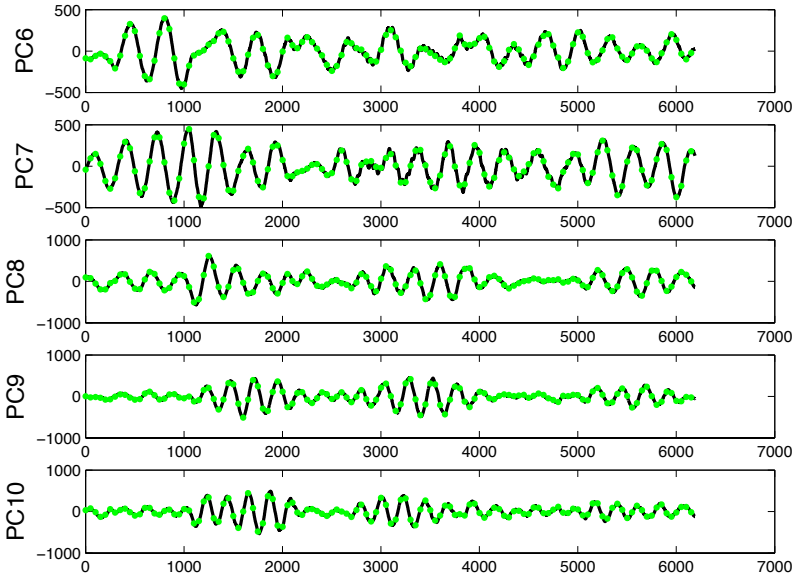




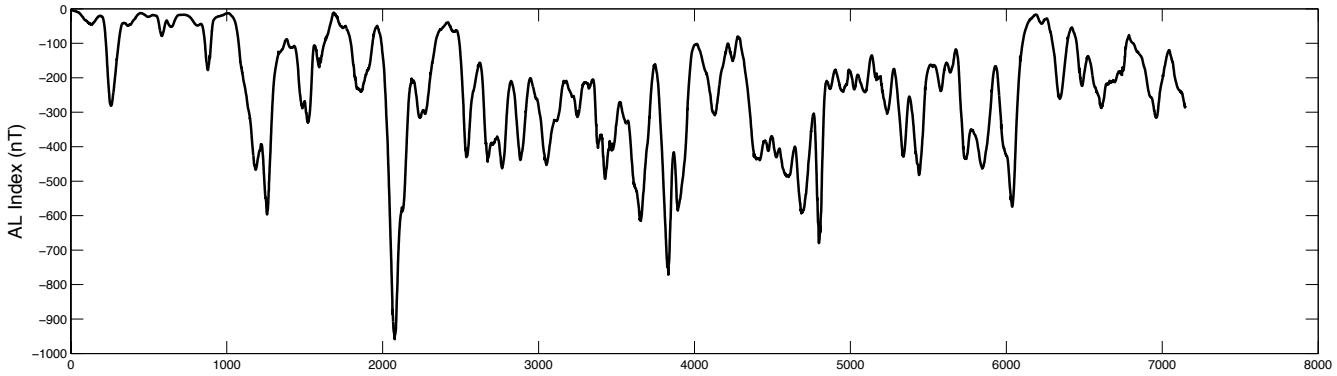
Observations of the Principal Components

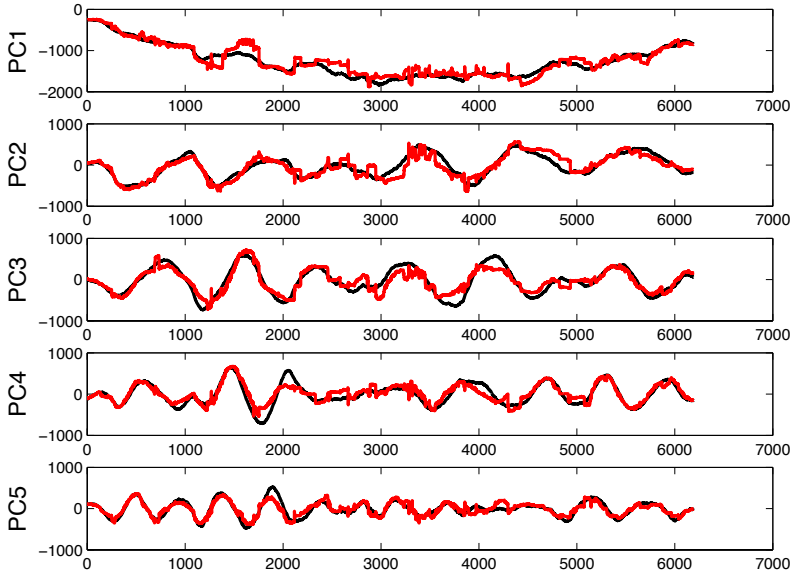


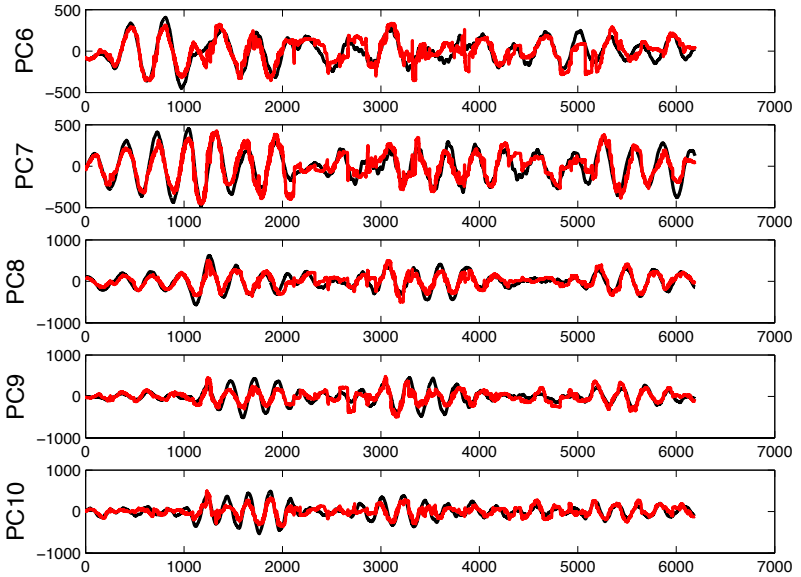




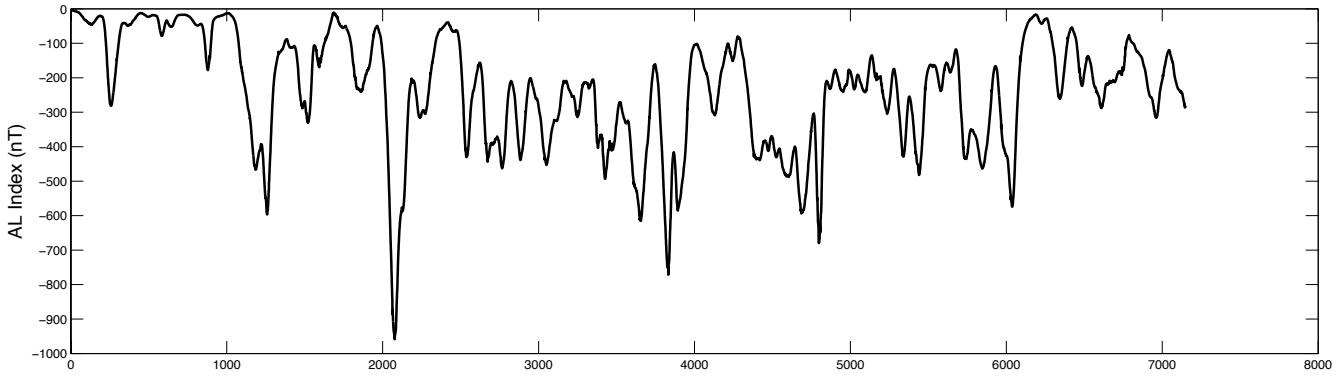
1 Minute Forecasts of the Principal Components

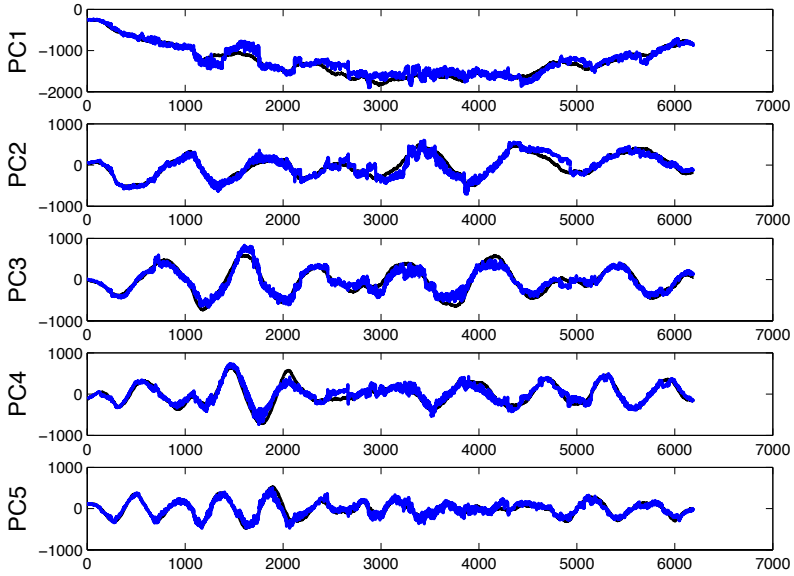


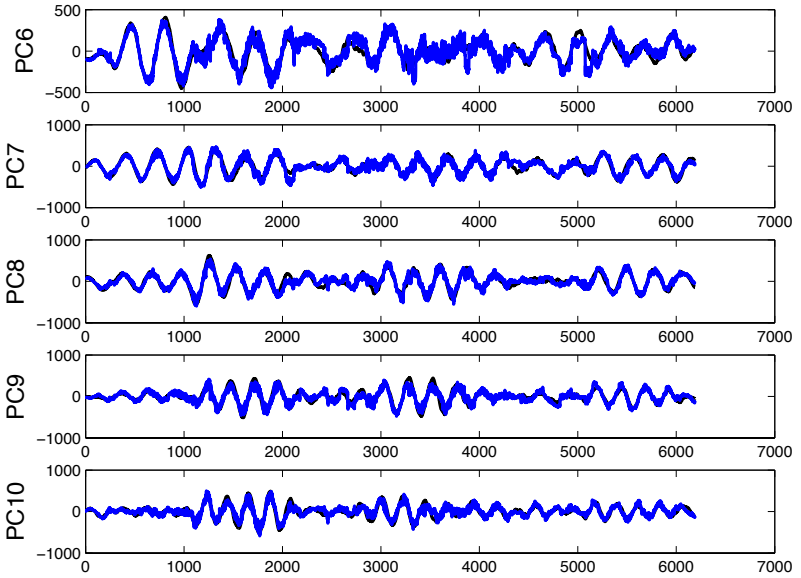




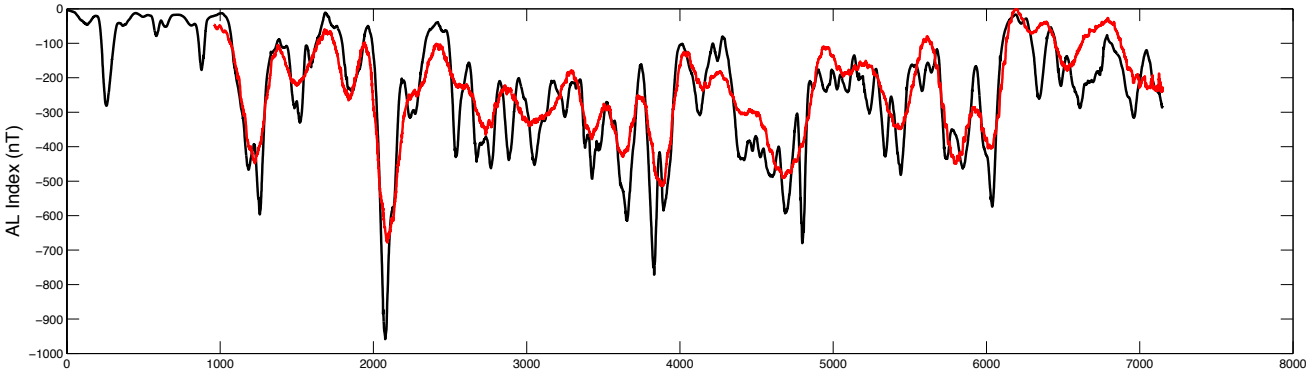
Analysis of the Principal Components

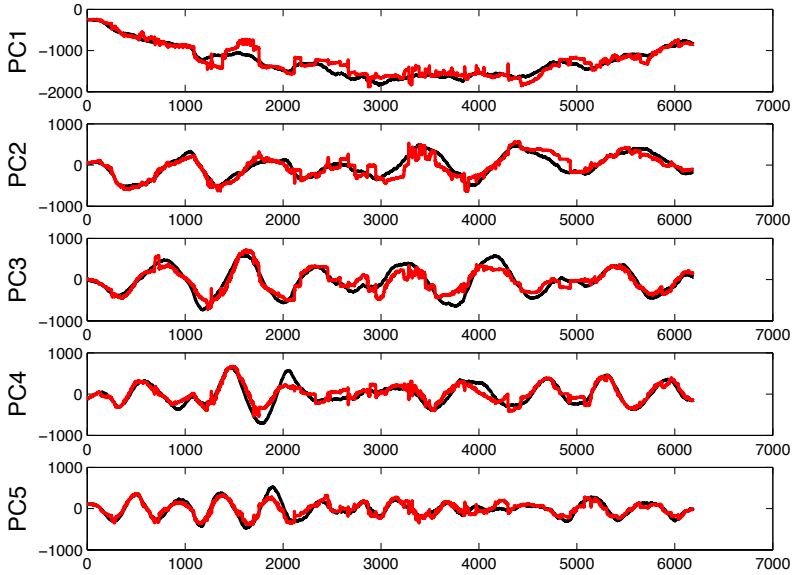


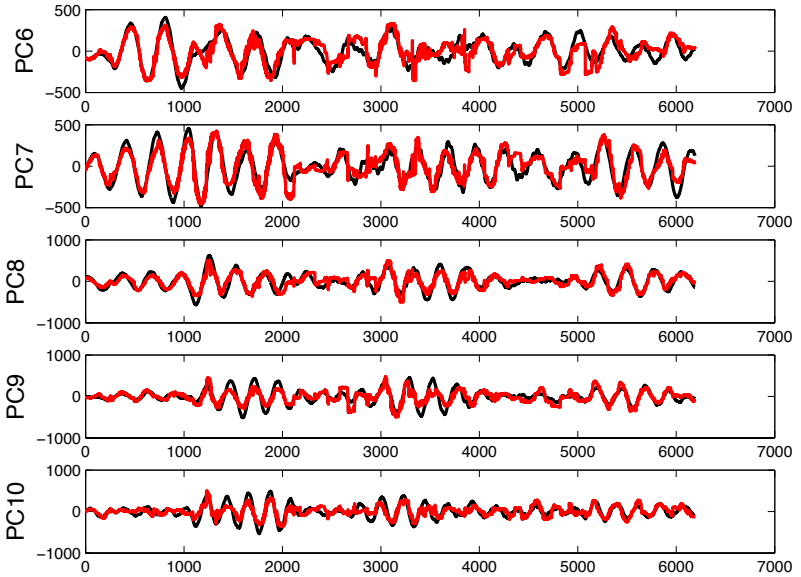


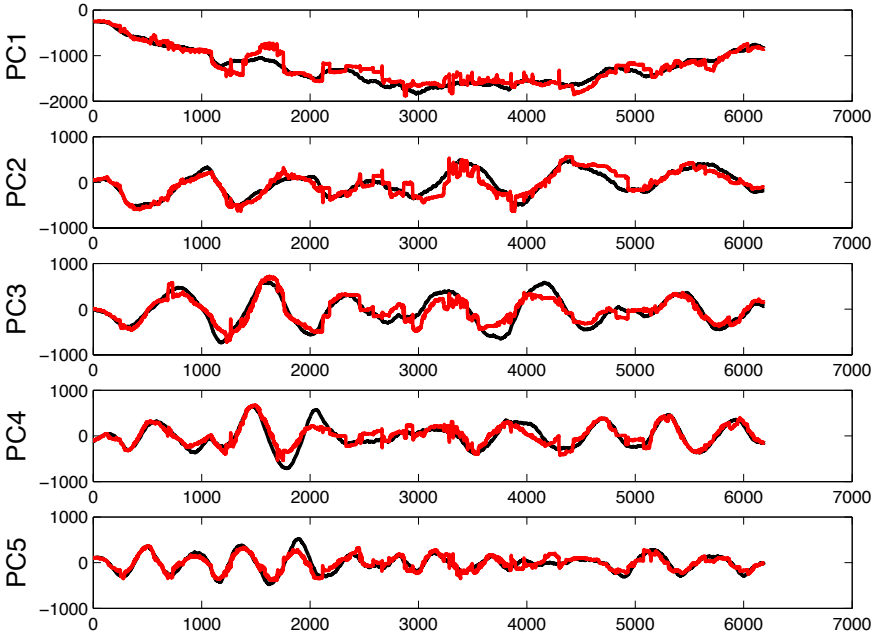


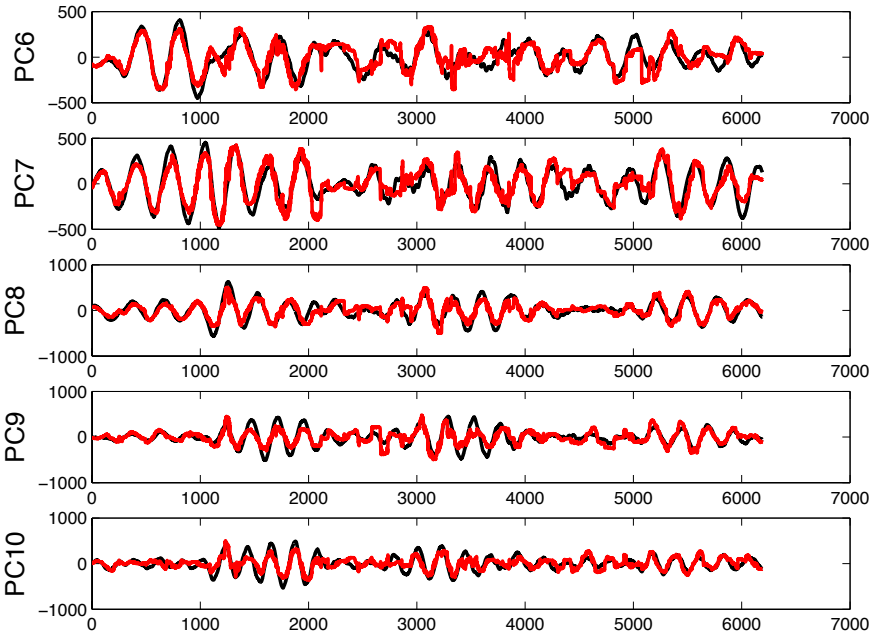
Reconstruction of the AL Index from Forecasts

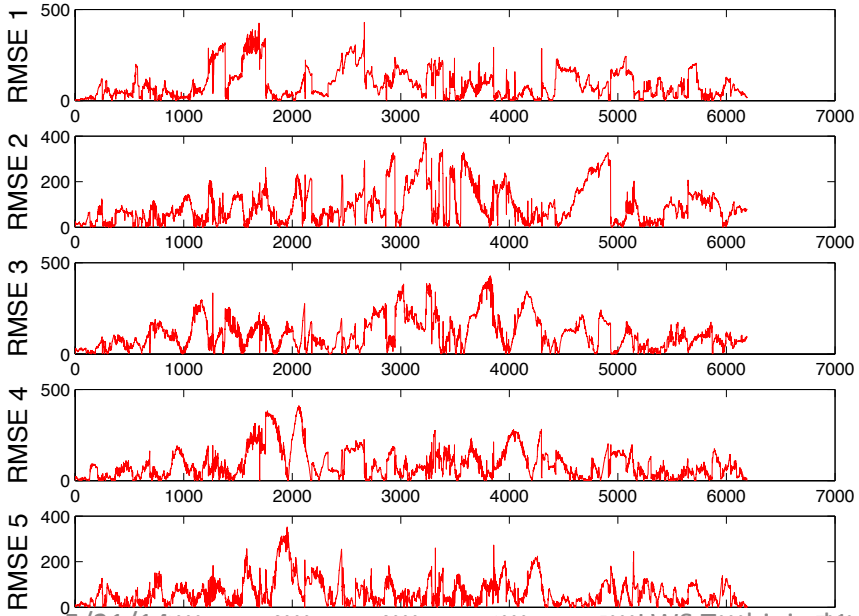


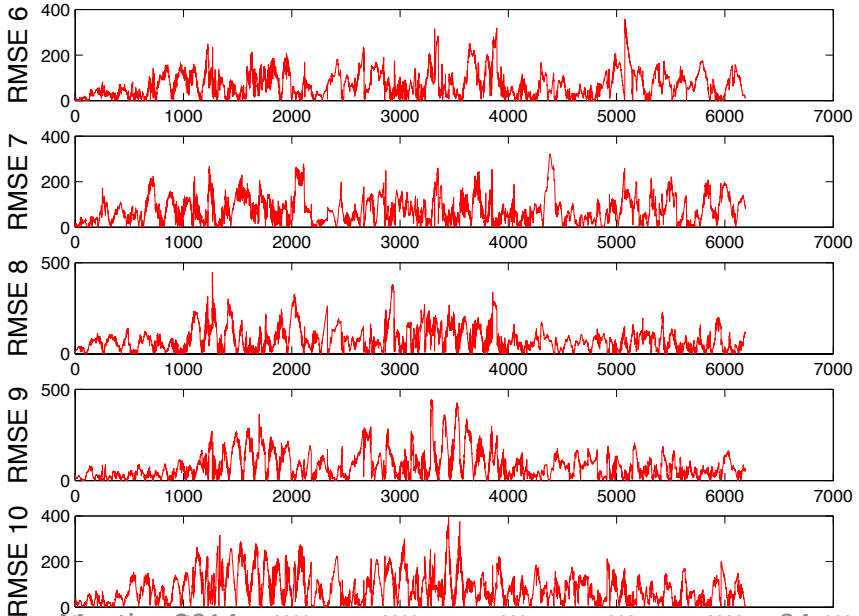




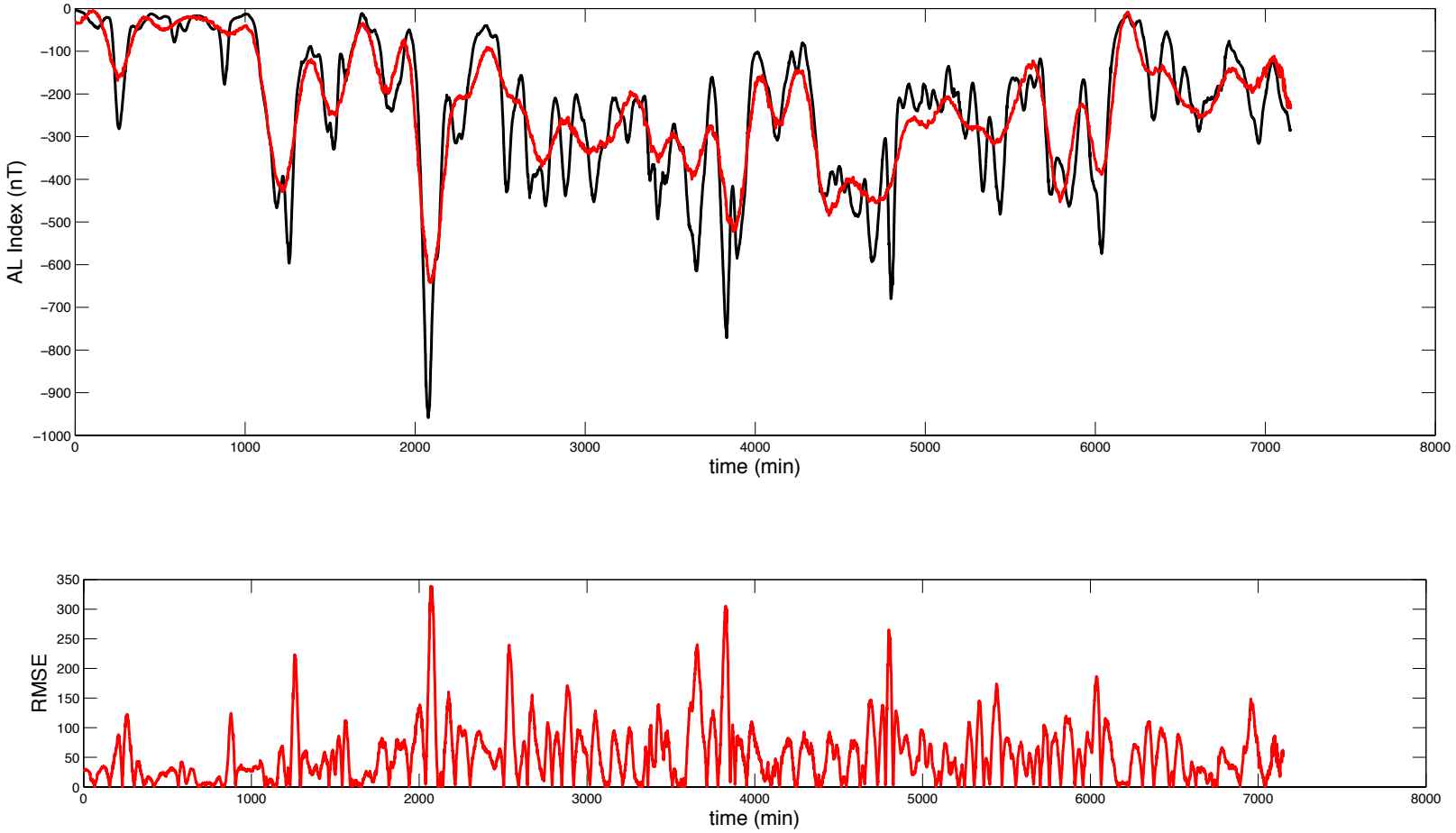

PC1
PC2
PC3
PC4
PC5


PC6
PC7
PC8
PC9
PC10

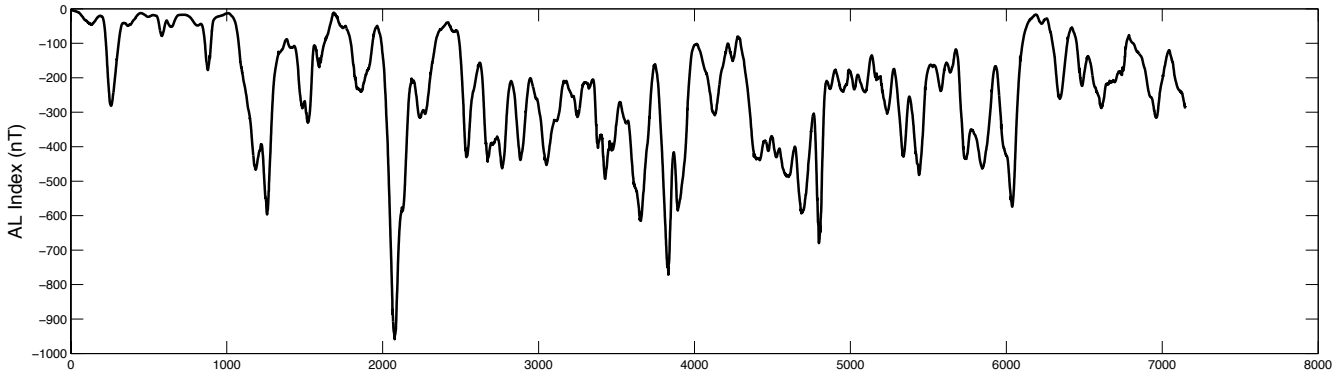

RMSE 1
RMSE 2
RMSE 3
RMSE 4
RMSE 5

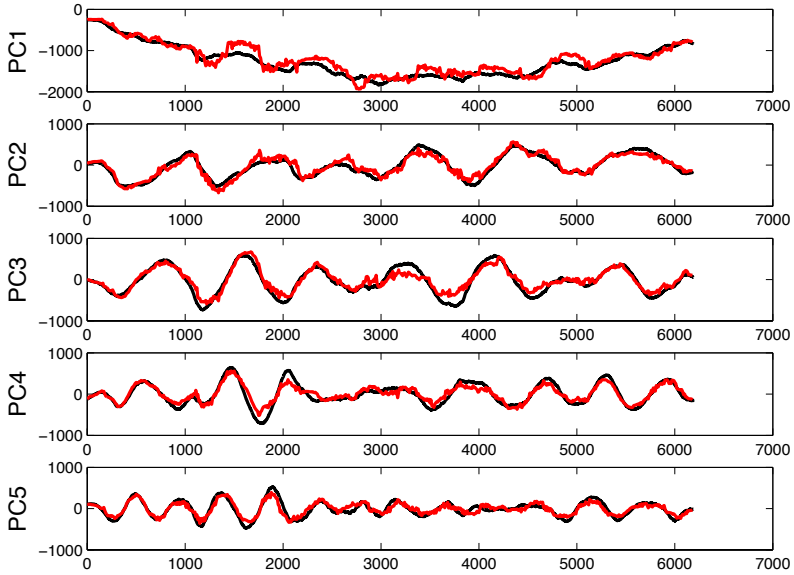

RMSE 6
RMSE 7
RMSE 8
RMSE 9
RMSE 10

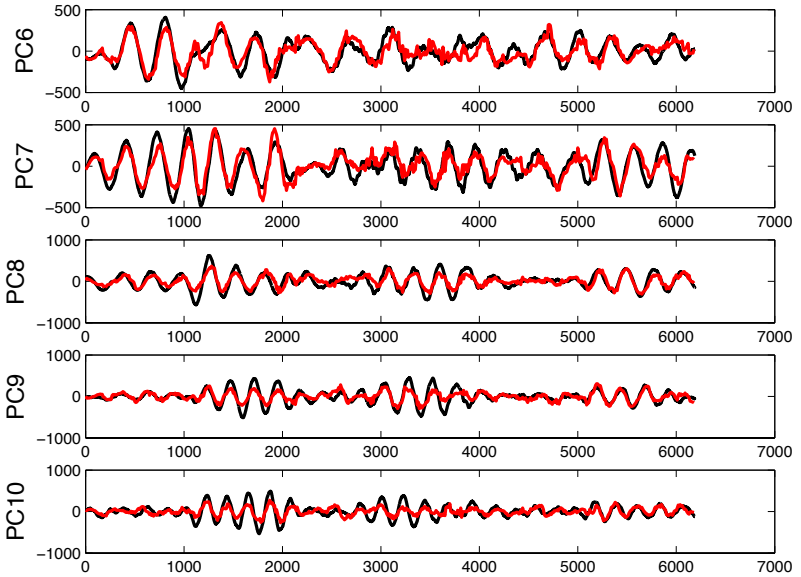
1 Minute Forecast of AL Index



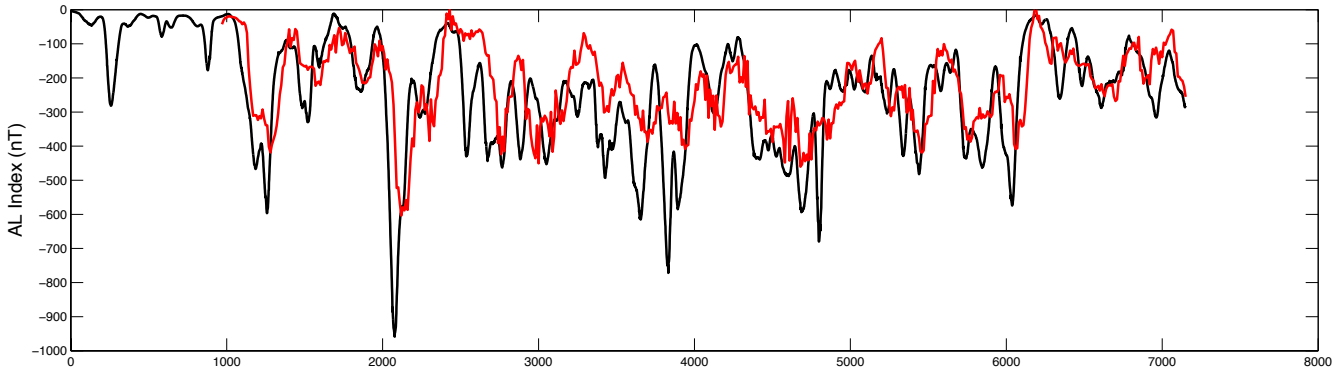
10 Minute Forecasts of the Principal Components

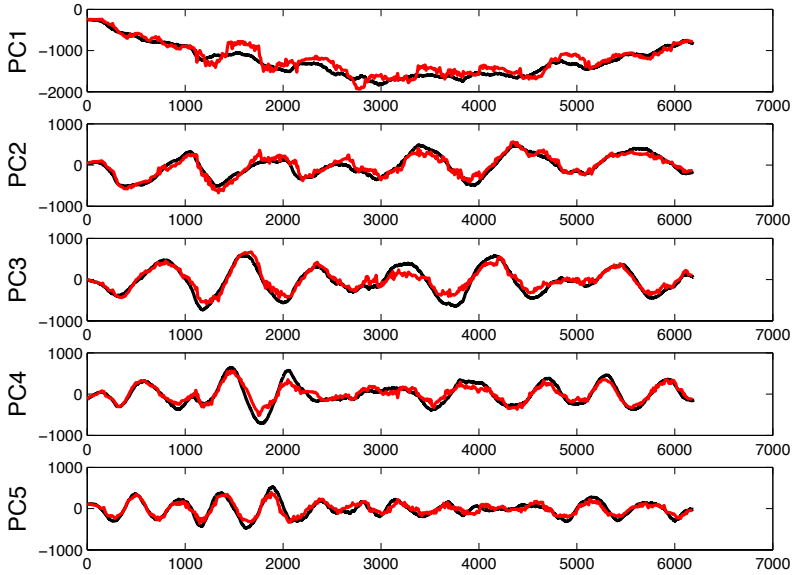


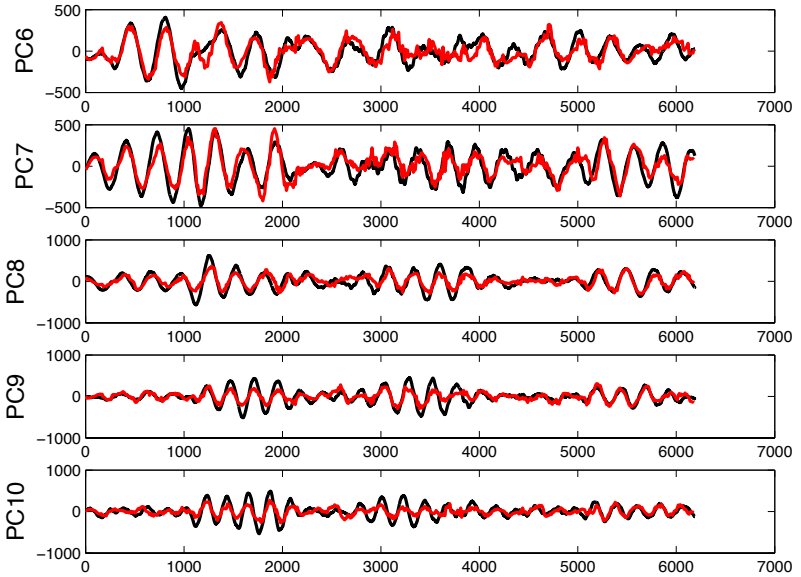




10 Minute Forecasts of the AL Index







Summary

- Dynamics of low dimensional dynamical systems can be reconstructed from a time series of a single variable
- For systems where there is an attractor, this technique allows for the estimate of the future state of the system in reconstructed phase space without use of a dynamical model
- Application of this technique to the time series of the AL Index results in forecasts that capture many features of the HSS event

Future Work

- Implement 4D-ETKF to take advantage of all observation for longer assimilation windows
- Continue to explore the distribution of points in model data by increasing number of model data points
- Improve search algorithms for nearest neighbors